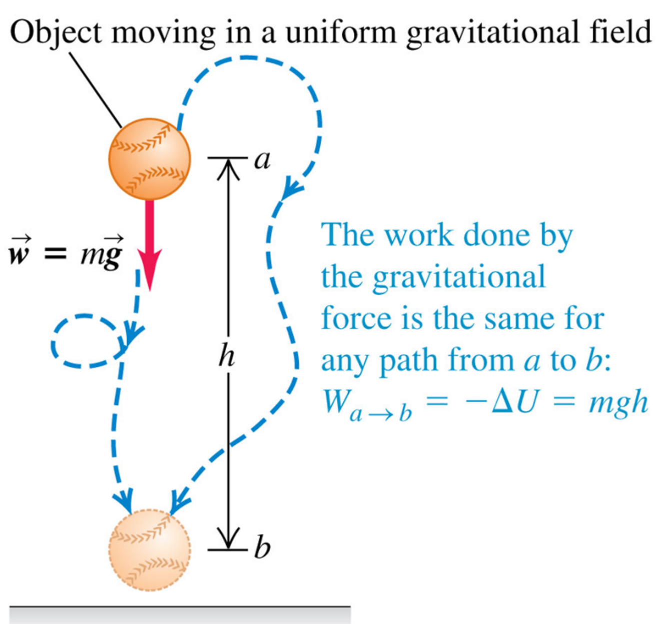
PHYS 122-Lecture 5:
Electric Potential Energy

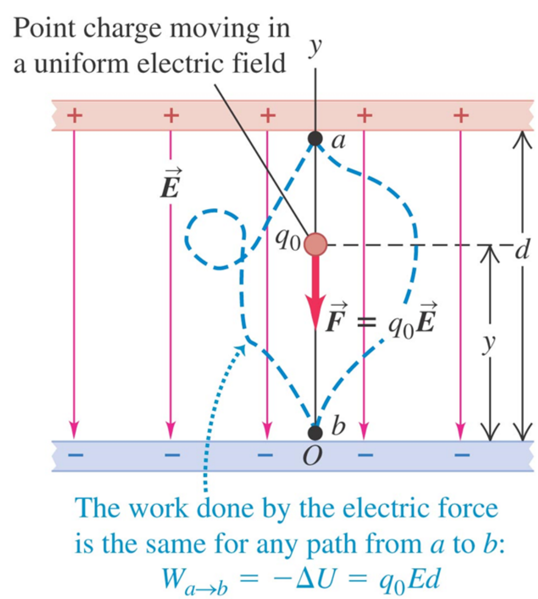
- Gravitational Potential Energy vs. Gravitational Potential

- Electric Potential Energy vs. Electric Potential

Electric potential energy in a uniform field

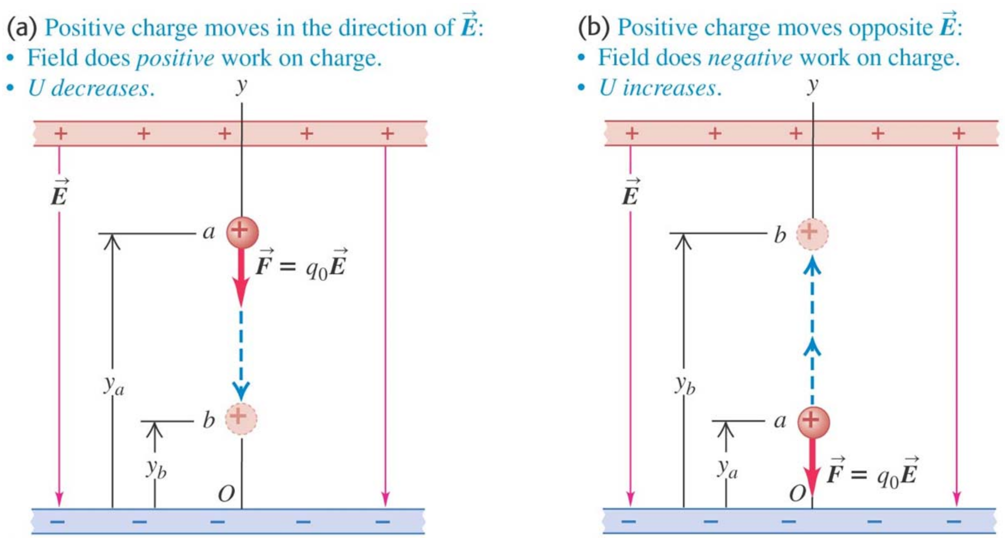
- The behavior of a point charge in a uniform electric field is analogous to the motion of a baseball in a uniform gravitational field.
- Figures 23.1 and 23.2 below illustrate this point.





