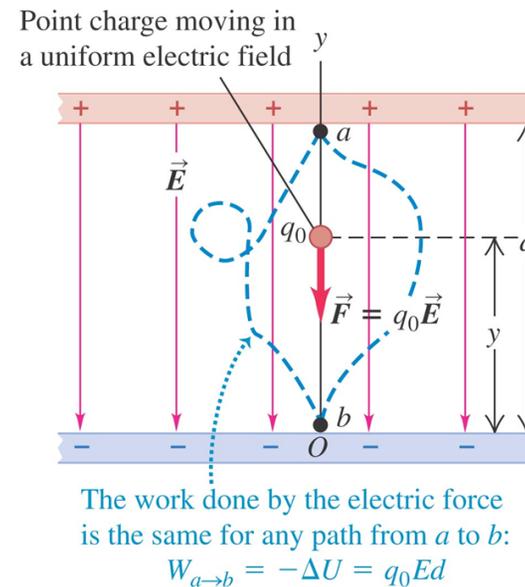
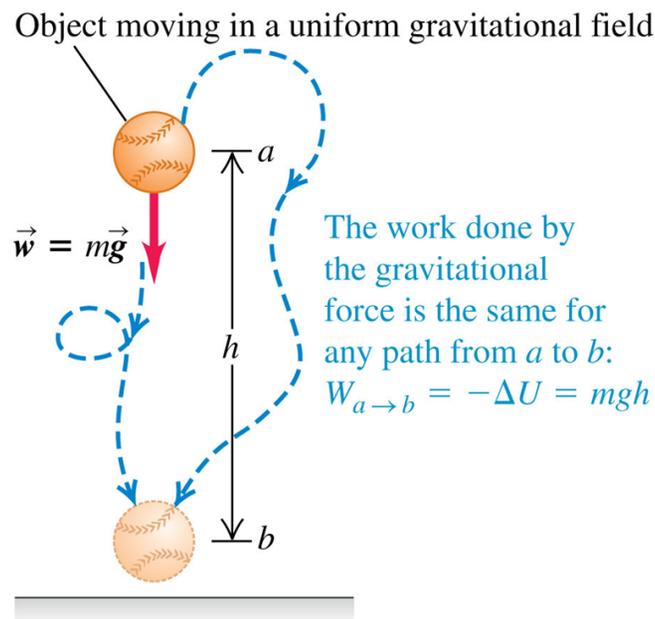


# PHYS 122-Lecture 5: Electric Potential Energy

- Gravitational Potential Energy vs. Gravitational Potential
- Electric Potential Energy vs. Electric Potential

# Electric potential energy in a uniform field

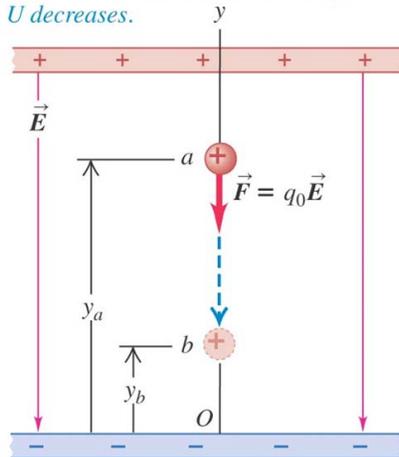
- The behavior of a point charge in a uniform electric field is analogous to the motion of a baseball in a uniform gravitational field.
- Figures 23.1 and 23.2 below illustrate this point.



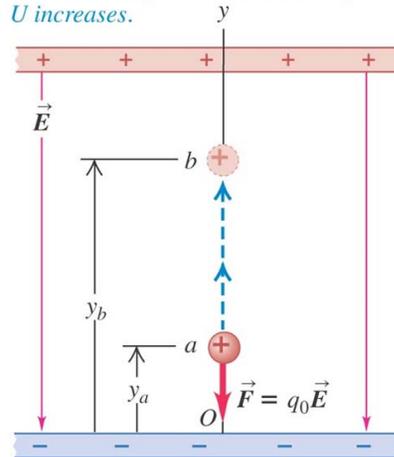
## A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the potential energy *decreases*, but if the charge moves opposite the field, the potential energy *increases*.
- Figure 23.3 below illustrates this point.

