Coverage Properties of Clustered Wireless Sensor Networks

RENITA MACHADO
Oklahoma Baptist University
WENSHENG ZHANG
Iowa State University
GUILING WANG
New Jersey Institute of Technology
and
SIRIN TEKINAY
Ozyegin University, Turkey

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This article studies clustered wireless sensor networks (WSNs), a realistic topology resulting from common deployment methods. We study coverage in *naturally clustered* networks of wireless sensor nodes, as opposed to WSNs where clustering is facilitated by selection. We show that along with increasing the vacancy in random placement of nodes in a WSN, it also alters the connectivity properties in the network. We analyze varying levels of redundancy to determine the probability of coverage in the network. The phenomenon of clustering in networks of wireless sensor nodes raises interesting questions for future research and development. The article provides a foundation for the design to optimize network performance with the constraint of sensing coverage.

Categories and Subject Descriptors: C.2 [**Computer Communication-Networks**]: Distribution and Maintenance—*documentation*; C.2.1 [**Computer Communication-Networks**]: Network Architecture and Design—*Distributed networks*; *network topology*

General Terms: Algorithm, Performance

Additional Key Words and Phrases: Wireless sensor networks, clustering, coverage

This work was supported in part by the National Science Foundations under Grants No. CNS-0834585 and CNS-0831906.
Authors' present addresses: R. Machado, Paul Dickinson School of Business, Oklahoma Baptist University, 500 West University, Shawnee, OK 74804; email: renita.machado@gmail.com; W. Zhang, Department of Computer Science, Iowa State University, Ames, IA 50011-1040; email: wzhang@iastate.edu; G. Wang, Department of Computer Science, New Jersey Institute of Technology, University Heights, Newark, NJ 07102; email: gwang@njit.edu; S. Tekinay, Vice-Rector for Research, Ozyegin University, Kuşbakişi Cad. No. 2 34662 Altunizade Üsküdar, Istanbul, Turkey; email: sirin.tekinay@ozyegin.edu.tr.

DOI 10.1145/1824766.1824769 http://doi.acm.org/10.1145/1824766.1824769

ACM Reference Format:
Machado, R., Zhang, W., Wang, G., and Tekinay, S. 2010. Coverage properties of clustered wireless sensor networks ACM Trans. Sens. Netw. 7, 2, Article 13 (August 2010), 21 pages.
DOI = 10.1145/1824766.1824769 http://doi.acm.org/10.1145/1824766.1824769

1. INTRODUCTION

Sensing coverage is an important functional metric to a wireless sensor network since it determines how well the sensor network can monitor the environment and generate corresponding data. In addition, the knowledge of coverage and redundancy is essential in developing algorithms [Heinzelman et al. 2002] to schedule the listening/sleeping cycle of sensors for optimizing the trade-off between coverage, communication connectivity and power consumption.

Currently the assumption of the uniform Poisson process for node distribution dominates the literature as the foundation of most algorithms designed to optimize network performance. However, the assumption is not practical in many situations. The nature and scale of most WSN applications makes it difficult to arrange the nodes in regular topologies across the deployment region. Node placements for environment monitoring [Biagioni and Bridges 2002; Mainwaring et al. 2002] or military applications [Winkler et al. 2007] is typically done through spraying nodes from an airborne device or other techniques that result in no control over the locations of deployed nodes. A significant consequence of this process is the clustering or clumping of nodes, where node positions form clusters resulting in redundancy of coverage in certain area and coverage *holes* or vacancy in the other area, where vacancy is defined as the region that is not covered by any of the circular coverage disks of sensor nodes in the deployment region. We call this scenario *naturally clustered* networks as opposed to the more prevalent notion of clustering by choice. Clustering is one of the widely prevalent topologies of nodes in random deployments of dense networks.

To emphasize the concept of clustered topologies in wireless sensor networks, a typical example of a clustered WSN is shown in Figure 1, as opposed to the widely-used measure of a stationary Poisson point process of node distribution in the deployment region (Figure 2). A quick examination of Figure 1 reveals the inherent feature of clustered networks: varying coverage in the deployment region. In fact, as we show in the next section, clustering has been shown to increase the area of vacancy compared to uniformly distributed nodes, and this preliminary observation signals the need for analysis of clustering properties to design protocols and algorithms that optimize WSN network performance.

In addition to the increased vacancy and vacancy distribution, which requires more study, the natural clustering of WSN also has impact on the design of those clustering algorithms [Heinzelman et al. 2002; Shu et al. 2005] for routing, data aggregation and energy conservation. An understanding of clustering properties can be efficiently leveraged to take advantage of the energy conservation properties offered by the clustering approach in dense WSNs.

To sum up, there are fundamental reasons why we investigate the properties of coverage in naturally clustered networks instead of using the widely used

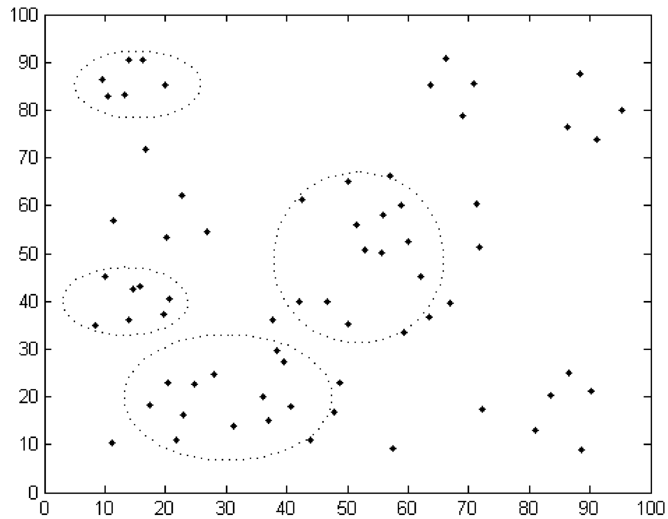


Fig. 1. Clustering of nodes in WSNs.