A positive charge moving in a uniform field

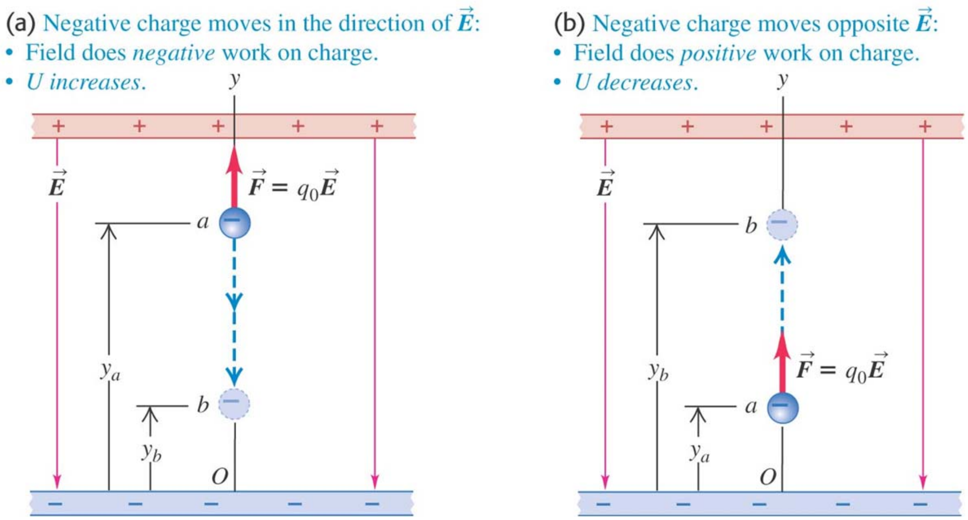
- If the positive charge moves in the direction of the field, the potential energy *decreases*, but if the charge moves opposite the field, the potential energy *increases*.
- Figure 23.3 below illustrates this point.



A negative charge moving in a uniform field

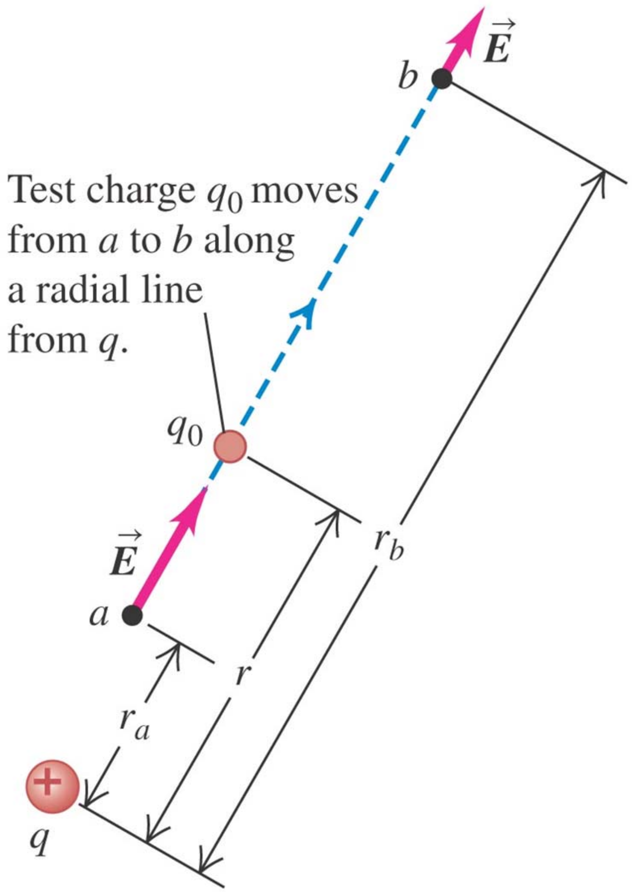
- If the negative charge moves in the direction of the field, the potential energy *increases*, but if the charge moves opposite the field, the potential energy *decreases*.

