PHYS 122-Lecture 6:
Capacitance and Dielectrics

- Capacitance [First half of Chap 24]
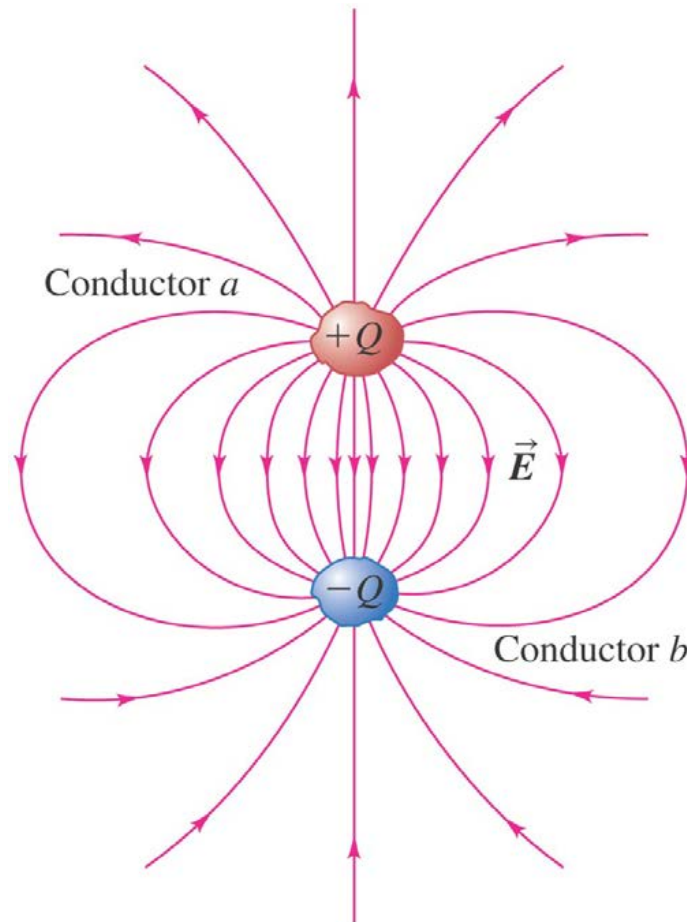
- Dielectrics [Second half of Chap 24]

Introduction

- How does a camera's flash unit store energy?

- Capacitors are devices that store electric potential energy.

- The energy of a capacitor is actually stored in the electric field.

Capacitors and capacitance

- Any two conductors separated by an insulator form a *capacitor*, as illustrated in Figure 24.1 below.
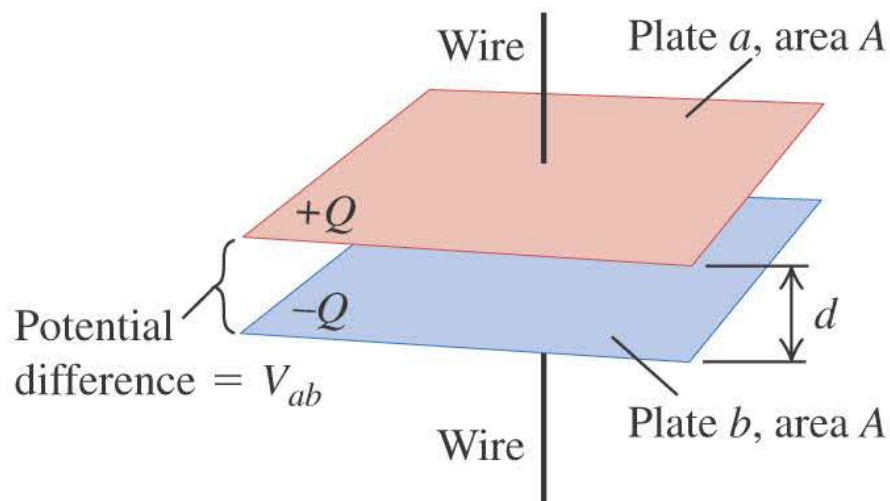
- The definition of capacitance is $C = Q/V_{ab}$.