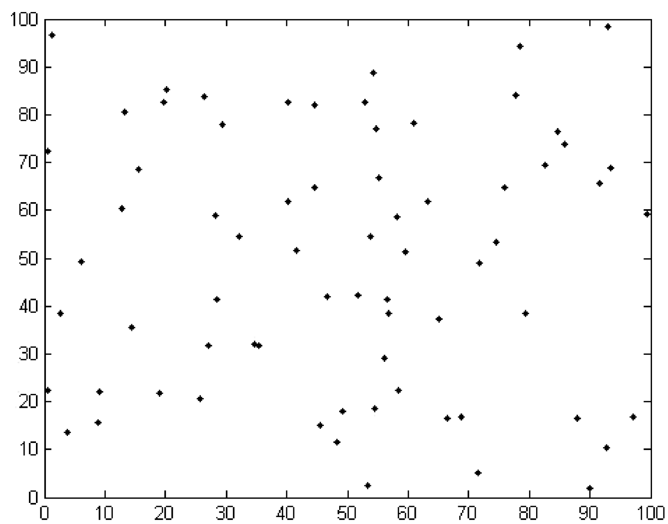


Fig. 2. Random distribution of nodes without clustering in WSNs.

theory of uniformly distributed nodes in a Poisson model of node distribution, some of which we list here.

— Models for naturally clustered networks show that coverage properties in clustered networks are significantly altered than those in Poisson models of uniform and random node distribution. As we show in the rest of this article, vacancy due to clustering is higher than in the widely used model of a stationary Poisson point process used to model node distribution.

—These models can provide a foundation for future work on placement of nodes in randomly deployed WSNs, thus making it easier to understand and extend the model of clustering.

—Even though clustering exhibits increased vacancy, this vacancy can be compensated by mobile nodes who can travel to the region of vacancy and cover the vacant region. Understanding the vacancy distribution can help design mobile sensor deployment and relocation protocols [Wang et al. 2005].

—With the insights from this work, researchers can create new models of clustering by customizing the properties that are appropriate to the WSN application as far as the network is deployed.

The objective of this article is to investigate how to model the randomness in the placement of nodes by examining the coverage attributes resulting from randomness in clustering. In particular, the attributes of coverage in naturally clustered networks are extensions of the Poisson model of node distribution widely used in modeling the distribution of nodes in WSNs. We show how to extend Poisson distribution of nodes to support clustering with an analytical framework using the Poisson cluster point process (PCPP). PCPP has been used to model natural environment, for example, the distribution of rainfall. To the best of our knowledge, we are the first one to employ PCPP in modeling the distribution of sensor clusters and vacancy.

We analyze the properties of naturally clustered WSNs using the theory of coverage processes [Hall 1988]. This analysis provides the foundation for a study of varying densities of nodes and cluster heads encountered in real-world deployment scenarios, where the densities of nodes and cluster heads are dictated by node failure due to device failure, battery energy exhaustion or other causes that can occur in remote and hostile deployment regions.

The article has the following contributions: (1) Study of the coverage property of clustered WSNs and (2) to provide a foundation for the design to optimize network performance.

The remainder of the article is organized as follows. Section 2 summarizes relevant literature on coverage and clustering in WSNs and highlights the differences between existing research and our work. Section 3 reviews background necessary for studying naturally clustered WSNs and explains prerequisite concepts, particularly the Poisson process, extension to clustering and coverage metrics of WSN performance. In Section 4, we discuss the applications of PCPP processes. In Section 5, we present the results of simulation. Finally, Section 6 concludes the article.

2. RELATED WORK

In this section, we provide an overview of related work in coverage, clustering in WSNs and coverage processes.

2.1 Coverage

General Coverage Metrics. Coverage in sensor networks has been extensively studied, in terms of the coverage resulting from various deployment patterns created by node placement [Ai and Abouzeid 2006; Ahmed and Wales 2005; Kumar et al. 2005]. The worst and best-case coverage has been studied in Megerian et al. [2005]. With the help of techniques from computational geometry and graph theory, the authors study the maximal breach paths and maximal support paths for the coverage problem, where the maximal breach path is the path with the minimum distance to a sensor and the maximal support path is the path with the maximum distance to a sensor. In Veltri et al. [2003], the authors study the minimal and maximal exposure paths corresponding to the worst and best-case coverage in WSNs. The authors propose a localized approximation algorithm for a WSN to determine its minimal exposure path. In Chin et al. [2005], the exposure metric has been further studied for collaboration in WSNs of mobile nodes in the presence of noise and obstacles. Using the definition of exposure as the least probability of target detection, the authors propose low-computationally intensive algorithms to obtain the upper and lower bounds on exposure.

Directional Coverage. Directional coverage has been studied in Yu et al. [2010], Adriaens et al. [2006], and Cai et al. [2007]. In Yu et al. [2010], the authors study the optimal patterns that provide connectivity in WSNs. The authors propose scheduling mechanisms to achieve higher connectivity and full coverage in WSNs with nodes equipped with directional antennas. In Adriaens et al. [2006], the authors study optimal worst-case coverage with sensors equipped with video cameras, and directionality is studied in terms of the field-of-view of sensors. In Cai et al. [2007], the authors propose the use of directional antennas for power-conservation and greater coverage in the WSN.

Coverage in Sensor Networks with Mobile Nodes. The coverage provided by mobile and static nodes has been studied in Liu et al. [2005], Lazos et al. [2007], Xing et al. [2008], and Tan et al. [2008]. The use of mobile nodes to provide improved coverage has been studied in Liu et al. [2005] and Wang et al. [2006, 2007]. In Xing et al. [2008], the authors study collaboration of mobile and static networks to meet stringent spatial and temporal application requirements of sensor networks deployed for surveillance applications. The authors propose a multi-sensor fusion and movement model to achieve three performance metrics: bounded detection delay, high detection probability and low false alarm rate. Collaboration and mobility in sensors has also been studied in Tan et al. [2008]. In Bisnik et al. [2006], the authors study the characteristics of mobility in a network of both mobile and static nodes. They study the scenarios under which the coverage provided by mobile sensors is higher than that provided by static sensors by analyzing the mobility framework of node velocity, mobility pattern, number of mobile sensors and dynamics of the phenomenon being sensed. The authors study this quality of coverage problem and propose motion planning algorithms to bound the probability of event loss in the network.