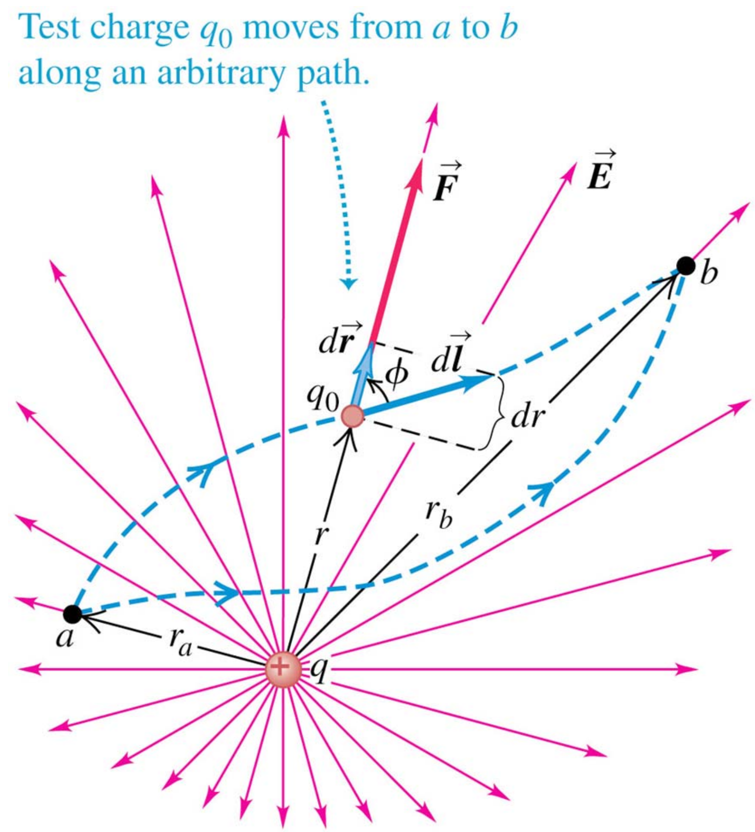
- Figure 23.4 below illustrates this point.



Electric potential energy of two point charges

- Follow the discussion of the motion of a test charge q_0 in the text.
- The electric potential is the same whether q_0 moves in a radial line (left figure) or along an arbitrary path (right figure).