(a) Positive charge moves in the direction of  $\vec{E}$ :  
• Field does *positive* work on charge.  
•  $U$  *decreases*.



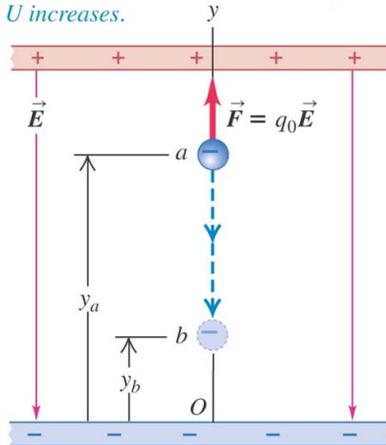
(b) Positive charge moves opposite  $\vec{E}$ :  
• Field does *negative* work on charge.  
•  $U$  *increases*.



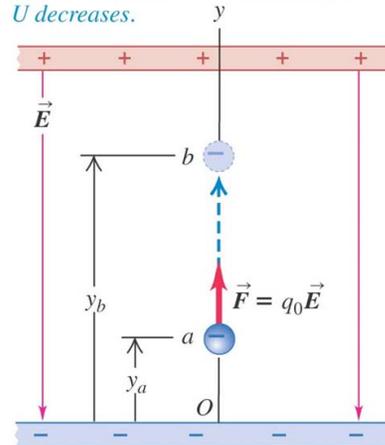
## A negative charge moving in a uniform field

- If the negative charge moves in the direction of the field, the potential energy *increases*, but if the charge moves opposite the field, the potential energy *decreases*.
- Figure 23.4 below illustrates this point.

(a) Negative charge moves in the direction of  $\vec{E}$ :  
• Field does *negative* work on charge.  
•  $U$  increases.

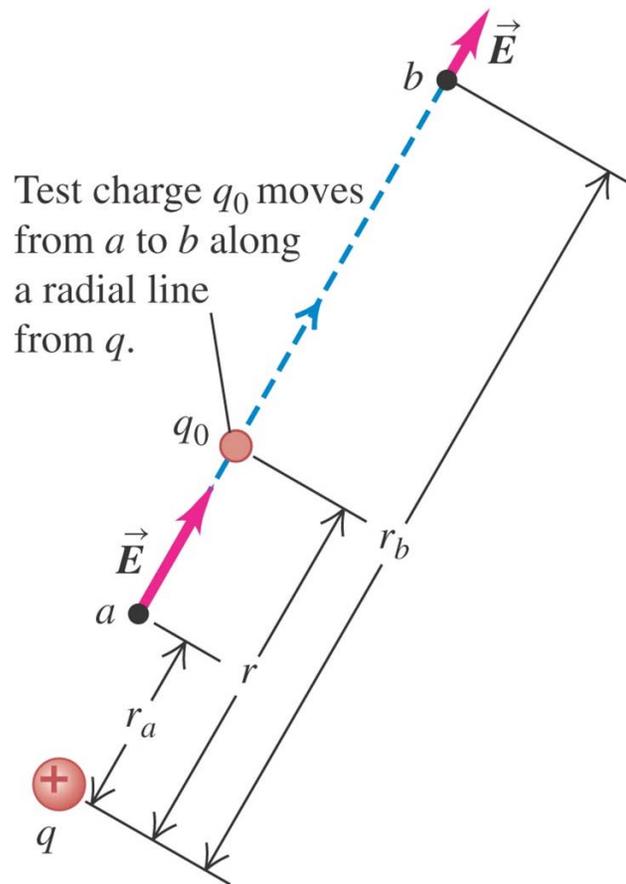


(b) Negative charge moves opposite  $\vec{E}$ :  
• Field does *positive* work on charge.  
•  $U$  decreases.

