PHYS 122-Lecture 6:
Capacitance and Dielectrics

- Capacitance [First half of Chap 24]
- Dielectrics [Second half of Chap 24]

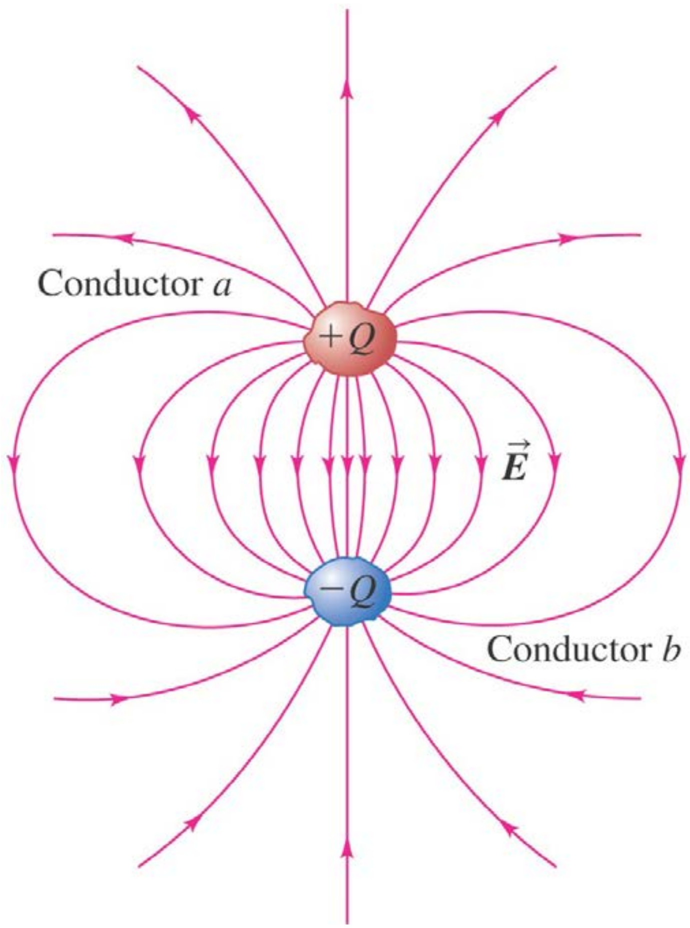
Introduction

- How does a camera's flash unit store energy?
- Capacitors are devices that store electric potential energy.
- The energy of a capacitor is actually stored in the electric field.

Capacitors and capacitance

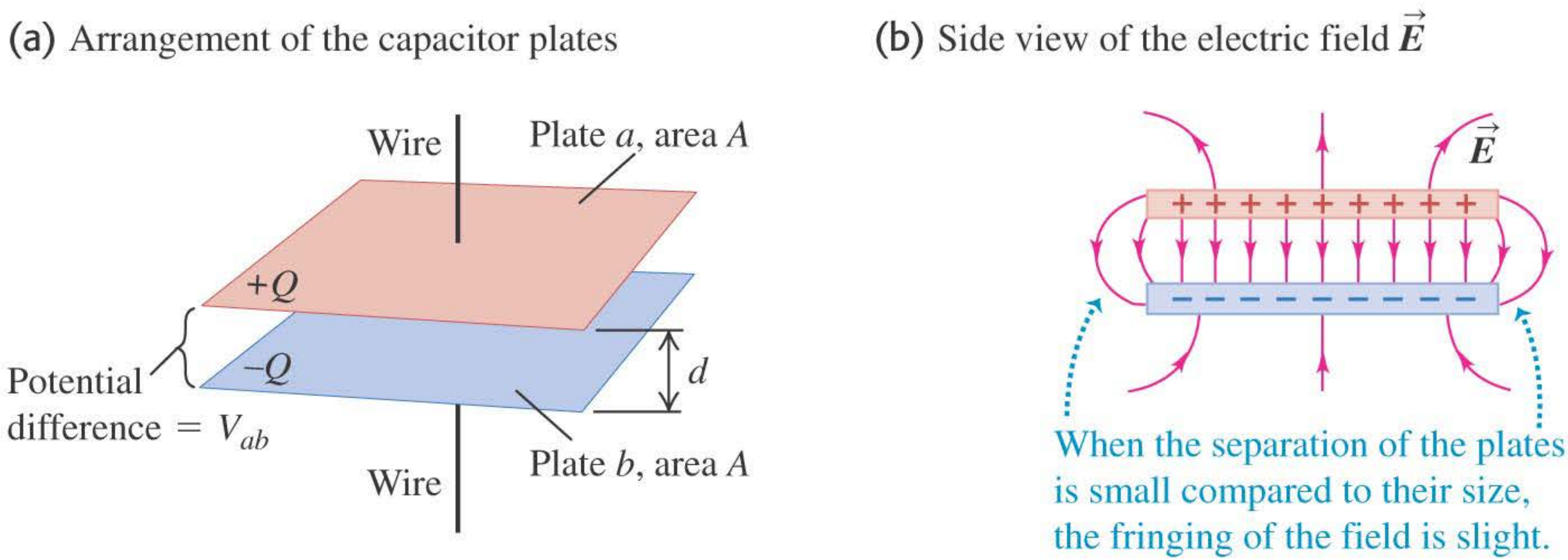
- Any two conductors separated by an insulator form a *capacitor*, as illustrated in Figure 24.1 below.
- The definition of capacitance is $C = Q/V_{ab}$.