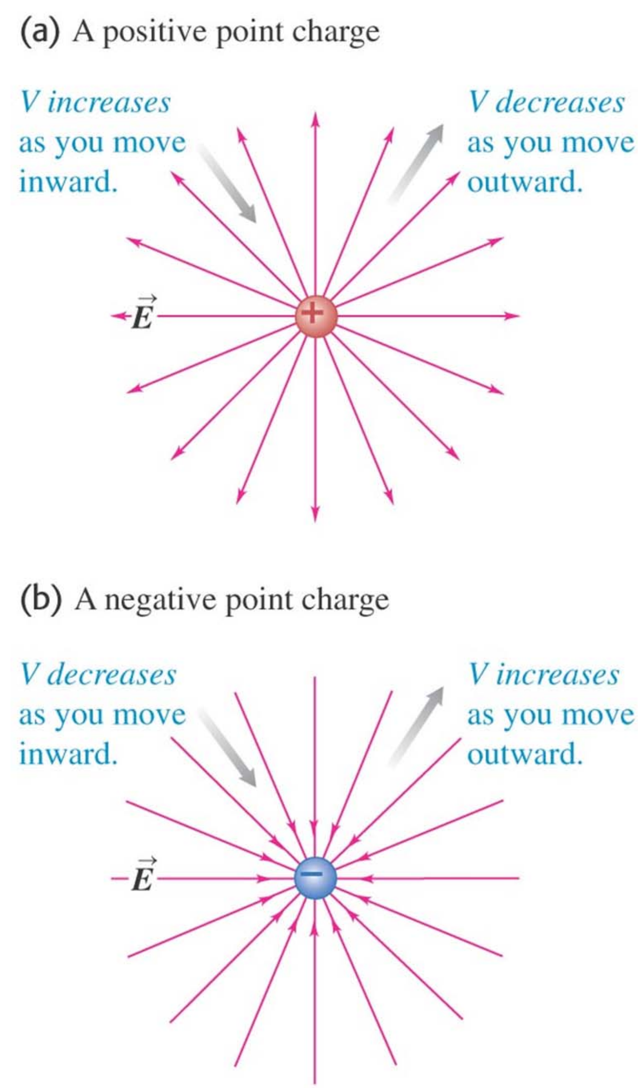


Electric potential

- *Potential* is *potential energy per unit charge.*
- We can think of the potential difference between points a and b in either of two ways. The potential of a with respect to b ($V_{ab} = V_a - V_b$) equals:
    - ✓ the work done by the electric force when a *unit* charge moves from a to b.
    - ✓ the work that must be done to move a *unit* charge slowly from b to a against the electric force.
- Follow the discussion in the text of how to calculate electric potential.

Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*. (See Figure 23.12 at the right.)
- Follow the discussion in the text.
- Follow Example 23.3.



if $F = \frac{1}{4\pi\epsilon_0}\frac{|q||q_0|}{r^2}$

$$W = \int_a^b \vec{F}\cdot\vec{dl} = \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0}\frac{|q||q_0|}{r^2}\underbrace{\cos\phi\, dl}_{dr} = \frac{|q||q_0|}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

if $r_b = \infty$

$$\therefore \quad U \equiv \frac{1}{4\pi\epsilon_0}\frac{|q||q_0|}{r} = \frac{1}{4\pi\epsilon_0}\frac{q\,q_0}{r}$$

$$U_T = \sum U_i$$

if $r_b = \infty$

$$\frac{U}{|q_0|} = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r} \equiv V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Note: No Vectors!

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$V = \sum V_i = \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{r_i} = \int \frac{1}{4\pi\epsilon_0} \frac{dq}{r}$$

Because $W = \int_a^b \vec{F} \cdot \vec{dl} = \int_a^b q_0 \vec{E} \cdot \vec{dl} =$

$$\frac{W}{q_0} = \boxed{V = \int_a^b E \cos\phi \, dl}$$

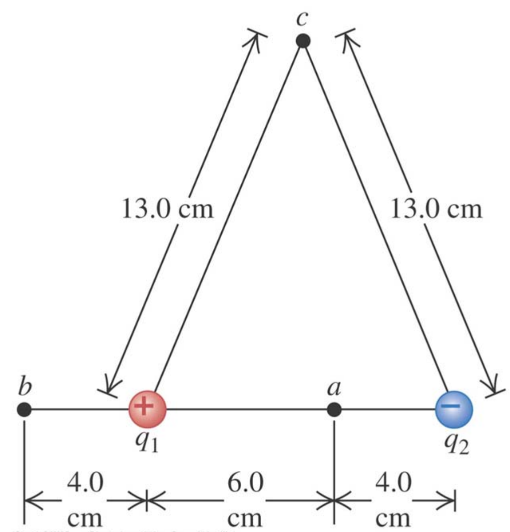
Have $E \longrightarrow$ get V

this is backwards to "real life"

$$\boxed{\vec{E} = -\nabla V}$$

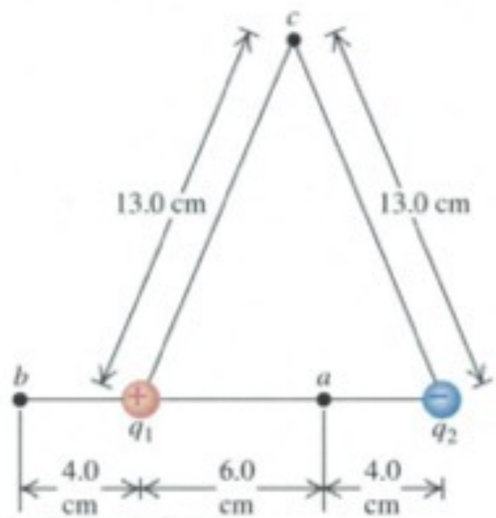
Potential due to two point charges

- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



Potential due to two point charges

- Follow Example 23.4 using Figure 23.13 at the right.
- Follow Example 23.5.