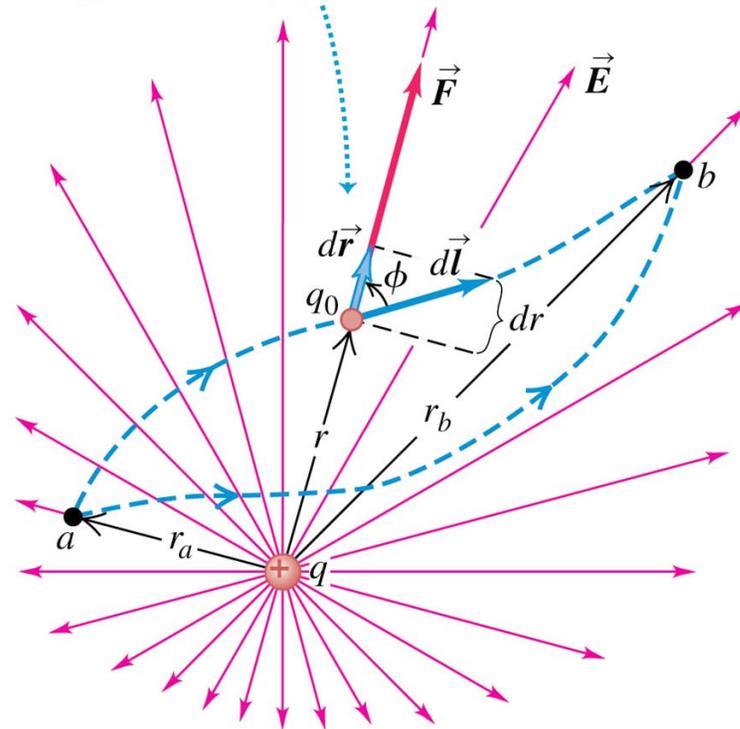


# Electric potential energy of two point charges

- Follow the discussion of the motion of a test charge  $q_0$  in the text.
- The electric potential is the same whether  $q_0$  moves in a radial line (left figure) or along an arbitrary path (right figure).



Test charge  $q_0$  moves from  $a$  to  $b$  along an arbitrary path.



## Electric potential

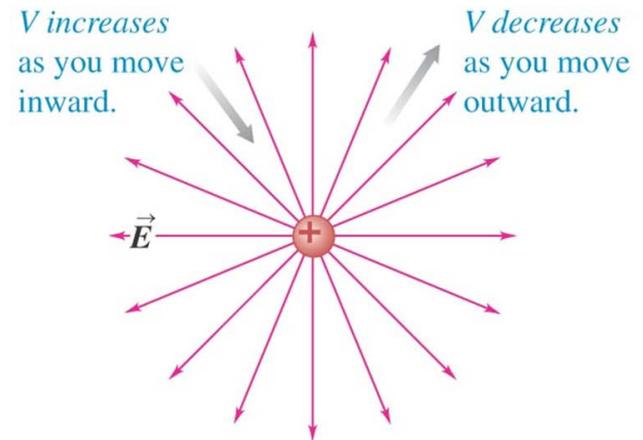
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- *Potential is potential energy per unit charge.*
- We can think of the potential difference between points  $a$  and  $b$  in either of two ways. The potential of  $a$  with respect to  $b$  ( $V_{ab} = V_a - V_b$ ) equals:
  - ✓ the work done by the electric force when a *unit* charge moves from  $a$  to  $b$ .
  - ✓ the work that must be done to move a *unit* charge slowly from  $b$  to  $a$  against the electric force.
- Follow the discussion in the text of how to calculate electric potential.

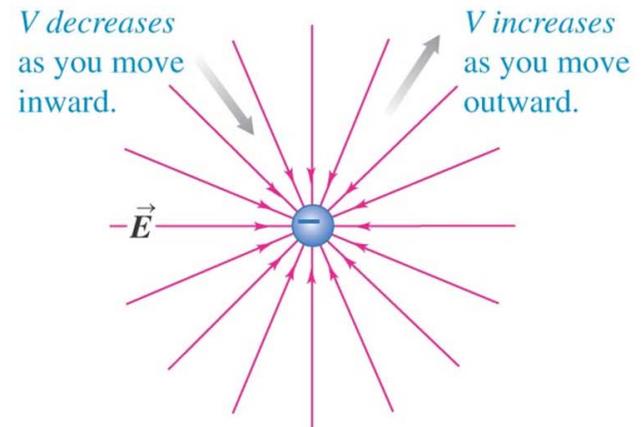
## Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*. (See Figure 23.12 at the right.)
- Follow the discussion in the text.
- Follow Example 23.3.