[C/V] = Farad [F]

Conductor a
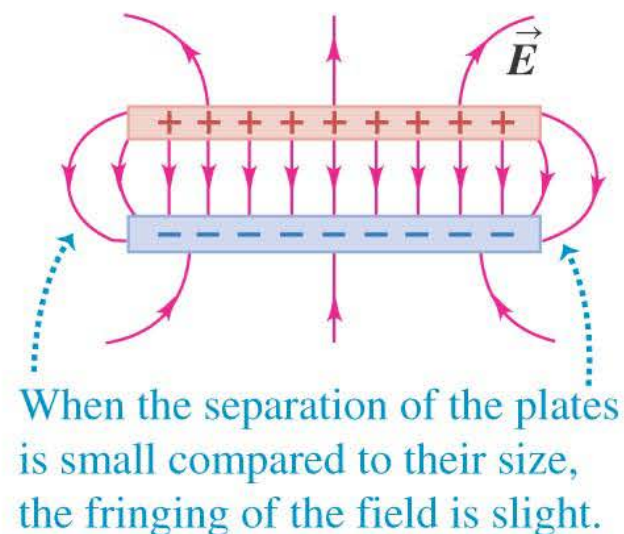
$+Q$

$\vec{E}$

$-Q$

Conductor b

Parallel-plate capacitor

- A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions. (See Figure 24.2 below.)

- The capacitance of a parallel-plate capacitor is $C = \varepsilon_0 A/d$.

- Follow Examples 24.1 and 24.2.

(a) Arrangement of the capacitor plates

Wire

Plate *a*, area A

$+Q$

$-Q$

Potential difference $= V_{ab}$

Wire

Plate *b*, area A

d

(b) Side view of the electric field $\vec{E}$

$\vec{E}$

+ + + + + + + +

– – – – – – – –

When the separation of the plates is small compared to their size, the fringing of the field is slight.

Parallel-plate capacitor

- A *parallel-plate capacitor* consists of two parallel conducting plates separated by a distance that is small compared to their dimensions. (See Figure 24.2 below.)

- The capacitance of a parallel-plate capacitor is $C = \varepsilon_0 A/d$.

- Follow Examples 24.1 and 24.2.
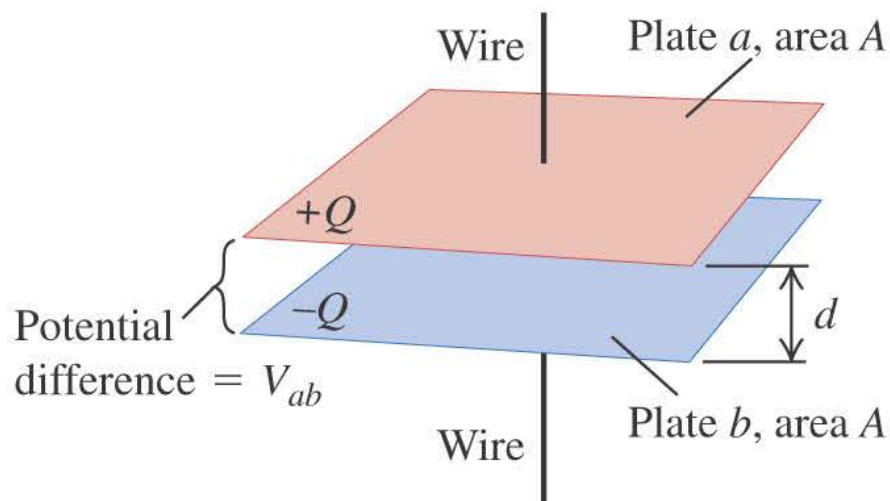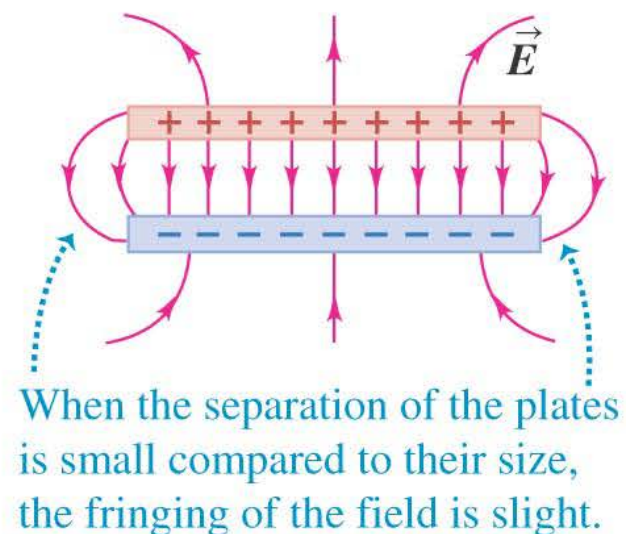
(a) Arrangement of the capacitor plates

Wire
Plate a, area A
$+Q$
Potential difference $= V_{ab}$
$-Q$
d
Wire
Plate b, area A

(b) Side view of the electric field $\vec{E}$

$\vec{E}$

+ + + + + + + +

− − − − − − − −

When the separation of the plates is small compared to their size, the fringing of the field is slight.

A spherical capacitor

- Follow Example 24.3 using Figure 24.5 to consider a spherical capacitor.