[C/V] = Farad [F]



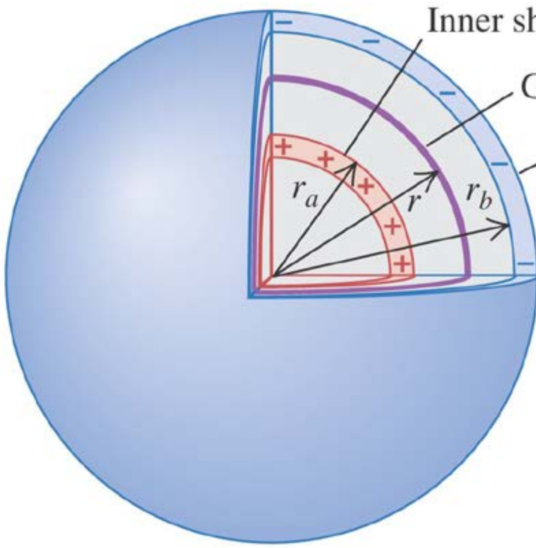
Parallel-plate capacitor

- A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions. (See Figure 24.2 below.)
- The capacitance of a parallel-plate capacitor is $C = \varepsilon_0 A/d$.
- Follow Examples 24.1 and 24.2.



A spherical capacitor

- Follow Example 24.3 using Figure 24.5 to consider a spherical capacitor.



A spherical capacitor

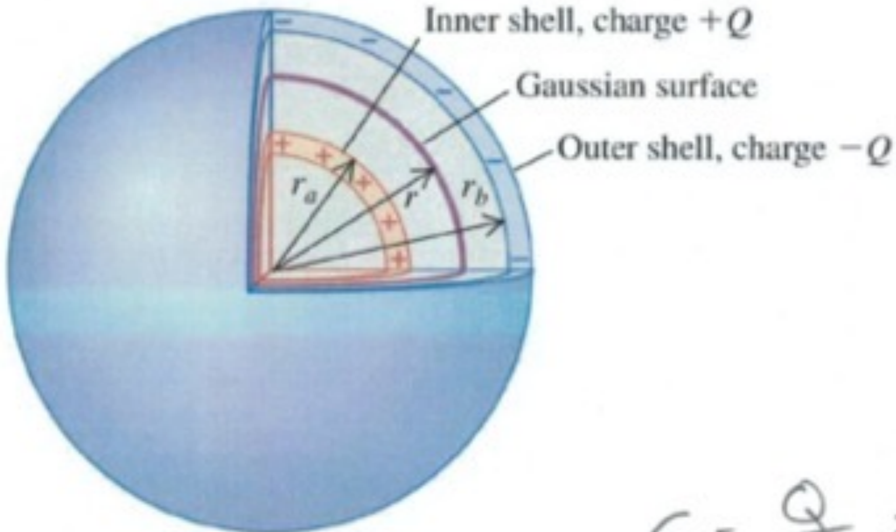
- Follow Example 24.3 using Figure 24.5 to consider a spherical capacitor.