a) $V_a = V_1 + V_2$

$= \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$

$= -900\,[V]$

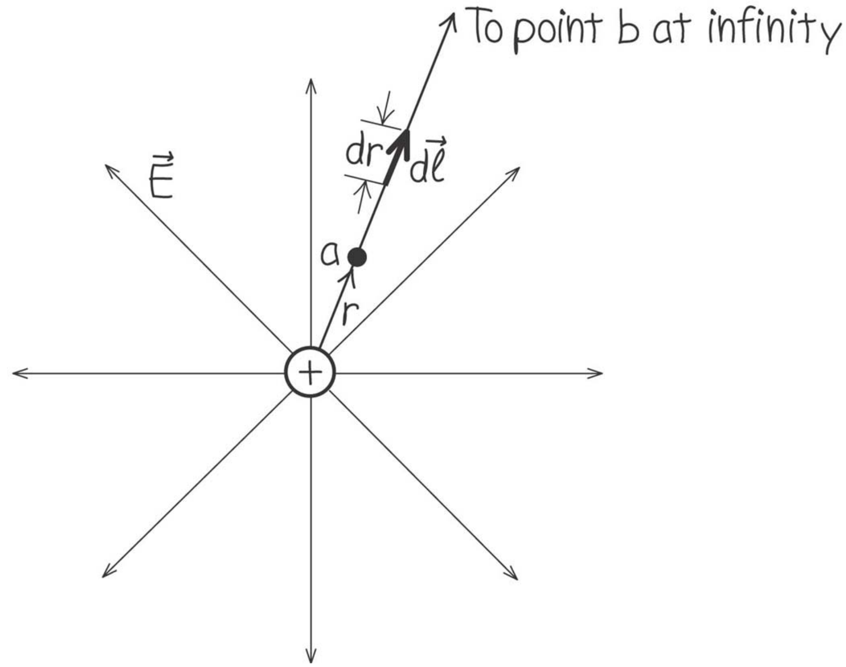
$\left(U_a = q V_a = -3.6\cdot10^{-6}\,[J] \right)$

c) $V_c = 0$

$\Longrightarrow$ NO Vectors!