Inner shell, charge $+Q$

Gaussian surface

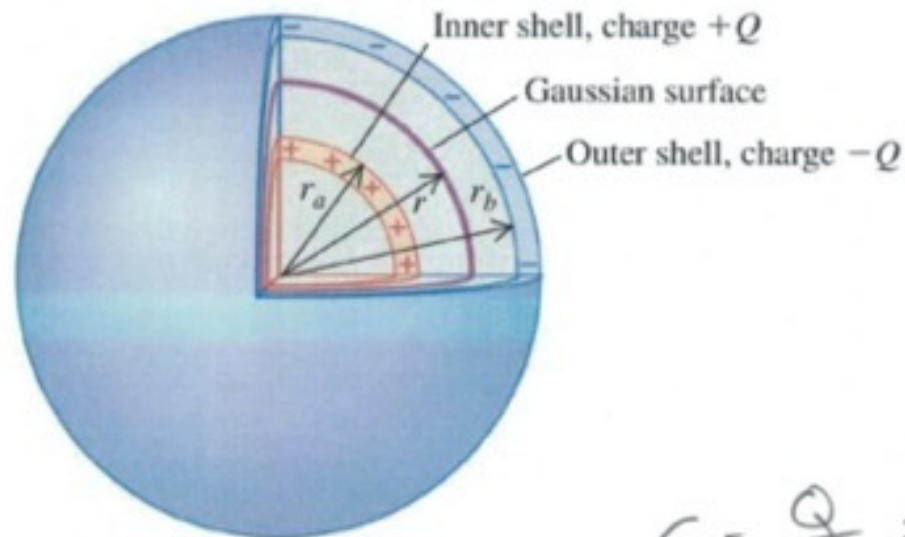Outer shell, charge $-Q$

r_a r' r_b

A spherical capacitor

- Follow Example 24.3 using Figure 24.5 to consider a spherical capacitor.

$$V_a = \frac{Q}{4\pi\epsilon_0 r_a}$$

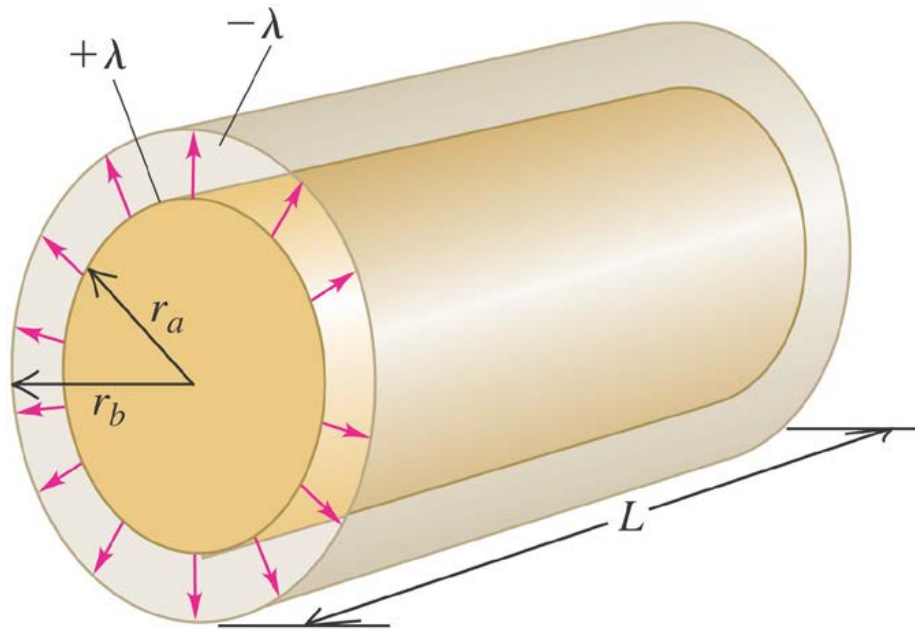$$V_b = \frac{Q}{4\pi\epsilon_0 r_b}$$

$$V_{ab} = V_a - V_b$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

Inner shell, charge $+Q$

Gaussian surface

Outer shell, charge $-Q$

r_a r' r_b

$$C = \frac{Q}{V_{ab}} = \frac{1}{\frac{1}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)}$$