Application-Specific Coverage. In [Luo et al. 2009], the authors study self-adjusting networks that provide surface coverage for sea-surface sensing application. The authors study two types of node mobility: Uncontrollable mobility (U-mobility) and Controllable mobility (C-mobility). U-mobility occurs when external forces, in this case, waves disrupt the coverage pattern in the networks which can be re-installed using the mobility of nodes provided by C-mobility.

The authors study this double mobility in terms of the coverage provided by the mobility of nodes, and propose a distributed algorithm using dominating sets that guarantees coverage on the sea surface.

The study of coverage in WSNs for surveillance applications has also been studied in Xing et al. [2008] and Amaldi et al. [2008]. In Amaldi et al. [2008], the authors study the problem of positioning sensors for optimal detection of mobile targets. The authors propose an optimization framework, wherein the distance of a node to the target is minimized for maximal exposure. The quality of coverage has also been studied in terms of the quality of surveillance in Gui and Mohapatra [2004], where the network topology has been exploited to achieve power-saving algorithms for nodes to track mobile targets. In Lazos et al. [2007], the authors study wireless sensor deployment for detection of mobile targets. They study the problem of detecting target presence in a Field of Interest (FoI). Specifically, the authors address the issue of maximum target detection probability in a FoI by N sensors. The authors study this problem in the context of deployment of a deterministic WSN with homogeneous and heterogeneous WSNs, where heterogeneity is explored in the form of varying sensing coverage areas of individual nodes. The authors study the target detection problem with the help of a line-set intersection problem and provide bounds on the detection probability. The authors show that sensor mobility can provide increased coverage and detection probability of targets.

2.2 Clustering

We provide an overview of clustering-related coverage studies in WSNs and clustering phenomena studied in other areas.

Clustering by Choice in WSNs. This section surveys related work on clustering to achieve energy efficiency, better organization and to suit the application needs in WSNs. First we review clustering by choice to achieve energy efficiency in WSNs. One of the earliest literature on clustering in WSNs is LEACH (Low Energy Adaptive Clustering Hierarchy) [Heinzelman et al. 2002], where cluster formation is designed to achieve prolonged network lifetime by local data processing, rotation of the cluster head (CH) position among nodes and low energy MAC access. The probability of becoming a CH is a function of the node energy level relative to the total residual energy level in the network. Since the CH is responsible for data aggregation and data transmission to the sink (communication and computation tasks) that are more energy-intensive than the tasks of sensing and communication to a CH that occur at a regular node, rotation of the CH position relative to node energy levels achieves distribution of the computation and communication as well as cluster maintenance tasks of the CH. Power balancing in clustered WSNs has been studied in Shu et al. [2005] in terms of maximizing the coverage time of CHs. In Younis and Fahmy [2003], the authors propose HEED (Hybrid Energy Efficient Distributed clustering), a distributed clustering protocol that uses residual node energy, cluster size and available power levels at a node for communication with the CH as parameters for CH selection and cluster formation. Clustering has also been employed to achieve better organization in WSNs. In Cha et al. [2007], the authors propose a clustering algorithm, SNOWCLUSTER that creates a 3-tiered hierarchy of nodes, clusters and regions. Clustering has also been studied based on application needs, for example, grouping nodes into clusters based on higher correlation in sensed data resulting form geographical proximity. In Vlajic and Xia [2006], the authors provide analytic results to validate the need for clustering in WSNs. They show that when the monitored phenomenon can be grouped as *isoclusters* (areas within the sensing field that have similar values of the monitored phenomenon), clustering nodes to lie within such isoclusters helps in achieving network objectives such as prolonged network lifetime.

Clustering in Other Areas: Clustering has been studied in other areas. The use of models to study environmental phenomena has been suggested in Cox and Isham [1980], Kingman [1993] and has been studied for modeling air temperature and rainfall in Onof et al. [2004], Kilsby et al. [2007], and Bilgin and Camurcu [2005]. A detailed study of applications of PCPP models to ecological modeling can be found in Cox and Isham [1980]. Various ecological models display clustering in the spatial and temporal domains. Many spatial cluster processes have been described and modeled in Neyman and Scott [1972]. Also, realistic deployment models result in clustering of nodes. One of the particular strengths of this article is that it can be used to pre-determine the degree of coverage required to study an ecological model that has been shown to display clustering [Onof et al. 2004; Kilsby et al. 2007; Bilgin and Camurcu 2005; Neyman and Scott 1972]. Since the spatial distribution of the phenomenon demonstrates clustering, deploying random topologies in a clustered pattern can help in effectively isolating and capturing the phenomenon.

2.3 Coverage Processes

The focus of coverage studies in WSNs deployed for environmental modeling has been on random topologies, and the model of choice for the topology is that of Poisson distributed nodes. The coverage properties of random topologies in a WSN have been studied with the help of coverage processes previously in Saito et al. [2008]. In Saito et al. [2008], the authors study the coverage in a Poisson process of node topology, where the WSN is deployed for target detection. They consider the scenario of Boolean model of coverage, where a target point is considered to be sensed if it lies within the coverage area of a sensor node and considered to be un-sensed otherwise. They extend this analysis to the interesting case of an expanding target, for example detecting oil spills or infected animals in a herd. Variations of the Poisson process in the study of coverage in WSNs have also been considered in Manohar et al. [2006]. The coverage problem has also been studied in terms of a set intersection problem using results from integral geometry in Lazos and Poovendran [2006].