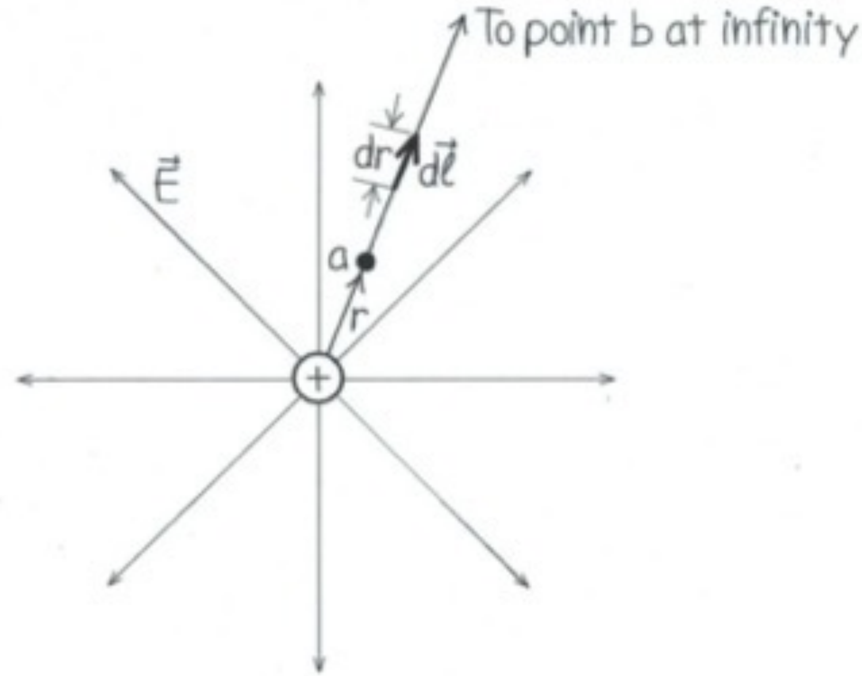
Finding potential by integration

- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



Finding potential by integration

- Example 23.6 shows how to find the potential by integration. Follow this example using Figure 23.14 at the right.



$$V = V_a - 0 = \int_r^{b=\infty} \vec{E} \cdot d\vec{l}$$

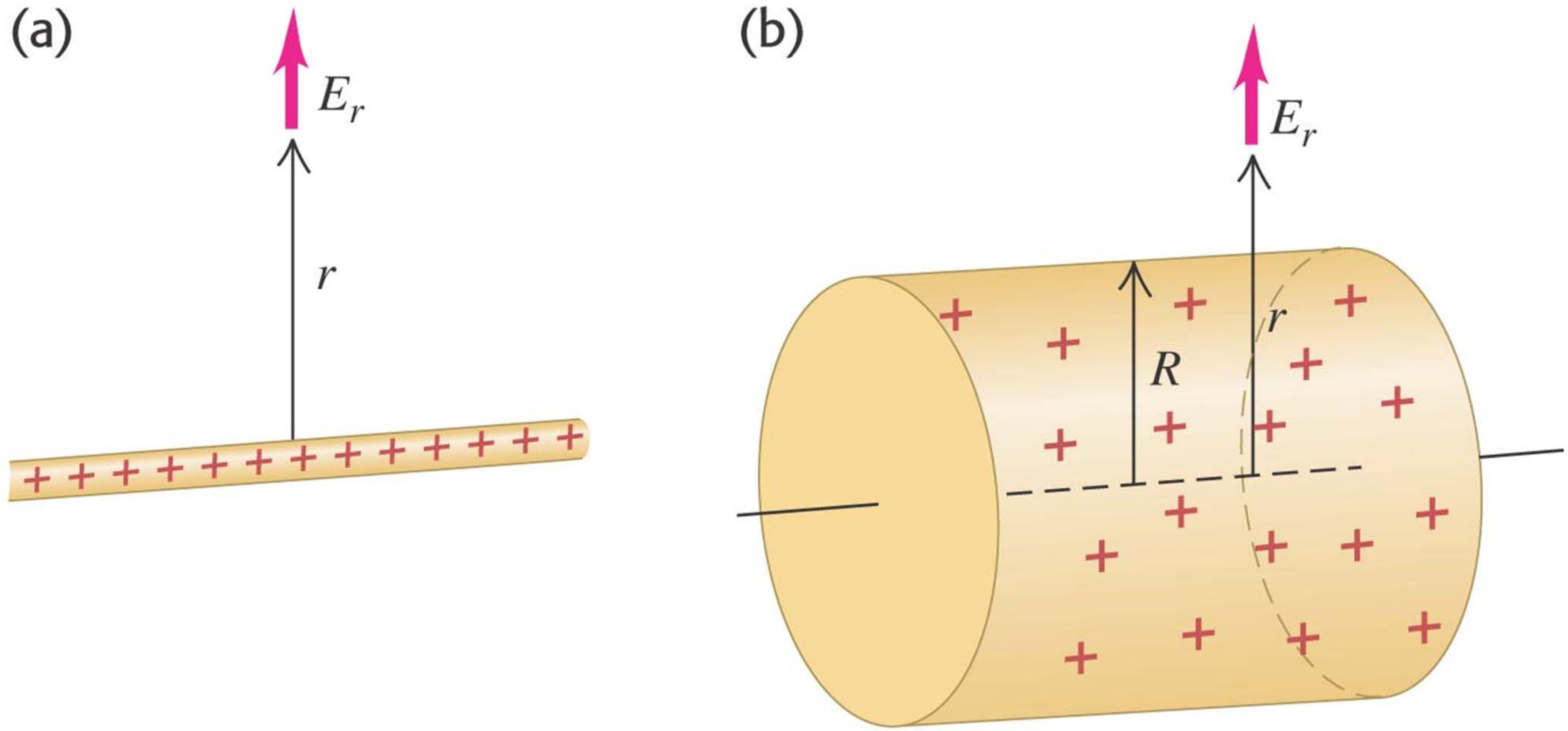
$$= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r} \, dr$$