$$= 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$$

A cylindrical capacitor

- Follow Example 24.4 and Figure 24.6 to investigate a cylindrical capacitor.
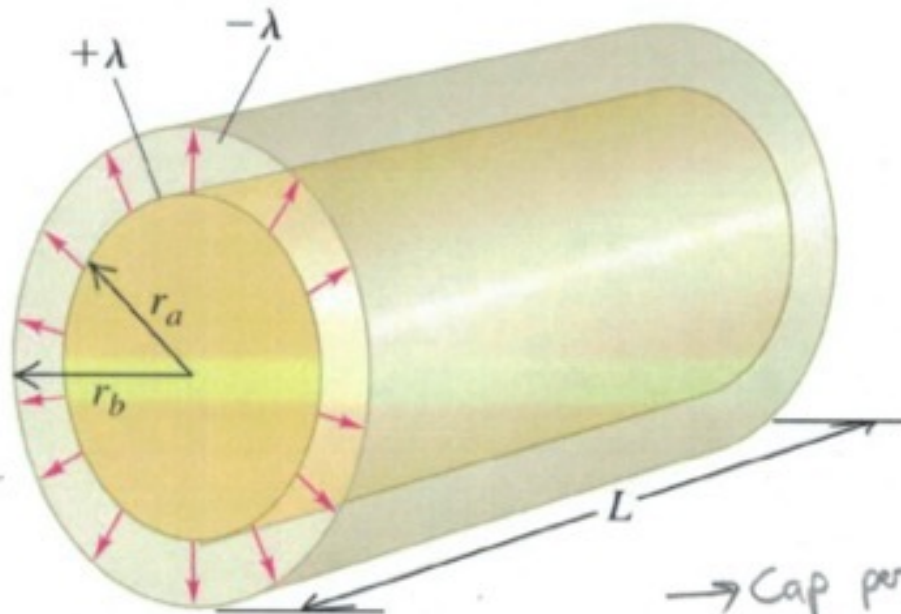
A cylindrical capacitor

- Follow Example 24.4 and Figure 24.6 to investigate a cylindrical capacitor.

$$V = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_0}{r}\right)$$

$$V_{ab} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_b}{r_a}\right)$$

$+\lambda$ $-\lambda$

r_a

r_b

L

→ Cap per unit length

$$C = \frac{Q}{V_{ab}} = \frac{\lambda L}{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_b}{r_a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{r_b}{r_a}\right)}$$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{r_b}{r_a}\right)}$$
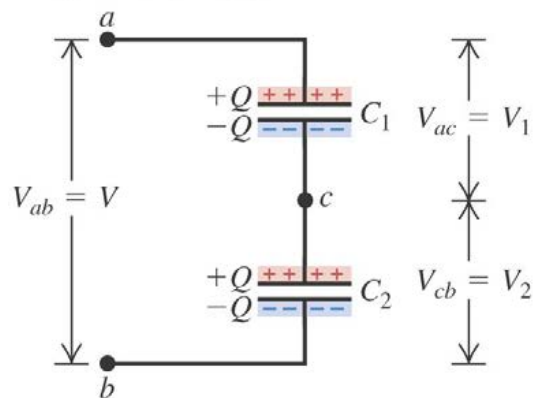
Capacitors in series

- Capacitors are in *series* if they are connected one after the other, as illustrated in Figure 24.8 below.

- The *equivalent capacitance* of a series combination is given by $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \ldots$
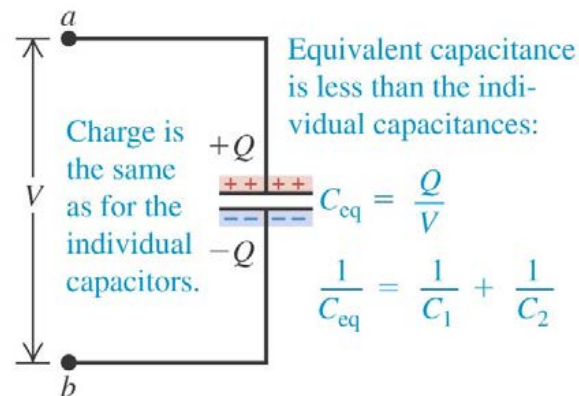
(a) Two capacitors in series

Capacitors in series:
- The capacitors have the same charge Q.
- Their potential differences add:
  $V_{ac} + V_{cb} = V_{ab}$.

a

$+Q$
$-Q$ C_1 $V_{ac} = V_1$

$V_{ab} = V$

c

$+Q$
$-Q$ C_2 $V_{cb} = V_2$

b

(b) The equivalent single capacitor

a

Charge is the same as for the individual capacitors.

$+Q$

$-Q$

Equivalent capacitance is less than the individual capacitances:

$C_{eq} = \dfrac{Q}{V}$

$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

b

Capacitors in parallel

- Capacitors are connected in *parallel* between *a* and *b* if the potential difference V_{ab} is the same for all the capacitors. (See Figure 24.9 below.)