(a) A positive point charge



(b) A negative point charge



$$\text{if } F = \frac{1}{4\pi\epsilon_0} \frac{|q||q_0|}{r^2}$$

$$W = \int_a^b \vec{F} \cdot d\vec{\ell} = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{|q||q_0|}{r^2} \underbrace{\cos\phi \, d\ell}_{dr} = \frac{|q||q_0|}{4\pi\epsilon_0} \left( \frac{1}{r_a} - \frac{1}{r_b} \right)$$

if  $r_b = \infty$

$$\therefore U \equiv \frac{1}{4\pi\epsilon_0} \frac{|q||q_0|}{r} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r}$$

$$U_T = \sum U_i$$

$$\text{if } r_b = \infty$$

$$\frac{U}{|q_0|} = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r} \equiv V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Note: No Vectors!

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V = \sum V_i = \sum \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

Because  $W = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b \epsilon_0 \vec{E} \cdot d\vec{l} =$

$$\frac{W}{q_0} = \boxed{V = \int_a^b E \cos \phi \, dl}$$

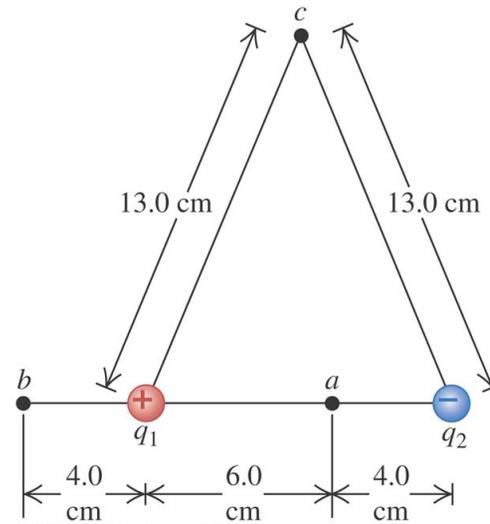
Have  $E \rightarrow$  get  $V$   
this is backwards to  
"real life"

$$\boxed{\vec{E} = -\nabla V}$$

## Potential due to two point charges

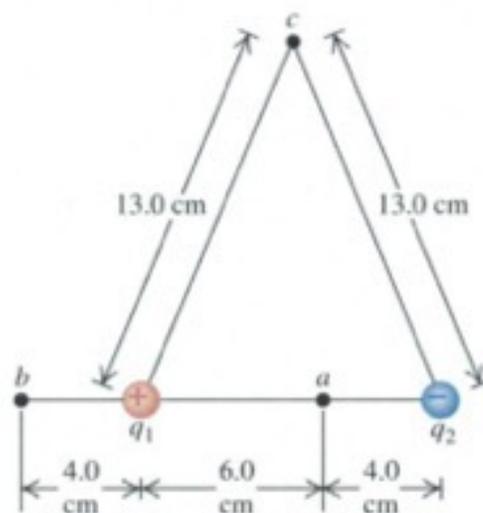
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- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



## Potential due to two point charges

- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



$$a) \quad V_a = V_1 + V_2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2}$$