Variations of the PCPP process, for example, Cox processes can be used to study networks of mobile sensor nodes. In this article, we use the results from coverage processes, specifically, Poisson cluster point processes (PCPP) to study random topologies of WSNs. For fixed networks of sensor nodes as we consider in this article, the PCPP process can accurately model the coverage properties of the WSN. Most real world deployments of random WSNs display clustering due to the deployment phenomenon and hence a PCPP is more appropriate to study the coverage properties in such topologies. Though the assumption of a Boolean model (Poisson process) offers ease of calculation, it is not reflective of the coverage and connectivity properties of a real-time random topology. The key assumptions in our article are the following:

— We study naturally clustered networks, where clustering is not facilitated by choice. We use the PCPP model to study coverage in naturally clustered networks. Rather, it is a consequence of the deployment process in large-scale, dense networks created by scattering/spraying of nodes.
— To investigate clustering, we leverage the concept of a Poisson cluster point process (PCPP) [Hall 1988] as opposed to the widely used Boolean model (Poisson process).

3. COVERAGE PROPERTIES OF CLUSTERED WSNS

3.1 Problem Statement and Motivation

The properties of coverage and vacancy in a 2-dimensional region $\mathcal{R}$ due to clustering processes vary significantly from that of the widely used Boolean model of node placement. As we show in the rest of this article, clustering results in increasing the expected vacancy per unit area of the region $\mathcal{R}$. We study the properties of a Poisson Cluster Point Process (PCPP) [Hall 1988], which possesses a degree of clustering not present in a Boolean model. Salient attributes of using PCPP models to study coverage in random topologies are evident, and help us to see the need for using cluster point processes. Our research is the first known work that analyzes the interaction between points of the coverage process resulting from a clustered topology. In this section, we present the problem statement for coverage in clustered topologies and study it with PCPP processes.

We study the problem where for a given placement pattern for wireless sensor nodes in a deployment region, we have to find the probability that every point in the deployment region is covered by m nodes with probability p, where $0 < p < 1$. We assume that the deployment process results in k-redundancy of nodes, and we find the probability that a given point (x, y) in the region is covered by m-redundancy, where $m < k$. The decrease in the degree of redundancy can be attributed to power management that turns off redundant sensors or to sensors that have run out of battery energy or suffered device failure. We assume nodes are randomly turned off to obtain m-redundancy of active coverage in the k-redundancy of nodes deployed at any given point. However, this work can be easily extended to incorporate power control algorithms for switching nodes in power-saving states.

3.2 Definition of a PCPP Process

Before we describe the PCPP model, we provide a brief introduction to the notation of the commonly used Poisson process.

A process $\mathcal{P}$ is said to be a stationary or homogenous Poisson point process $\mathcal{P}$ with intensity λ if:

(1) the number of points ξ_i in any Borel subset [Hall 1988] $\mathcal{S}$ of $\mathcal{R}$ is Poisson distributed with mean $\lambda \|S\|$ and
(2) the numbers of points in any number of disjoint Borel subsets are independent random variables.

A process is called stationary if and only if the function $\lambda(x)$ is constant almost everywhere. For the rest of this article, we denote $\lambda(x)$ as λ. A Boolean model in k-dimension Euclidean space is just the coverage pattern created by a Poisson-distributed sequence of random sets. Specifically, let $P \equiv \{\xi_i, i \geq 1\}$ be a stationary Poisson process of intensity λ in $\mathcal{R}$, the points ξ_i being indexed in any systematic order. Let $S_1, S_2, \ldots$ be independent and identically distributed random sets, independent of $\mathcal{P}$. Then

$$C \equiv \{\xi_i + S_i, i \geq 1\} \tag{1}$$

is a Boolean model, where the Poisson process $\mathcal{P}$ drives the Boolean model and the shapes S_i are said to generate the model.

Definition of a PCPP Process. We now introduce the PCPP model for clustered topologies. In a PCPP $\mathcal{P}$, the points of $\mathcal{P}$ are the *children* of parent points [Hall 1988]. The parent points form a stationary Poisson process $\mathcal{P}'$ in $\mathcal{R}$ with intensity λ_0 given by $\{\eta_i, i \geq 1\}$. Each parent point produces progeny represented by points in space in an independent and identically distributed manner. The number N_i of progeny born to a parent point η_i which is independent of i. Let $p_n = \Pr(N = n)$.

The jth child of η_i is the point $\eta_i + \eta_{ij},\ \ 1 \leq j \leq N_i$. Conditional on all N_i and η_j and on the locations of all progeny of all parents other than the ith, the vectors η_{ij} are independent and identically distributed with density *h(x)* defined on $\mathcal{R}$. The points $\left\{\eta_i + \eta_{ij},\ \ i \geq 1,\ 1 \leq j \leq N_i\right\}$ comprise a PCPP $\mathcal{R} \equiv \left\{\xi_i,\ \ i \geq 1\right\}$. Since the points η_i comprise a stationary Poisson point process $\mathcal{P}'$ with intensity λ_0, the number $\mathcal{M}$ of points η_i which have at least one child lying inside $-\mathcal{S}$ must be Poisson distributed with mean λ_0, where $-\mathcal{S}$ is the complement of the region S in R. We assume the particular case where the progeny $\mathcal{N}$ has a Poisson distribution with mean μ. The total density of random sets in $\mathcal{C}$ per unit content of R^2 equals $\mu\lambda_o$, which we henceforth call the clump factor. This is also the average intensity of the driving point process $\mathcal{P}$. A point (x, y) in the deployment region is said to be covered if the point lies within the circular sensing region of a node. We assume that the sensing disk is a closed set. The total density of random coverage disks in the coverage process per unit area of the deployment region equals the clump factor times the intensity of distribution of the original Poisson process for parent points. Therefore, the expected vacancy of the Boolean model has the same type of set and the same density of sets in the Poisson cluster point process as the Poisson point process. Hence, we extend this definition to obtain the vacancy in PCPP process.