- The *equivalent capacitance* of a parallel combination is the *sum* of the individual capacitances: $C_{eq} = C_1 + C_2 + C_3 + \dots$.
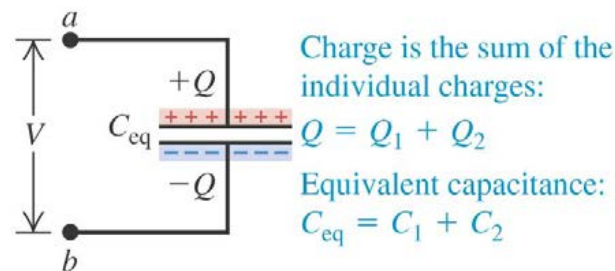
(a) Two capacitors in parallel

Capacitors in parallel:
- The capacitors have the same potential *V*.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.
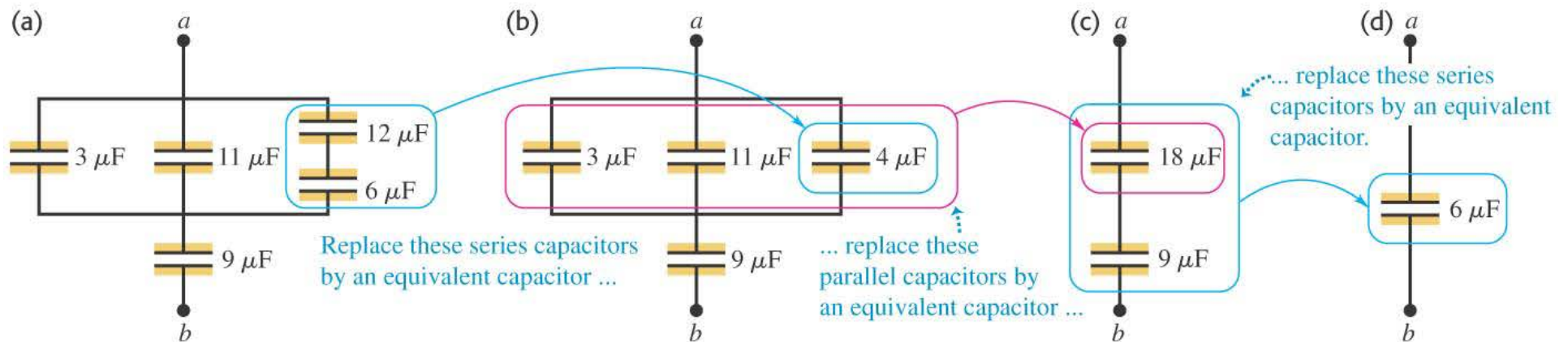
(b) The equivalent single capacitor

Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$
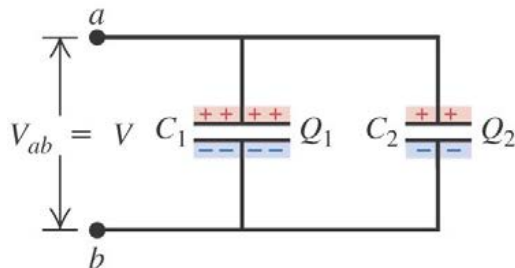
Equivalent capacitance:
$C_{eq} = C_1 + C_2$

Calculations of capacitance

- Refer to Problem-Solving Strategy 24.1.

- Follow Example 24.5.

- Follow Example 24.6, a capacitor network, using Figure 24.10 below.

(a) a — $3\,\mu F$, $11\,\mu F$, $12\,\mu F$, $6\,\mu F$, $9\,\mu F$ — b
Replace these series capacitors by an equivalent capacitor ...

(b) a — $3\,\mu F$, $11\,\mu F$, $4\,\mu F$, $9\,\mu F$ — b
... replace these parallel capacitors by an equivalent capacitor ...