$$V_a = \frac{Q}{4\pi\epsilon_0 r_a}$$

$$V_b = \frac{Q}{4\pi\epsilon_0 r_b}$$

$$V_{ab} = V_a - V_b$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

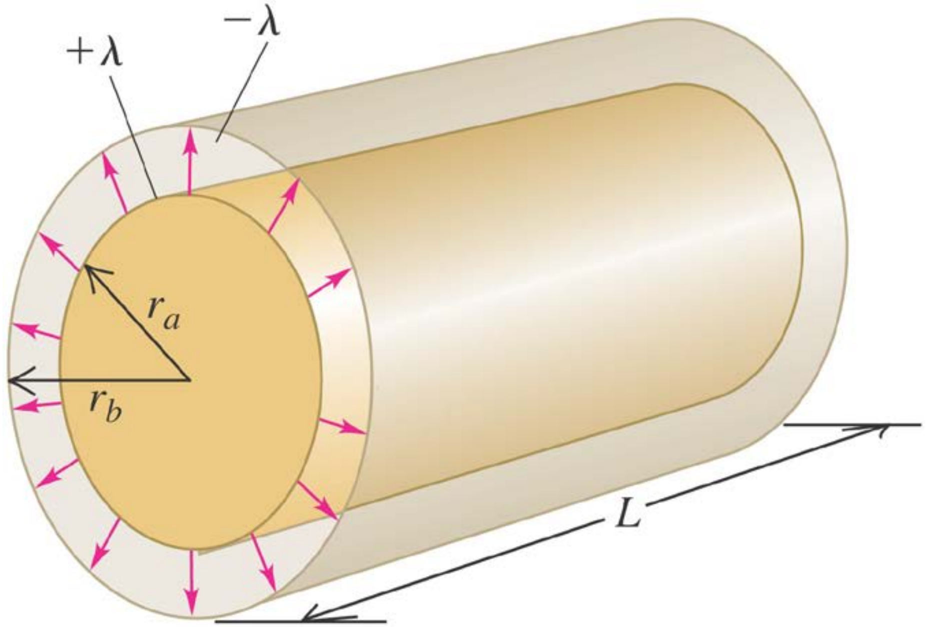


$$C = \frac{Q}{V_{ab}} = \frac{1}{\frac{1}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

$$= 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

A cylindrical capacitor

- Follow Example 24.4 and Figure 24.6 to investigate a cylindrical capacitor.

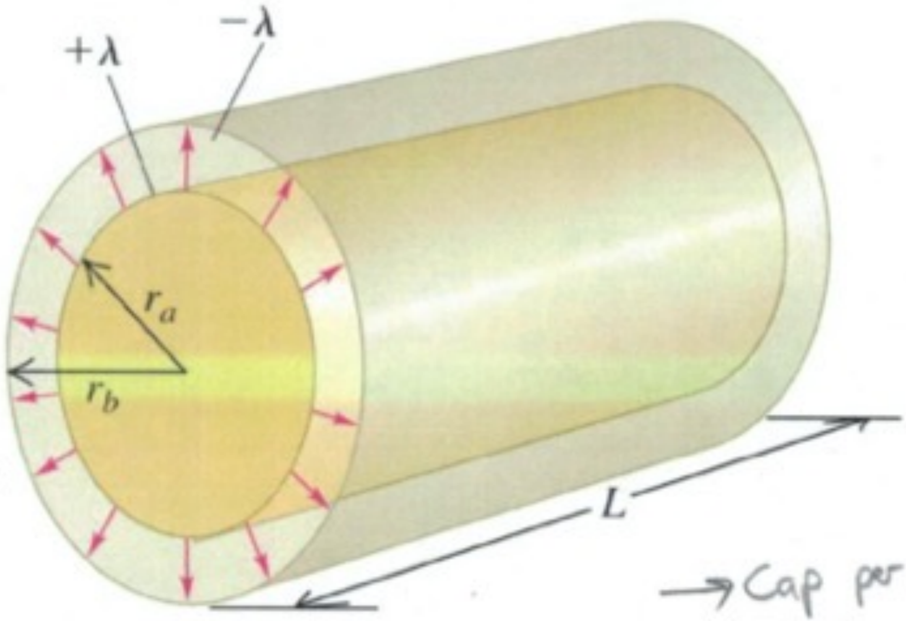


A cylindrical capacitor

- Follow Example 24.4 and Figure 24.6 to investigate a cylindrical capacitor.