In dense networks, for any point that is sensed by k ($k>1$) nodes, it results in k-redundancy. However, power management or node failure can result in decreased quality of sensing such that any m, $(m<k)$ sensors are sensing that point resulting in actual m-redundancy of coverage in the region. A Boolean model of coverage in 2-dimensional Euclidean space $\mathcal{R}$ is just the coverage pattern created by a Poisson-distributed sequence of random sets. Similarly, k-connectivity exists when for any given two nodes, a and b, multiple k paths exist between them. Dense networks through their topology create conditions for both k or m-redundancy and connectivity. A systematic evaluation of the maximum likelihood estimation for a PCPP and demonstration of the convergence of the procedure with a sample small data set has been presented in Castelloe and Zimmerman [2002]. We refer the interested reader to Castelloe and Zimmerman [2002] for an analysis of empirical quantification of similarity in actual node distribution to the one predicted by the PCPP. In the rest of this article, we will obtain analytical solutions for the probability of coverage in both k- or m-redundancy and expected number of connected sensors in the WSN of PCPP process.

3.3 Vacancy Estimation in a Clustered Network

We obtain the analytical solution of the coverage in a clustered topology of nodes in a 2-D deployment region. Suppose λ is the intensity of the Poisson point process for nodes. Then, the expected vacancy within a region $\mathcal{R}$ denoted by $E(V)$ [Hall 1988] is

$$E(V) = \|R\| \exp\left(-\lambda \|S\|\right), \tag{2}$$

where λ is the intensity of the Poisson point process for nodes, $\|R\|$ is the area of the deployment region and $\|S\|$ is the Lebesgue measure of the node coverage disk S. This vacancy denotes that part of deployment region that is not covered by any node. Let $v(S_o)$ be the mean number of coverage disks of nodes intersecting any fixed coverage disk S_o in the deployment region $\mathcal{R}$. Let $\mu(S_o, S)$ be the mean area of the region into which centers of coverage disks intersecting S_o must fall. Consider the set $\mathcal{A}$ of all points $\mathbf{x}$, $x \in R$ such that $\mathbf{x}+ S$ intersects S_o. The expected value of the quantity μ(So; S) is the expected number of random sets intersecting S_o and is used to estimate the vacancy. If the coverage disks distributed as S are centered at points of a stationary point process with intensity λ, then expected number of random sets intersecting S_o equals

$$v(S_o) = \lambda E\left\{\int_R f(\mathbf{x}, S)\, d\mathbf{x}\right\}, \tag{3}$$

where $f(\mathbf{x}, S)$ is a coverage function denoted as

$$f(\mathbf{x}, S) = \begin{cases} 1 & \mathit{if}\ (\mathbf{x}+S)\cap S_0 \neq \phi \\ 0 & \mathit{otherwise} \end{cases}$$

Hence,

$$v(S_o) = \lambda E\left\{\mu\left(S_o, S\right)\right\}.$$

Let $\alpha = E\|S\|_2$ be the area of the coverage disk and $\beta = E\left(\|\partial S\|_1\right)$ denote the perimeter of the coverage disk. Then, the probability that no disks intersect

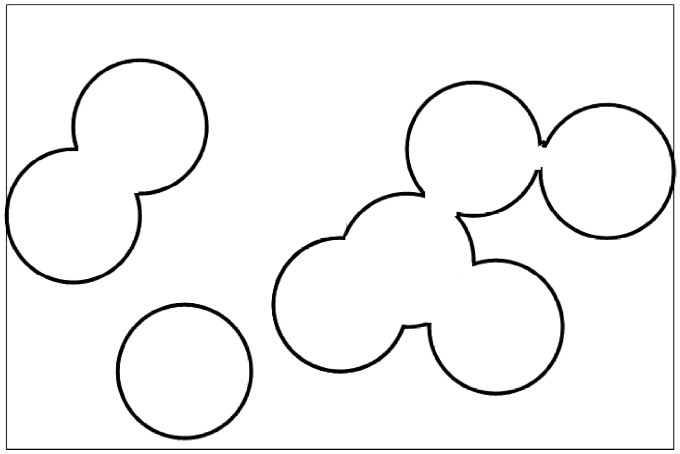


Fig. 3. Pattern of clumps of coverage areas of sensor nodes and coverage voids due to absence of sensors.

the coverage disk S_o is

$$e^{-v(S_o)} = e^{-\alpha\lambda} \exp[-\lambda\{\|S_o\|_2 + (2\pi)^{-1}\|\partial S_o\|_1\beta\}]. \tag{4}$$

In this article, we study clustering of nodes in random deployments with the help of the PCPP process. Modifying this analysis to account for the PCPP process, we substitute the intensity λ for the intensity of the PCPP process $\mu\lambda_0$, which we call the clump factor, the probability that no disks intersect the coverage disk S_o is

$$\lambda\{\|S_o\|_2 + E\|S\|_2 + (2\pi)^{-1}\|\partial S_o\|_1 E(\|\partial S\|_1)\}. \tag{5}$$

Hence, in a PCPP with average intensity $\mu\lambda_0$, the expected vacancy is given by [Hall 1988]

$$E(V) = \|R\|\, e^{-\mu\lambda_0} E[\exp\{-\lambda_o\mu\|S\|_2 + (2\pi)^{-1}\beta\|\partial S\|_1\}], \tag{6}$$

where S is a disk of sensing radius r. E($\|S\|_2$) is area of S and is equal to $\pi E(r^2)$. We use α to denote $E(\|S\|_2)$. $E(\|\partial S\|_1)$ is perimeter of S and is equal to $2\pi E(r)$. We use β to denote $\pi E(r^2)$.