$$= -900 \text{ [V]}$$

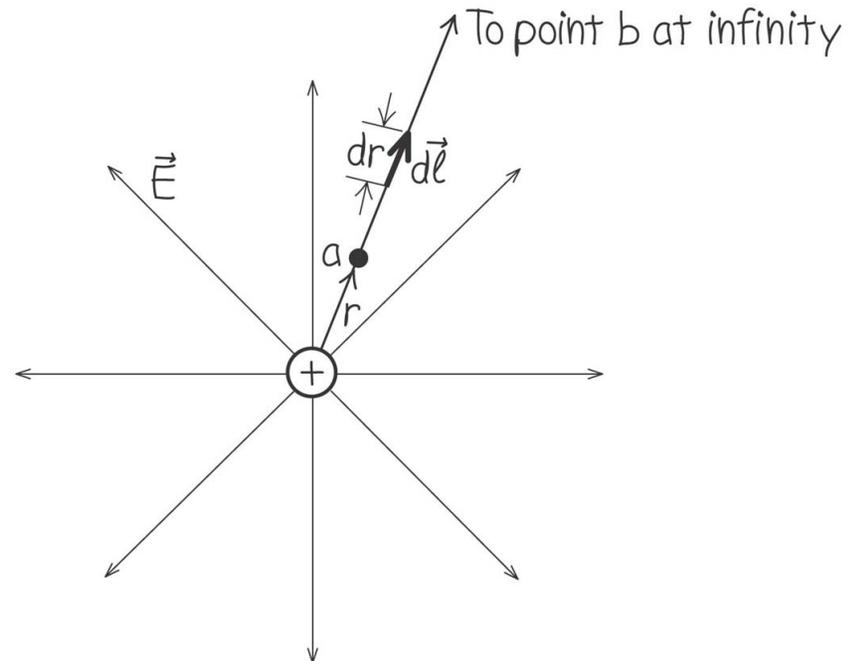
$$\left( U_a = q V_a = -3.6 \cdot 10^{-6} \text{ [J]} \right)$$

$$c) \quad V_c = 0$$

$\Rightarrow$  NO Vectors!