(c) a — $18\,\mu F$, $9\,\mu F$ — b
... replace these series capacitors by an equivalent capacitor.
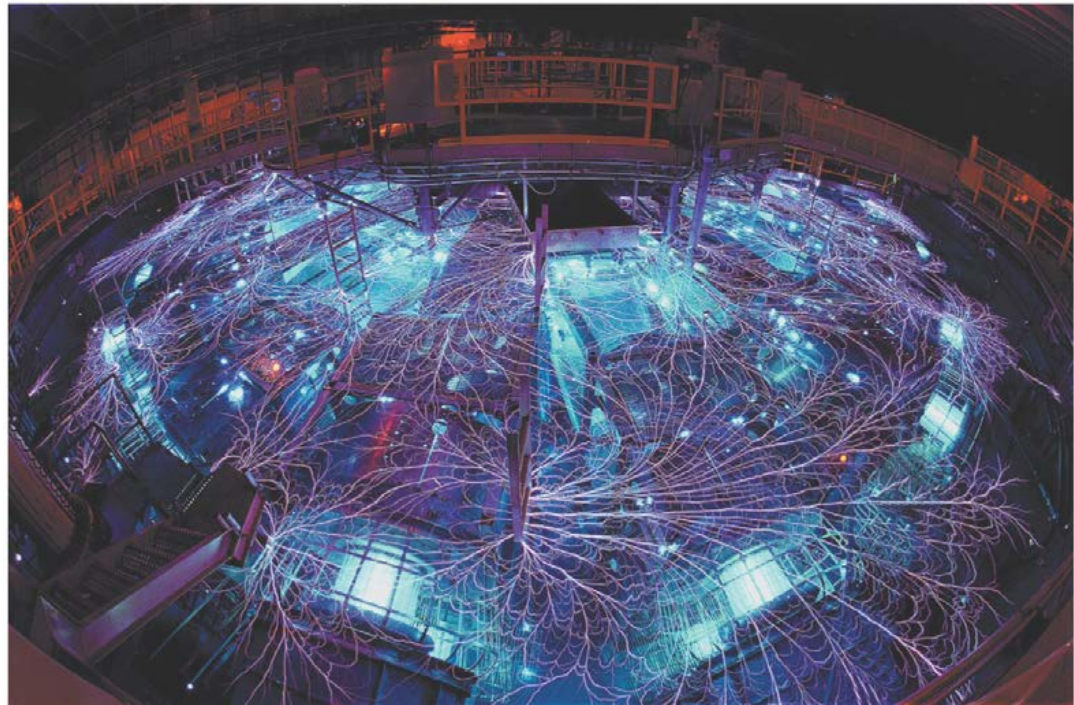
(d) a — $6\,\mu F$ — b

Energy stored in a capacitor

- The potential energy stored in a capacitor is
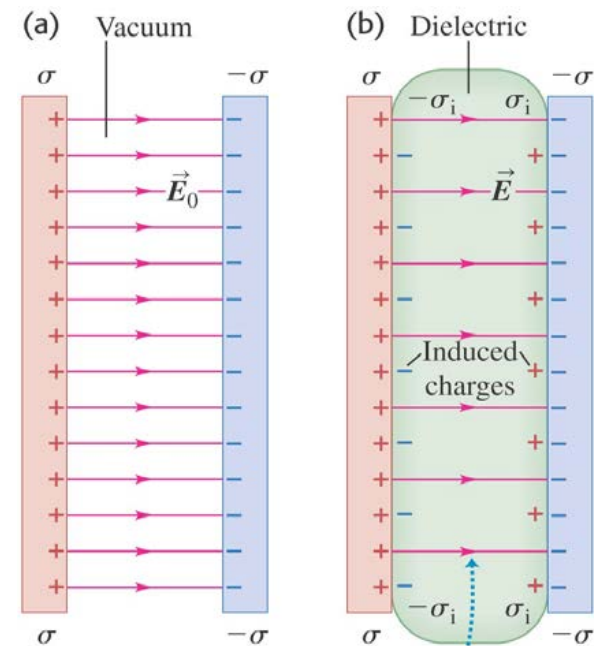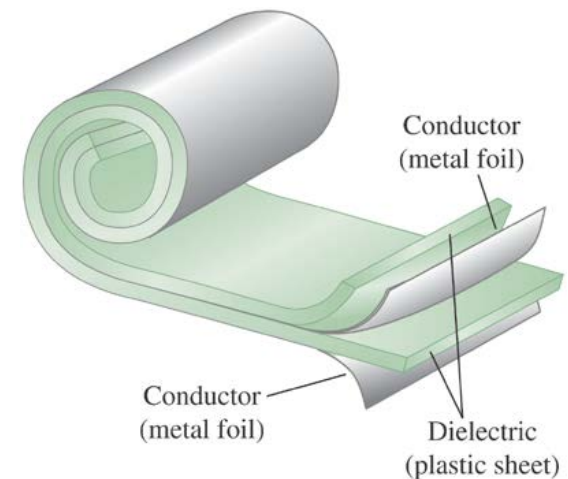
$$U = Q^2/2C = 1/2 \; CV^2 = 1/2 \; QV.$$

- The capacitor energy is stored in the *electric field* between the plates. The *energy density* is $u = 1/2 \; \varepsilon_0 E^2$.

- The Z machine shown below can produce up to 2.9×10^{14} W using capacitors in parallel!
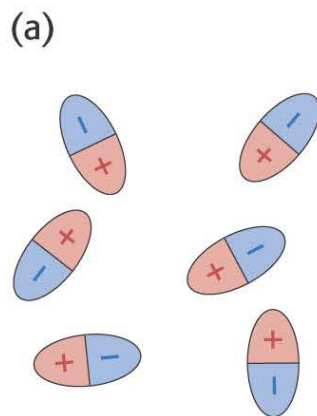
Dielectrics

- A *dielectric* is a nonconducting material. Most capacitors have dielectric between their plates. (See Figure 24.13 at upper right.)