3.4 Impact of Clustering on Coverage Properties

In this section, we obtain the properties of clumping of the nodes scattered as a PCPP. This information can be used to obtain the redundancy in deployment, and then use the redundancy to selectively turn sensors in the sleep mode for power management. A PCPP model is a pattern of voids and clumps. We denote *clumps* as the coverage area of a pattern of redundant nodes sensing a given point in the region, the shape of which is shown in Figure 3. We define the m-redundancy for a given point as the order of the clump, where the order of the clump is the expected area of a clump divided by the number of coverage disks that overlap to form the clump. In a perfect Poisson process of nodes, nodes are deployed throughout the sensing region to achieve uniformity of coverage properties. Though this assumption offers ease of calculation, it does not accurately model the increased vacancy resulting from clustering. Since a random deployment can easily result in cluster formation of clumps (redundancy) and voids (sensing holes) in the region, the PCPP process readily yields the m-redundancy resulting from clustering. The voids denote coverage voids due to sensors that are no longer sensing due to possible battery energy exhaustion, sensor nodes in power-saving sleep states or sensors that have stopped working due to device failure. Since k-redundancy denotes the number of sensors present at any point in the deployment region, it also contains the voids due to absence of sensor nodes at that point. However, the m-redundancy studies the coverage properties in the presence of newly generated voids due to battery failure or power management.

Since the coverage disks might overlap due to the clustering process, one way to calculate the m-redundancy is by using the Euler characteristic of a clump and using it to find the visible curvature of the clump. The Euler characteristic of a figure is a simple topological invariant that describes the properties of a shape [Kinsey 1997]. It is a number that describes the topological properties by considering the number of clumps and voids in the figure.

Once again, we start with the derivation for a Poisson model for the nodes and then extend it to a PCPP model to reflect the clumping. The Euler characteristic of a figure equals the number of disjoint components minus the number of voids. In a deployment region, the Euler characteristic is thus the number of isolated (disjoint) coverage disks (those that do not intersect with other disks due to overlapping coverage areas) minus the number of areas that are vacant and bounded by the perimeters of the coverage disks of surrounding nodes. The Euler characteristic is helpful in determining if the active coverage can be increased by deploying mobile sensors, or by turning nodes in the *on* state.

Assuming that the coverage areas are isotropic, we now obtain the expected curvature per unit area resulting from the Euler characteristic of the Boolean model. Since the intensity of coverage disks is Poisson with intensity λ, the expected visible curvature of the coverage disks per unit area is

$$2\pi\lambda\chi e^{-\alpha\lambda}, \tag{7}$$

where $\chi = x(S)$ denotes the expected value of Euler characteristic of the set S. This value of expected curvature does not consider uncovered crossings. For our model of coverage disks of sensor nodes, uncovered crossings denote the intersection of perimeter of two coverage disks, that is not covered by another coverage disk. If we know that the coverage disks have smooth boundaries, then any discontinuities in the Boolean model can be identified as an uncovered region (sensing void) which can be removed from the calculation of total curvature. The expected total curvature of clumps in the deployment region is given by

$$\chi\lambda, \tag{8}$$

The expected total curvature from uncovered crossings of random coverage disks centered in the deployment region R is given by,

$$-\frac{1}{2}(\beta\lambda)^2 e^{-\alpha\lambda}. \tag{9}$$

Thus the expected number of clumps minus voids in the deployment region of Boolean model of nodes following the Poisson distribution is given by subtracting (9) from (8),

$$2\pi(\chi\lambda - (4\pi)^{-1}(\beta\lambda)^2)\, e^{-\alpha\lambda}. \tag{10}$$

So the expected number of clumps minus voids (m-redundancy) in the deployment region with PCPP distributed nodes is a straightforward extension of the above equation by substituting the clump factor $\mu\lambda_0$ for λ. The expected m-redundancy per unit area in the coverage in a deployment region with PCPP distribution of nodes is given by

$$[\chi\mu\lambda_0 - (4\pi)^{-1}(\beta\chi\lambda_0)^2]e^{-\alpha\chi\lambda_0}. \tag{11}$$

For a 2-D region $\mathcal{R}$ populated with PCPP nodes, expected area of coverage α, perimeter of node coverage β, and binary function for coverage χ denoting the presence/absence of a node that covers a point (x, y), and λ is the intensity of the Poisson process of Boolean model denoted by $C(\delta, \lambda)$, where δS is the distribution of coverage areas and η, δ and λ are related as $\eta = \delta^2\lambda$.

Finally, we obtain the expected number of clumps of coverage disks of sensors per unit area (denoting the k-redundancy in the region). The expected number of coverage disks in a Boolean model C that intersect a fixed coverage disk S is given by Eq. (3). The probability that no sets from C intersect S is given by

$$\exp\left\{-\frac{1}{2}v(S)\right\}.$$

Hence, the mean number of coverage disks per unit area with no sets intersecting them is given by the expected number of clumps per unit area is given by

$$v_1 = \lambda e^{-0.5\alpha\eta} E[\exp\{-0.5\eta(\|S\|_2 + (2\pi)^{-1}\beta\|\partial S\|_1)\}]. \tag{12}$$

Once again, we extend this analysis to that of the PCPP model for clustered nodes and present the formula for the k-redundancy in a region with the clump factor given by

$$v_1 = \mu\lambda_0 e^{-0.5\alpha\eta} E[\exp\{-0.5\eta(\|S\|_2 + (2\pi)^{-1}\beta\|\partial S\|_1)\}]. \tag{13}$$

Next, we obtain the number of neighbors of a node to obtain the connectivity properties of PCPP process of nodes. The expected number of neighbors of a node is a useful measure of the connectivity of the network. The expected number of neighbors of a node N is given by the mean number of coverage area disks of other nodes intersecting a given node's coverage area (Eq. (3)). Modifying it for the PCPP process, the number of neighbors is given by

$$N = \mu\lambda_0\{\|S\|_2 + \alpha + (2\pi)^{-1}\beta\,\|\partial S\|_1\}. \tag{14}$$

With N neighbors, the probability that a sensor has at least k neighbors, where $1 \leq k \leq N$ is given by 1-Pr (no neighbors) $= 1-$ Pr$(N = 0)$. Here, $N = 0$ implies

$$N = \mu\lambda_o\{\|S\|_2 + \alpha + (2\pi)^{-1}\beta\,\|\partial S\|_1\} = 0. \tag{15}$$

This implies that a) $\lambda_0 = 0$ representing only one sensor in the entire region or b)$\{\|S\|_2 + \alpha + (2\pi)^{-1}\beta\,\|\partial S\|_1\}$ denoting that the coverage area of a node is a point. This verifies the connectivity properties of the clustered WSN.