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_a}{r}\right)$$

$$V_{ab} = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)$$



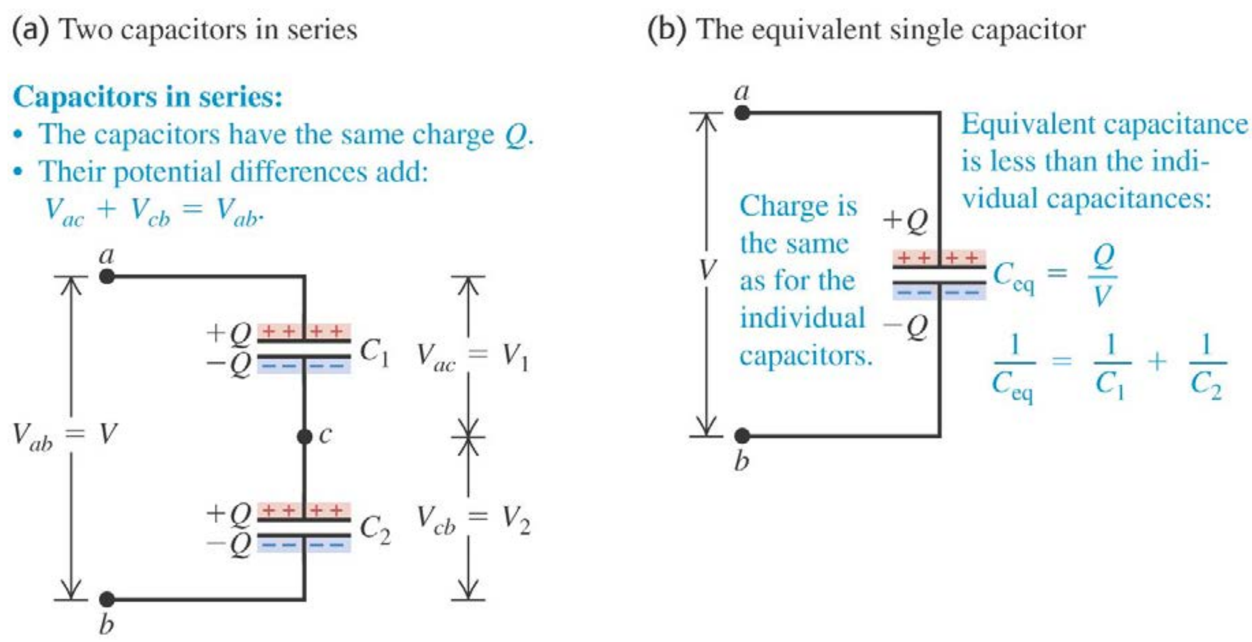
→ Cap per unit length

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{r_b}{r_a}\right)}$$

$$\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}$$

Capacitors in series

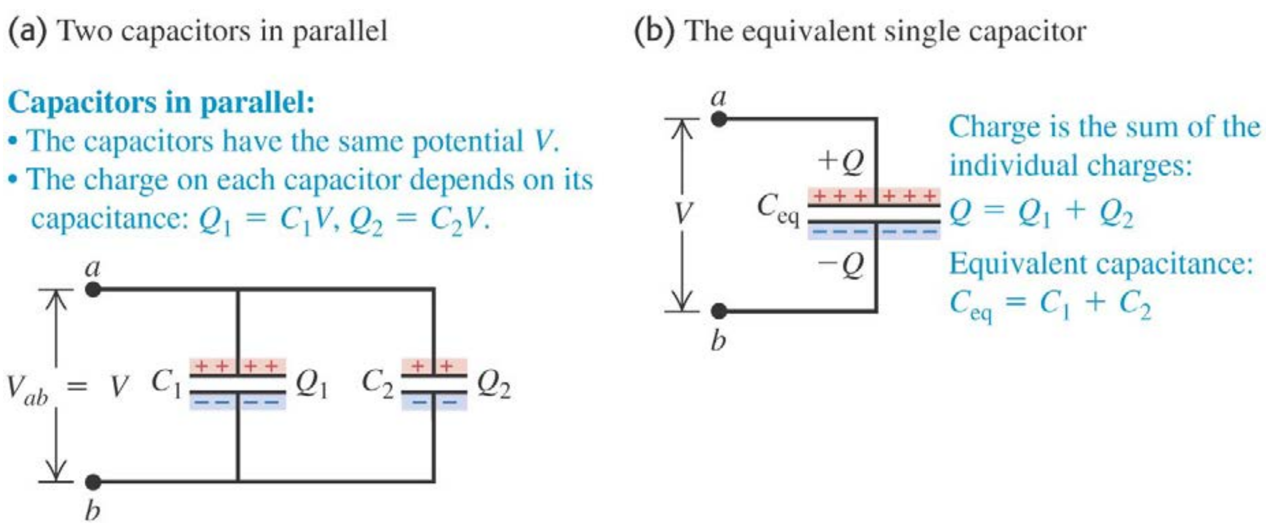
- Capacitors are in *series* if they are connected one after the other, as illustrated in Figure 24.8 below.
- The *equivalent capacitance* of a series combination is given by $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$