$$= \int_r^{\infty} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \, dr$$

$$= -\frac{q}{4\pi\epsilon_0 r} \Big|_r^{\infty} = 0 - \left(-\frac{q}{4\pi\epsilon_0 r}\right) = \frac{q}{4\pi\epsilon_0 r}$$

An infinite line charge or conducting cylinder

- Follow Example 23.10 using Figure 23.19 below.



An infinite line charge or conducting cylinder

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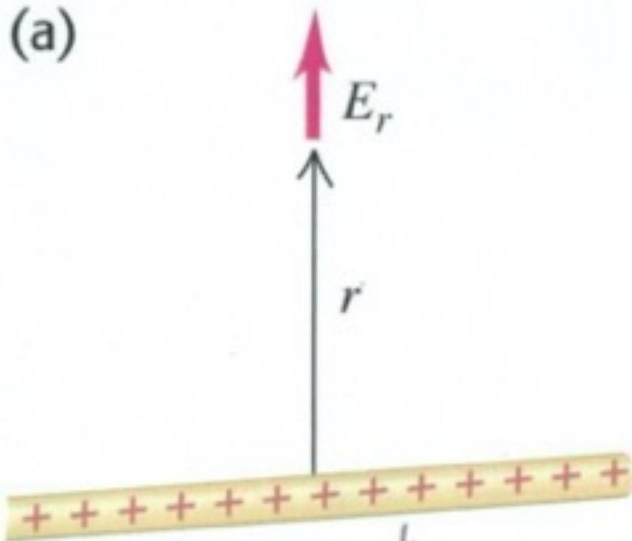
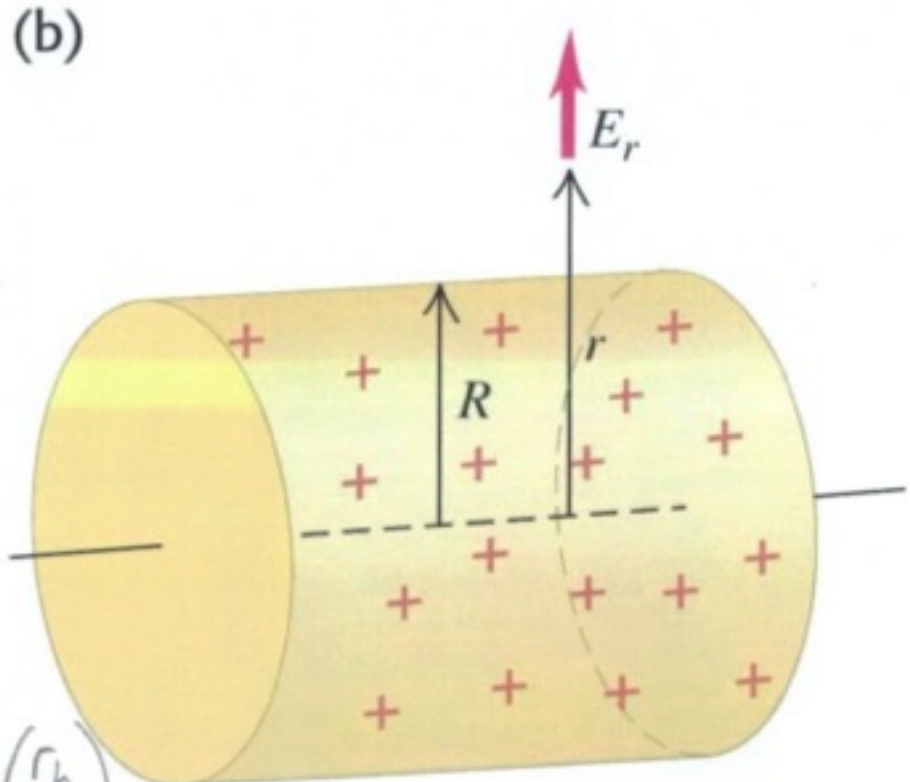