4. APPLICATIONS

In this section, we describe applications of the measures of coverage and connectivity obtained in this article for clustered topologies.

4.1 Density Control

Random deployment of dense networks through node scattering or spraying has the advantage of eliminating the overhead of planning for deterministic placement of nodes. However, the clustering resulting from such topologies creates redundancy and sensing holes in the deployment region. The knowledge of redundancy in node deployment and node coverage helps in implementing network density control. Network density control satisfies sensing objectives such as extending network lifetime and increasing the reliability of operation by selectively increasing the density of awake sensors. With the help of the analytical framework described in this article, the k-redundancy and m-redundancy can be used to determine parameters such as level of active coverage for various monitoring applications. Knowing the redundancy can also help us deploy additional sensors in regions that need increased coverage for target tracking applications.

4.2 Routing

Energy efficiency in WSNs can be implemented in various ways such as routing, transceiver efficiency, power saving states for the node and efficient data processing. The expected number of neighbors of a node can be used to compute optimal routing tables at individual nodes for objectives such as lower latency of data transfer from nodes to the central sink, or finding the most energy efficient path from the set of known neighbors.

4.3 Extensions to Tiered Architectures for WSNs

Although this article studied coverage and connectivity in a WSN of homogenous nodes, the results can readily be extended to multi-tiered models in WSNs. For example, in heterogeneous networks of sensor nodes and stronger processing nodes, the coverage results can be used to optimize the ratio of number of nodes to the number of processing nodes for optimum cluster size. The knowledge of coverage in clustered topologies can also be used to set up optimization problems of maximizing coverage versus network lifetime in both homogeneous and heterogeneous [Machado et al. 2010] networks.

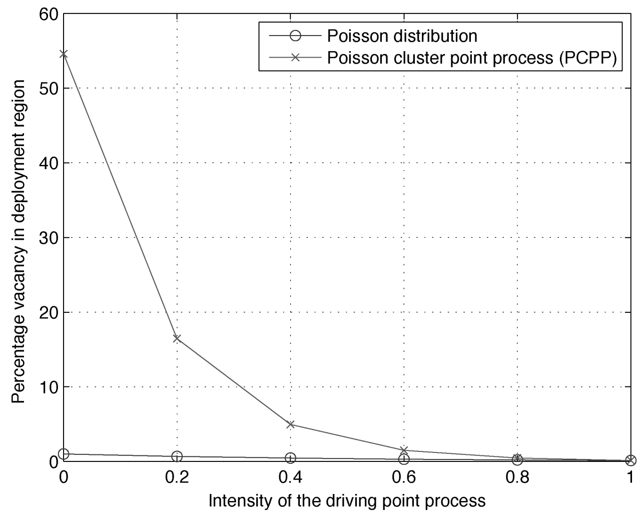


Fig. 4. Expected vacancy in Poisson and PCPP distributions of 300 nodes.

4.4 Edge Effects in Channel Access and Routing

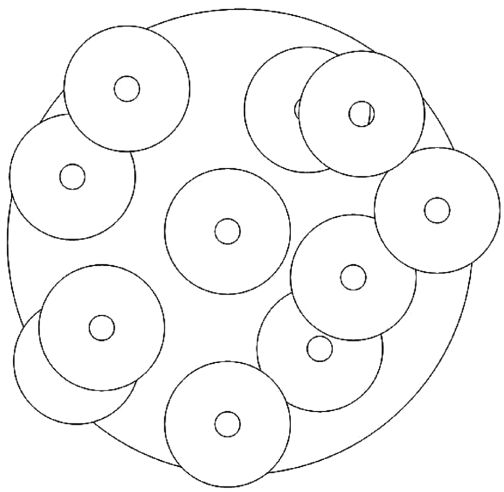
Dense randomly deployed networks exhibit edge effects in channel access, due to known problem of lesser interference [Durvey et al. 2008] from fewer neighbors in the edge of the deployment region. Similar edge effects occur at the edges of cluster coverage in WSNs that employ clustering to achieve lower redundancy in data processing and extension in network lifetime. Specifically, edge effects occur in routing from border nodes to cluster-heads, where the goal is to reduce the packets from edge nodes from transmitting redundant data to the cluster-heads for processing.

5. PERFORMANCE EVALUATION

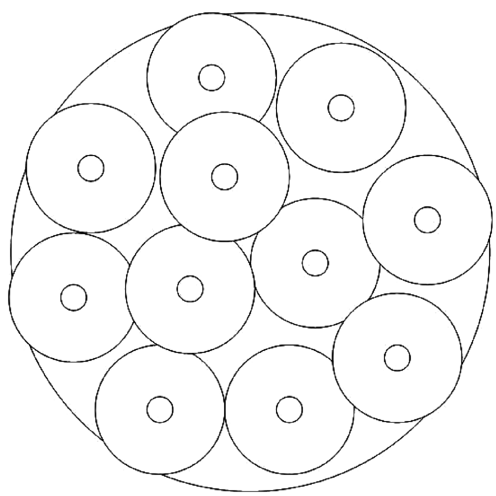
The objective of this section is to study the variation of the following coverage properties: vacancy, m-redundancy and the expected number of neighbors of a node as a function of the intensity of the driving PCPP and Poisson point processes. The vacancy estimation is compared in both Poisson and PCPP processes, and the m-redundancy is estimated as the expected number of clumps minus voids in the deployment region. Both m-redundancy and expected number of neighbors of a node increase with intensity of the driving process, while vacancy decreases. These results are shown in Figures 4–8.

In all our simulations, we use the following parameters. The number of nodes in the deployment region varies according to the intensity of the PCPP process from 2 to 20 in steps of two. The sensing radius is varied from 0.01 to 0.7 to study the impact on redundancy.