- The *dielectric constant* of the material is $K = C/C_0 > 1$.

- Dielectric *increases* the capacitance and the energy density by a factor K.

- Figure 24.15 (lower right) shows how the dielectric affects the electric field between the plates.

- Table 24.1 on the next slide shows some values of the dielectric constant.

Conductor (metal foil)

Conductor (metal foil)

Dielectric (plastic sheet)

(a) Vacuum (b) Dielectric

σ $-\sigma$ σ $-\sigma$

$-\sigma_i$ σ_i

$\vec{E}_0$

$\vec{E}$

Induced charges

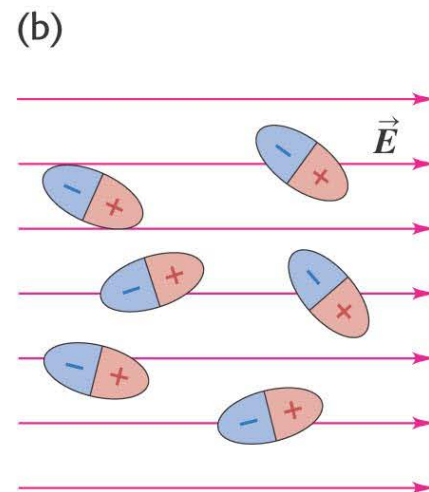σ $-\sigma$ σ $-\sigma$

$-\sigma_i$ σ_i

For a given charge density σ, the induced charges on the dielectric's surfaces reduce the electric field between the plates.
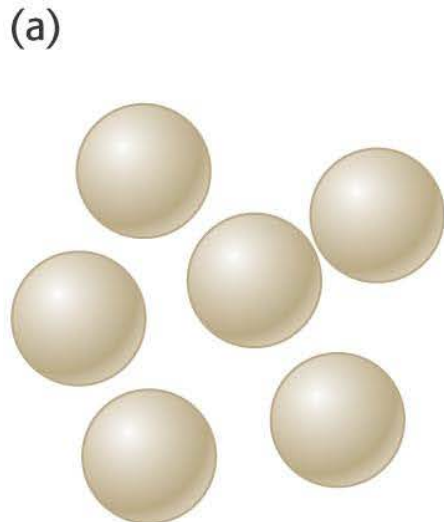
Molecular model of induced charge - I

- Figures 24.17 (right) and 24.18 (below) show the effect of an applied electric field on polar and nonpolar molecules.

(a)
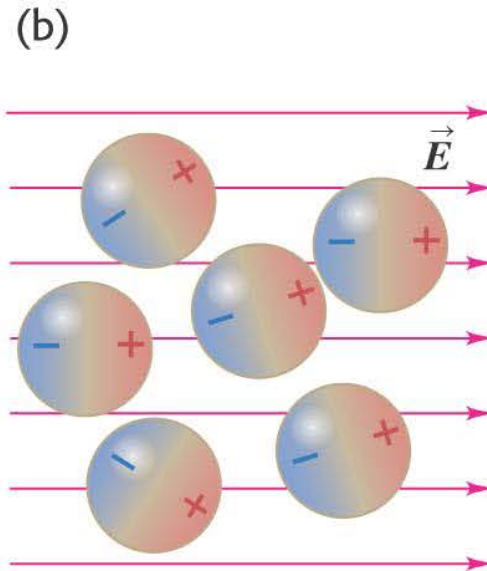
In the absence of an electric field, polar molecules orient randomly.

(b)

$\vec{E}$

When an electric field is applied, the molecules tend to align with it.

(a)

In the absence of an electric field, nonpolar molecules are not electric dipoles.

(b)

$\vec{E}$

An electric field causes the molecules' positive and negative charges to separate slightly, making the molecule effectively polar.

Molecular model of induced charge - II

- Figure 24.20 below shows *polarization* of the dielectric and how the induced charges reduce the magnitude of the resultant electric field.