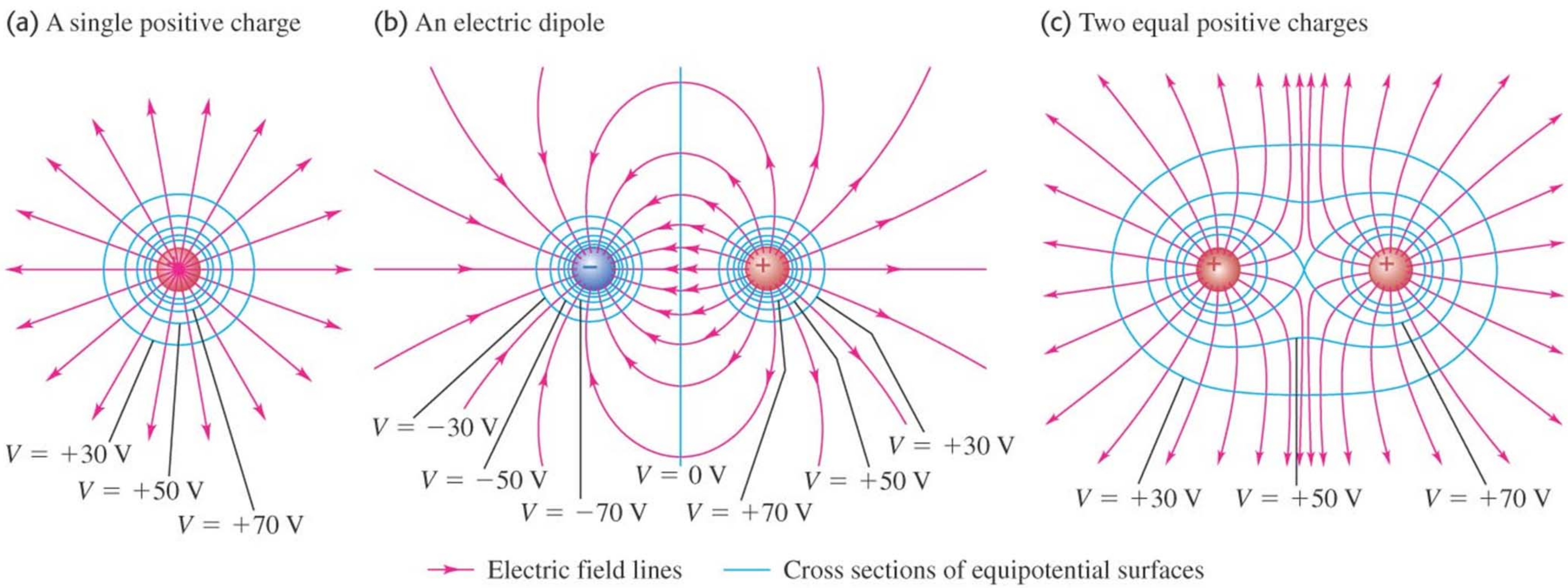


$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E_r\, dr$$

$$= \int_a^b \frac{\lambda}{2\pi\epsilon_0}\cdot\frac{1}{r}\, dr = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_b}{r_a}\right)$$

Equipotential surfaces and field lines

- An *equipotential surface* is a surface on which the electric potential is the same at every point.
- Figure 23.23 below shows the equipotential surfaces and electric field lines for assemblies of point charges.
- Field lines and equipotential surfaces are always mutually perpendicular.



Equipotentials and conductors

- When all charges are at rest:
  - ✓ the surface of a conductor is always an equipotential surface.
  - ✓ the electric field just outside a conductor is always perpendicular to the surface (see figures below).
  - ✓ the entire solid volume of a conductor is at the same potential.