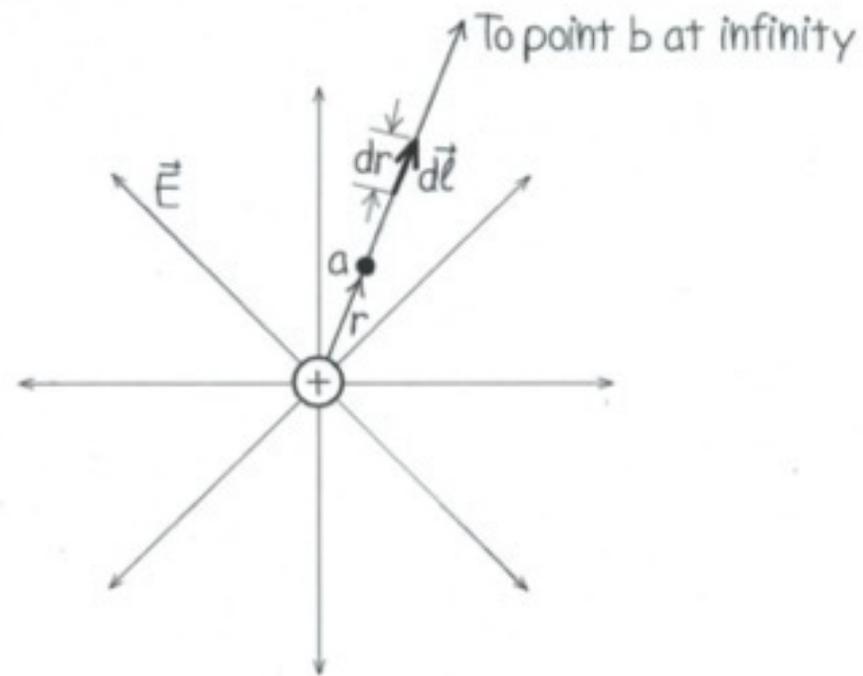
## Finding potential by integration

- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



## Finding potential by integration

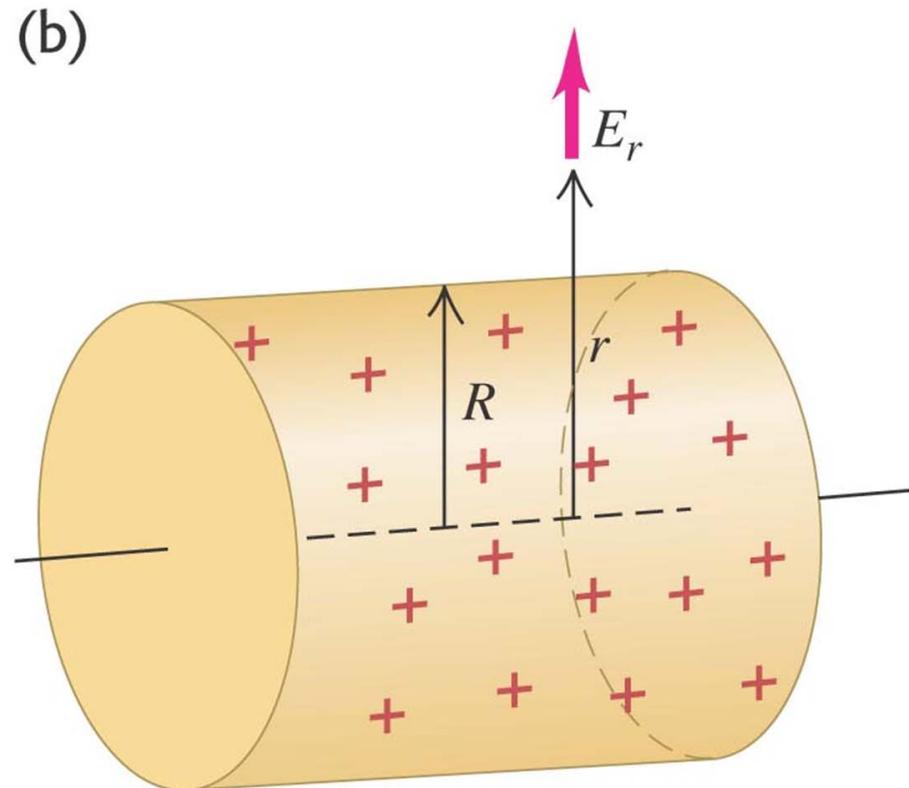
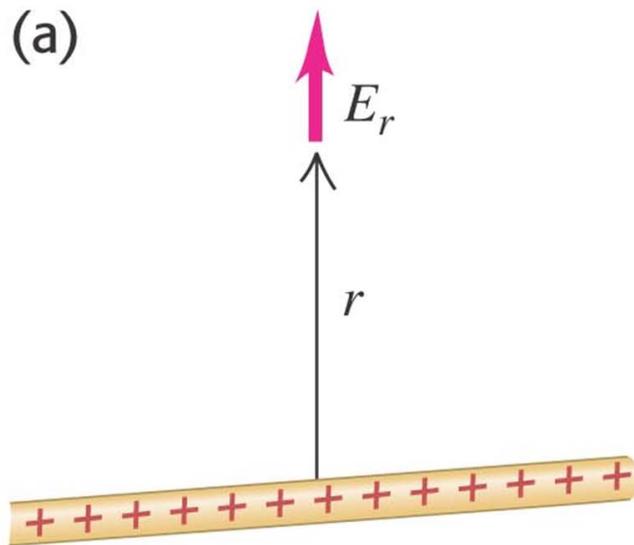
- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



$$\begin{aligned} V &= qV_a - 0 = \int_r^{b=\infty} \vec{E} \cdot d\vec{\ell} \\ &= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r} dr \\ &= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr \\ &= -\frac{q}{4\pi\epsilon_0 r} \Big|_r^{\infty} = 0 - \left( -\frac{q}{4\pi\epsilon_0 r} \right) = \frac{q}{4\pi\epsilon_0 r} \end{aligned}$$

## An infinite line charge or conducting cylinder

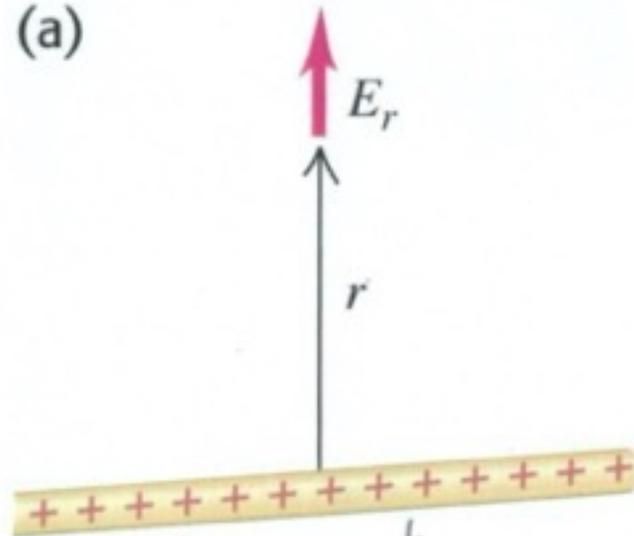
- Follow Example 23.10 using Figure 23.19 below.