Capacitors in parallel

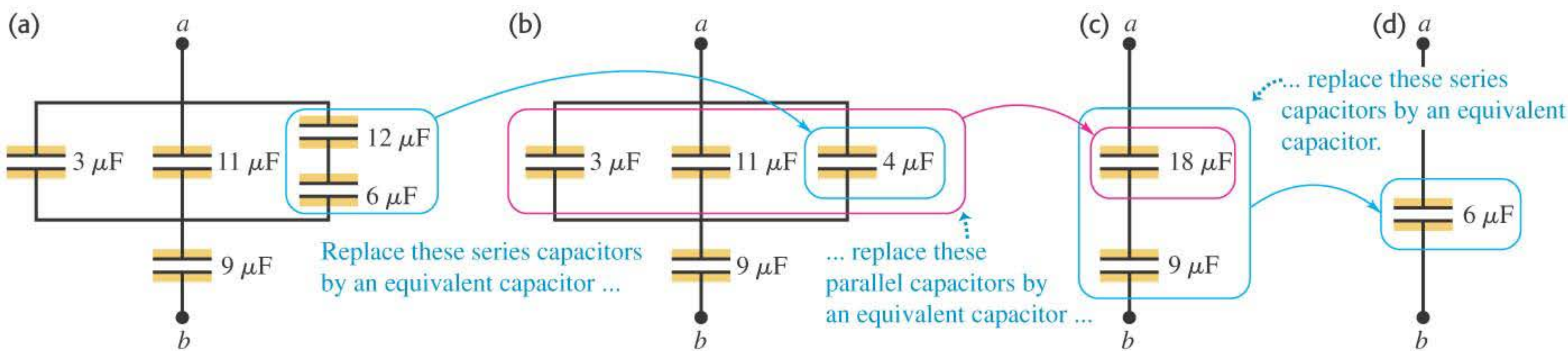
- Capacitors are connected in *parallel* between *a* and *b* if the potential difference V_{ab} is the same for all the capacitors. (See Figure 24.9 below.)

- The *equivalent capacitance* of a parallel combination is the *sum* of the individual capacitances: $C_{eq} = C_1 + C_2 + C_3 + \ldots$.



Calculations of capacitance

- Refer to Problem-Solving Strategy 24.1.
- Follow Example 24.5.
- Follow Example 24.6, a capacitor network, using Figure 24.10 below.