(a) No dielectric

(b) Dielectric just inserted

(c) Induced charges create electric field

(d) Resultant field

Original electric field

Weaker field in dielectric due to induced (bound) charges

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$
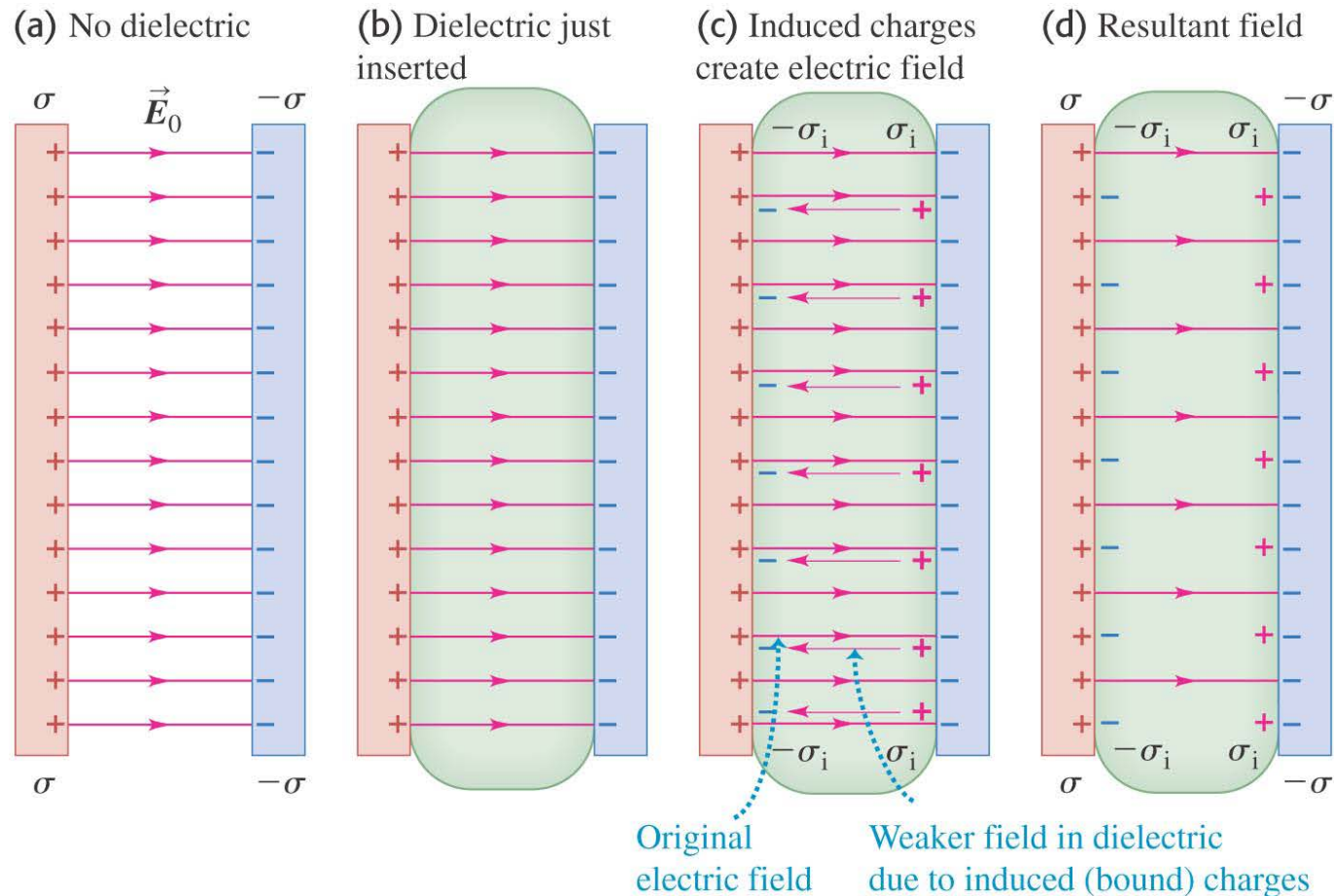
$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}$$ *Gauss' Law in Integral Form*

- -

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$ *Gauss' Law in Material*

- -

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$ *(Divergence Theorem)*

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$ *Gauss' Law in a Material in Differential Form*

EE's often call D the "Electric Field!"

$$\nabla \cdot \vec{D} = \rho$$ *Gauss' Law in a Material in Differential Form for EE's*

$$\vec{D} = \epsilon \vec{E}$$

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\phi \, dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}$$

Gauss' Law in Integral Form

$$\boldsymbol{\epsilon = K\epsilon_o}$$

K is the "dielectric constant"… also called the "relative permittivity."

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho \, dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

EE's often call D the "Electric Field!"

$$\nabla \cdot \vec{D} = \rho$$

Gauss' Law in a Material in Differential Form for EE's

$$\vec{D} = \epsilon \vec{E}$$

Table 24.1—Some dielectric constants

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

Dielectric breakdown

- If the electric field is strong enough, *dielectric breakdown* occurs and the dielectric becomes a conductor.

- The *dielectric strength* is the maximum electric field the material can withstand before breakdown occurs.

- Table 24.2 shows the dielectric strength of some insulators.

Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7