## An infinite line charge or conducting cylinder

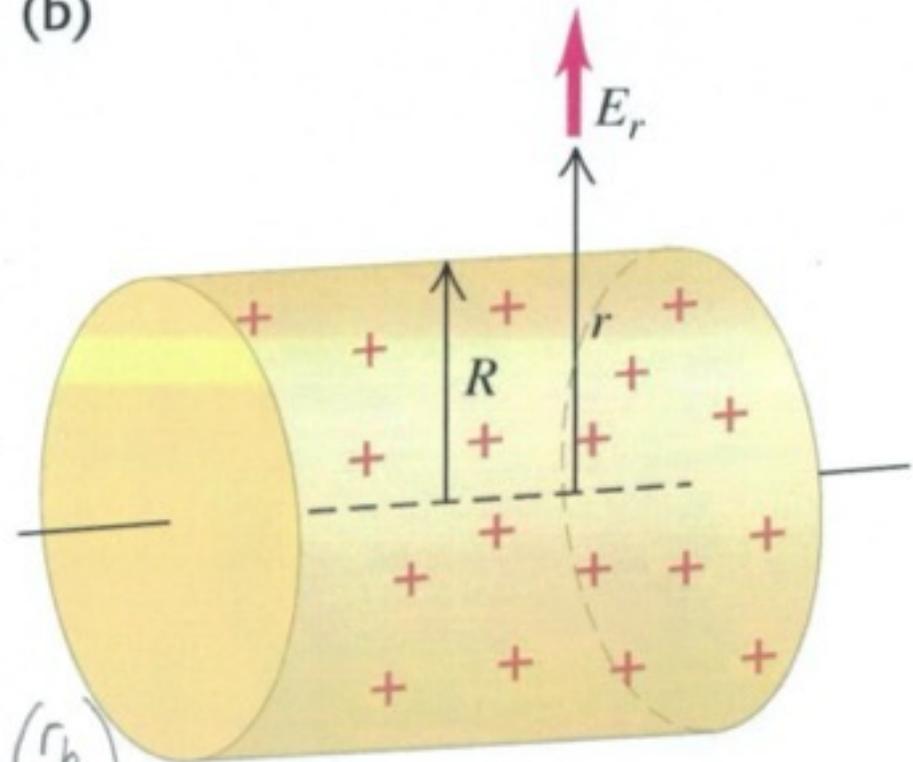
- Follow Example 23.10 using Figure 23.19 below.

(a)



$$\begin{aligned} V_a - V_b &= \int_a^b \vec{E} \cdot d\vec{l} = \int_a^b E_r dr \\ &= \int_a^b \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{r} dr = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right) \end{aligned}$$

(b)