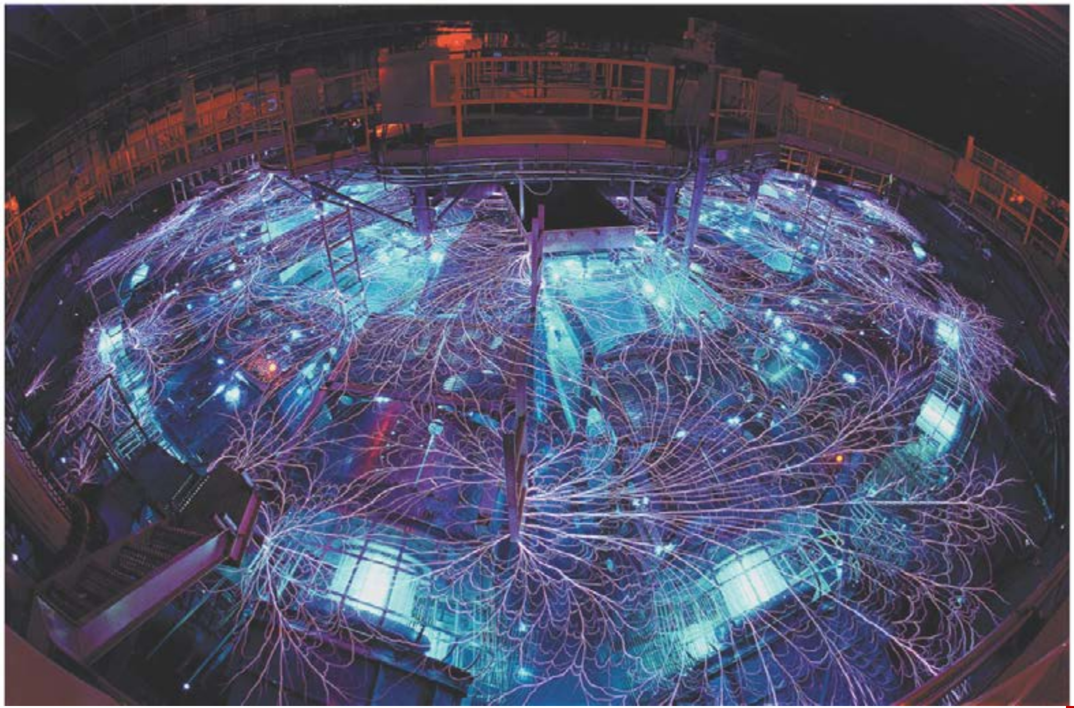
Energy stored in a capacitor

- The potential energy stored in a capacitor is

$$U = Q^2/2C = 1/2\ CV^2 = 1/2\ QV.$$

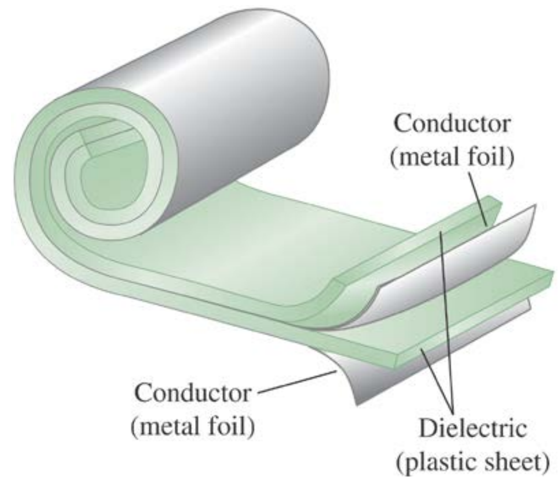
- The capacitor energy is stored in the *electric field* between the plates. The *energy density* is $u = 1/2\ \varepsilon_0 E^2$.

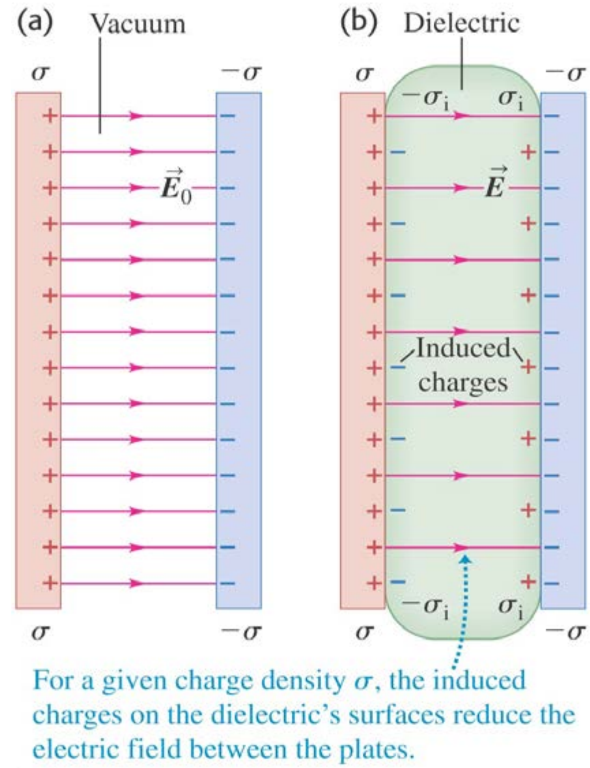
- The Z machine shown below can produce up to 2.9×10^{14} W using capacitors in parallel!



