

The 2009 NJIT Integration Bee

April 16, 2009

1. **Problem:** $\int e^{2009x} dx$

Solution:

$$\frac{1}{2009} e^{2009x}$$

This example would be too easy for a real competition problem.

2. **Problem:** $\int_4^{15} 2009 dx$

Solution:

$$22099$$

$2009x|_4^{15} = 2009 \cdot (15 - 4) = 2009 \cdot 11 = 22099$
Again, too easy for the competition.

3. **Problem:** $\int \frac{1}{1+x^2} dx$

Solution:

$$\tan^{-1} x$$

Very basic Math 111 integral. Now things will get tougher...

4. **Problem:** $\int \frac{x^3 - 3x^2 + 3x - 1}{x^2} dx$

Solution:

$$\frac{x^2}{2} - 3x + 3 \log|x| + \frac{1}{x}$$

Split into four pieces.

5. **Problem:** $\int xe^{-x} dx$

Solution:

$$-xe^{-x} - e^{-x}$$

Integration by parts.

6. **Problem:** $\int 7^x dx$

Solution:

$$\frac{1}{\ln 7} 7^x$$

Recall $7^x = e^{x \ln 7}$.

7. **Problem:** $\int 5x\sqrt{x-3} dx$

Solution:

$$2(x-3)^{5/2} + 10(x-3)^{3/2}$$

Substitute $u=x-3$

8. **Problem:** $\int \frac{3x+6}{\sqrt{2x^2+8x+3}} dx$

Solution:

$$\frac{3}{2} \sqrt{2x^2 + 8x + 3}$$

u-substitution

9. **Problem:** $\int_{-1}^1 \tan x dx$

Solution:

$$0$$

Odd integrand over symmetric interval.

10. **Problem:** $\int x^3 e^{-x^2} dx$

Solution:

$$\frac{1}{2} e^{-x^2} (-x^2 - 1)$$

Substitute $u = x^2$.

11. **Problem:** $\int_0^1 \sqrt{1-x^2} dx$

Solution:

$$\frac{\pi}{4}$$

Area of a quarter circle of unit radius

12. **Problem:** $\int \tan^3 x \sec^2 x dx$

Solution:

$$\frac{1}{4} \tan^4 x$$

13. **Problem:** $\int \tan^3 x \sec^3 x dx$

Solution:

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Note that

$$\begin{aligned}\tan^3 x \sec^3 x &= \tan x (1 - \sec^2 x) \sec^3 x \\ &= (\sec x \tan x)(\sec^2 x - \sec^4 x)\end{aligned}$$

and substitute $u = \sec x$.

14. **Problem:** $\int \frac{1}{\sqrt{1-x^2+\sin^{-1}x-x^2\sin^{-1}x}} dx$

Solution:

$$2\sqrt{1+\sin^{-1}x}$$

Factor inside the radicand and substitute $u = \sin^{-1}x$.

15. **Problem:** $\int \frac{x}{x+1} dx$

Solution:

$$x - \ln|x+1|$$

Add and subtract 1 from the numerator

16. **Problem:** $\int f(x)f'''(x) dx$

Solution:

$$f(x)f''(x) - \frac{1}{2}f'(x)^2$$

Integrate by parts, then substitute $u = f'(x)$