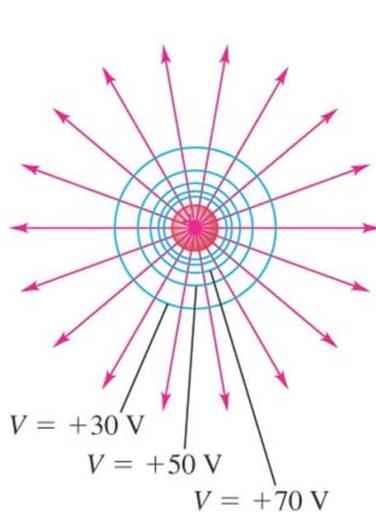




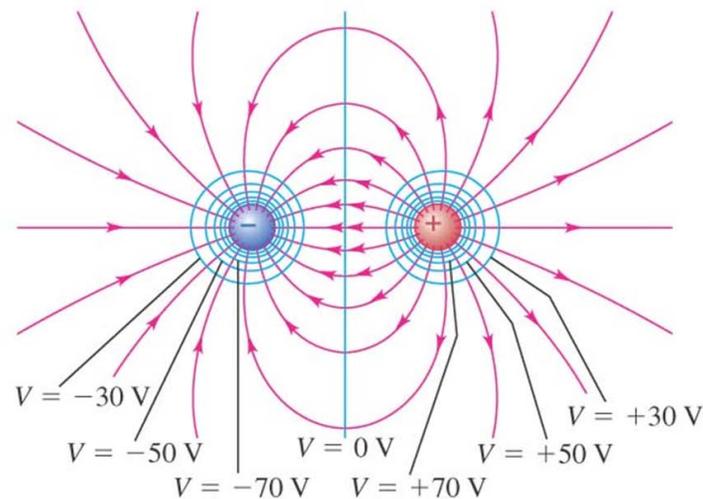
# Equipotential surfaces and field lines

- An *equipotential surface* is a surface on which the electric potential is the same at every point.
- Figure 23.23 below shows the equipotential surfaces and electric field lines for assemblies of point charges.
- Field lines and equipotential surfaces are always mutually perpendicular.

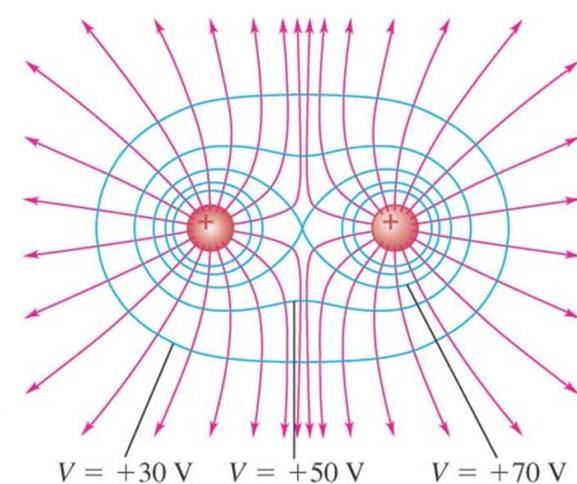
(a) A single positive charge



(b) An electric dipole



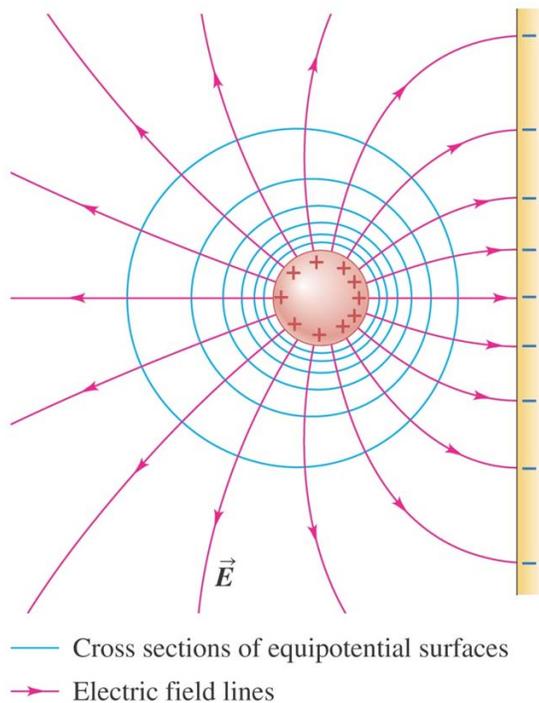
(c) Two equal positive charges



→ Electric field lines    — Cross sections of equipotential surfaces

# Equipotentials and conductors

- When all charges are at rest:
  - ✓ the surface of a conductor is always an equipotential surface.
  - ✓ the electric field just outside a conductor is always perpendicular to the surface (see figures below).
  - ✓ the entire solid volume of a conductor is at the same potential.



**An impossible electric field**  
If the electric field just outside a conductor had a tangential component  $E_{\parallel}$ , a charge could move in a loop with net work done.