Dielectrics

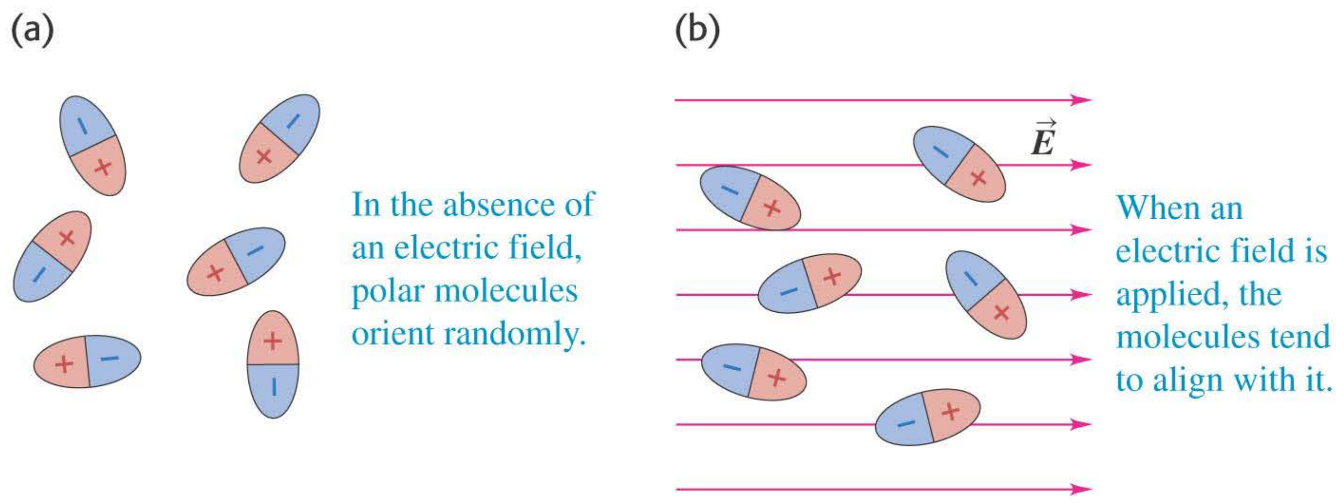
- A *dielectric* is a nonconducting material. Most capacitors have dielectric between their plates. (See Figure 24.13 at upper right.)
- The *dielectric constant* of the material is $K = C/C_0 > 1$.
- Dielectric *increases* the capacitance and the energy density by a factor K.
- Figure 24.15 (lower right) shows how the dielectric affects the electric field between the plates.
- Table 24.1 on the next slide shows some values of the dielectric constant.

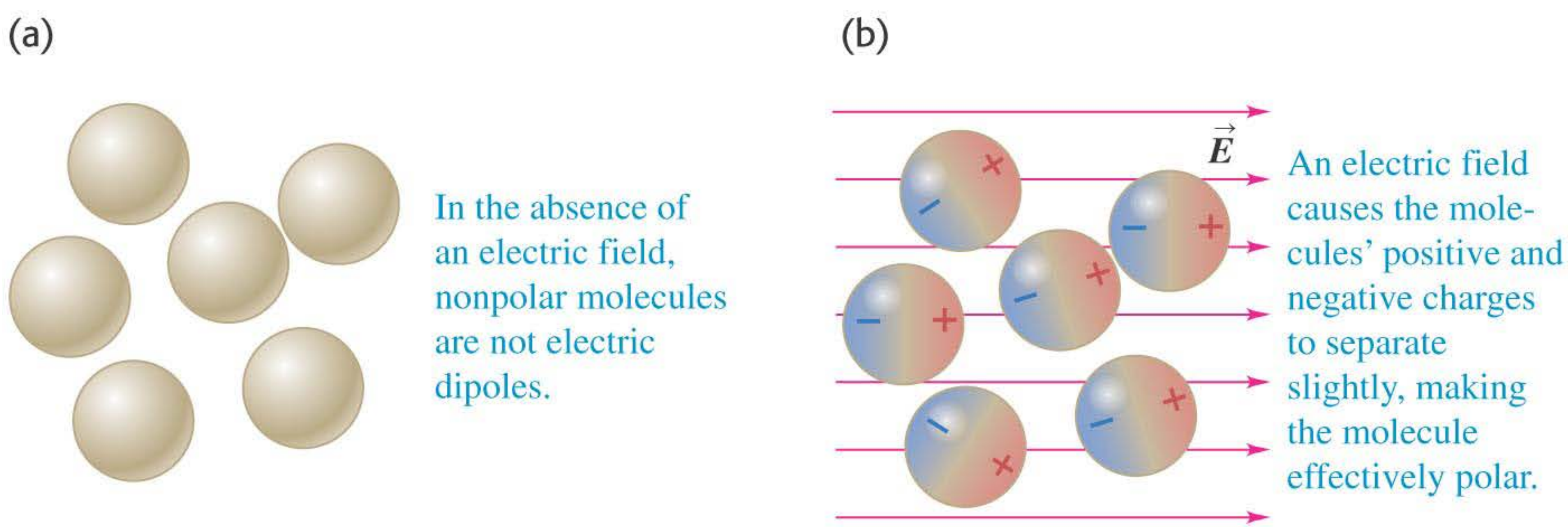




Molecular model of induced charge - I

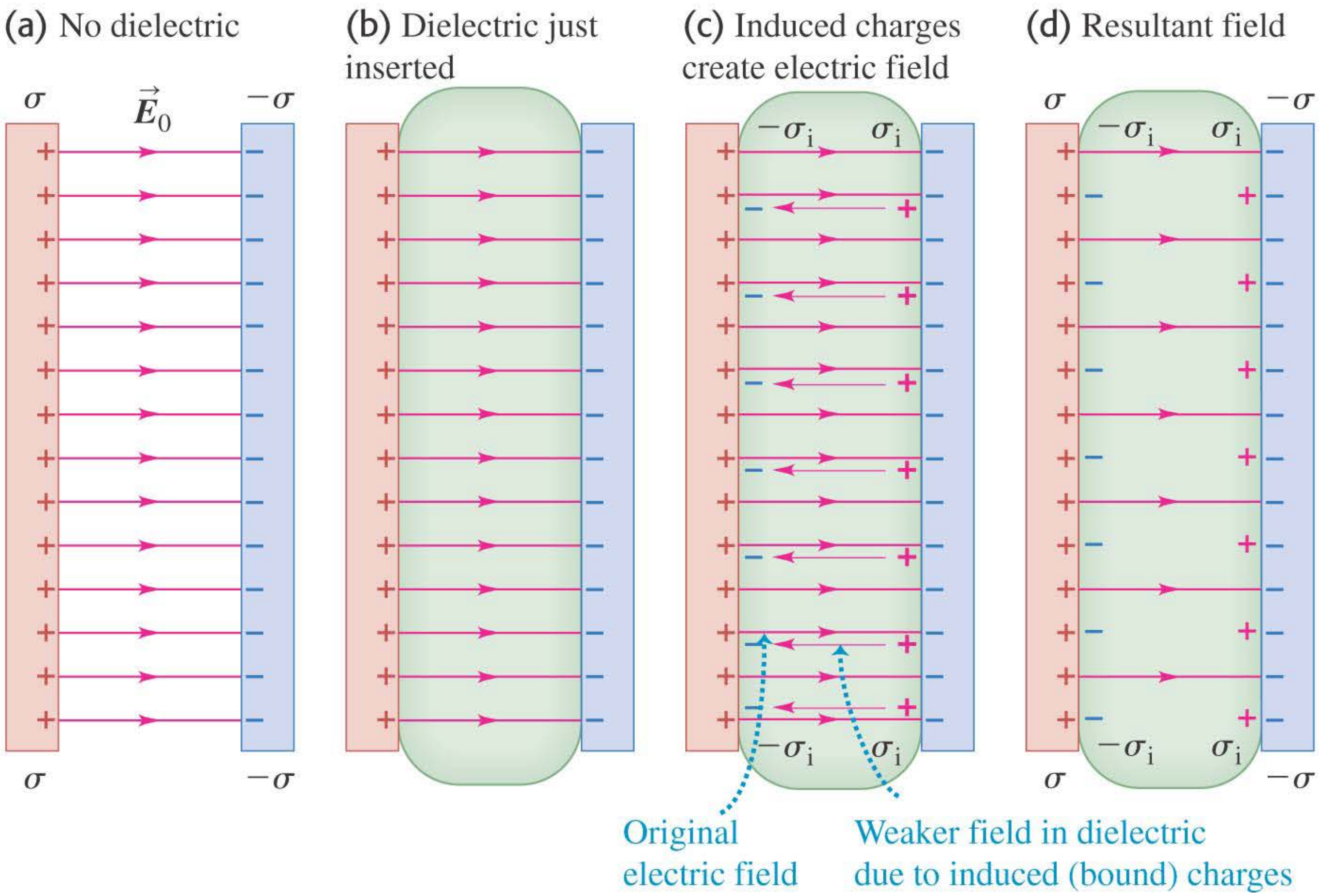
- Figures 24.17 (right) and 24.18 (below) show the effect of an applied electric field on polar and nonpolar molecules.





Molecular model of induced charge - II

- Figure 24.20 below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.



Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

EE's often call D the "Electric Field!"

$$\nabla \cdot \vec{D} = \rho$$

Gauss' Law in a Material in Differential Form for EE's

$$\vec{D} = \epsilon \vec{E}$$

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}$$

Gauss' Law in Integral Form

$$\epsilon = K\epsilon_o$$

K is the "dielectric constant"… also called the "relative permittivity."

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

EE's often call D the "Electric Field!"

$$\nabla \cdot \vec{D} = \rho$$

$$\vec{D} = \epsilon \vec{E}$$

Gauss' Law in a Material in Differential Form for EE's

Table 24.1—Some dielectric constants

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric breakdown

- If the electric field is strong enough, *dielectric breakdown* occurs and the dielectric becomes a conductor.
- The *dielectric strength* is the maximum electric field the material can withstand before breakdown occurs.
- Table 24.2 shows the dielectric strength of some insulators.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7