To evaluate the expected vacancy in a deployment region, we simulate a WSN of 300 nodes as a function and plot the variation vacancy as a function of the intensity of the deployment process. We use the sensing range of a circular coverage disk with radius equal to 6m. The deployment region is assumed to be circular with radius of 1000m.



(a) The vacancy in PCPP model is 17% of deployment region.



(b) The vacancy in Poisson model is 2% of deployment region.

Fig. 5. Vacancy comparison in PCPP and Poisson models with intensity equal to 0.2. This figure shows that clustering can be accurately modeled using a PCPP instead of a Poisson process.

Figure 4 shows the expected vacancy in a region, assuming the Poisson process and the PCPP distribution of node placement and shows the decrease in vacancy with increase in the intensity of nodes in the region. We see that the PCPP process exhibits higher vacancy in the region than a Poisson process of node placement. The vacancy decreases with increase in the intensity of nodes. This vacancy can be further decreased by increasing the degree of clustering. From Figure 4, we can also see that the PCPP model captures different coverage property compared to that captured by the Poisson distribution, especially when the node density is low.

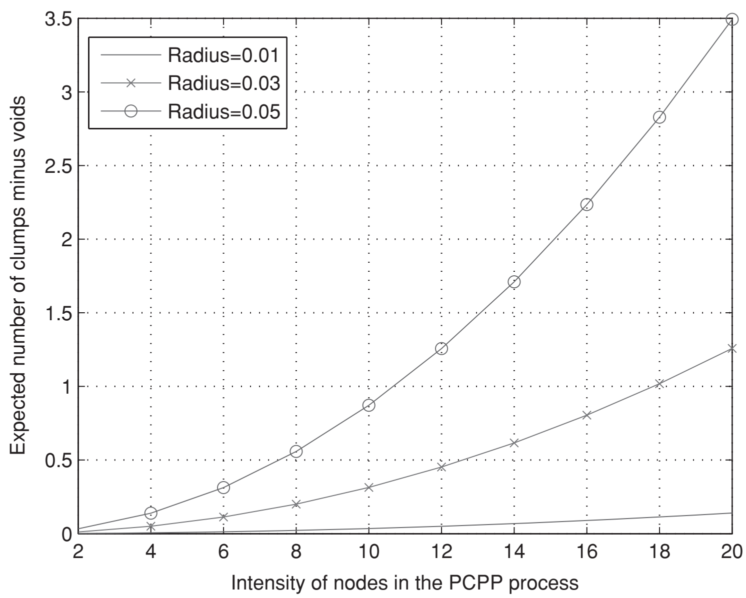


Fig. 6. Expected m-redundancy in node coverage.

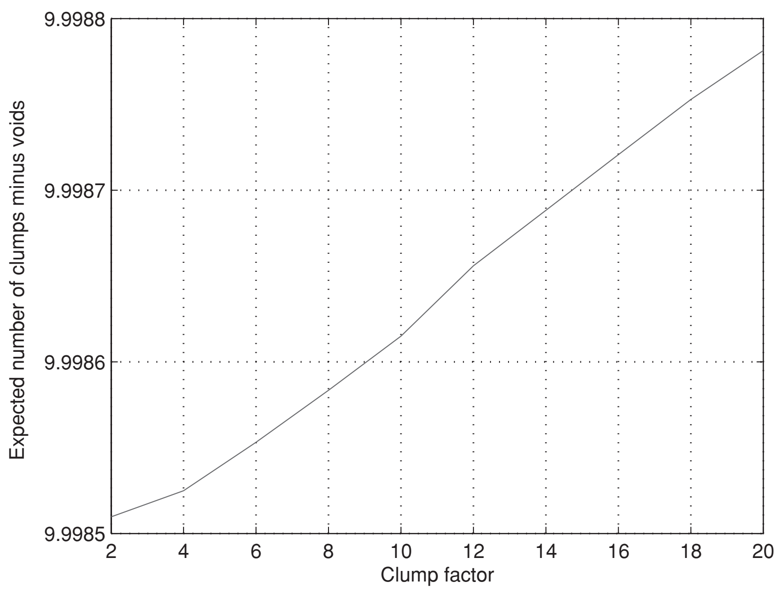


Fig. 7. Expected m-redundancy as a function of the clump factor.

We show a representative case in Figures 5(a) and 5(b). Figure 5 shows the positions and coverage of a sensor network, whose process intensity is 0.2. The vacancy in a PCPP modeled topology as seen from Figure 5(a) is approximately 17%, while Poisson model can only report 1% vacancy. The clustered topology is evident in the grouping of nodes. Figure 5 shows the vacancy in

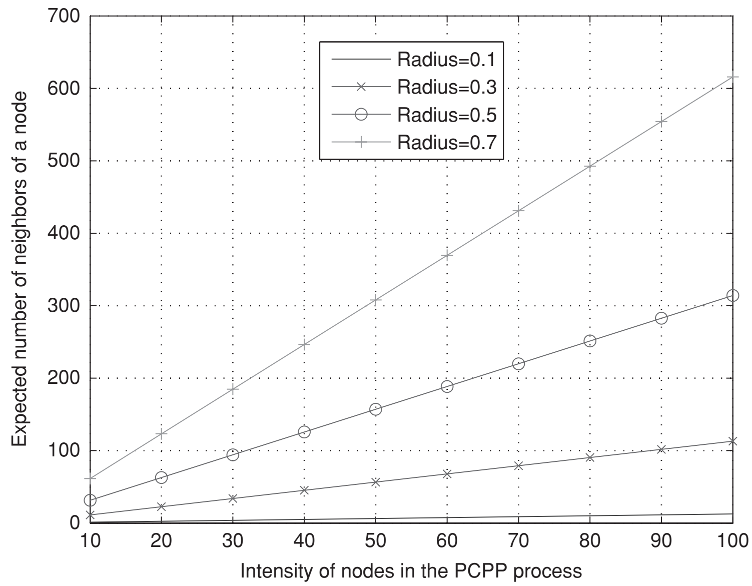


Fig. 8. Expected number of neighbors of a node.

Poisson and PCPP topology of nodes. As seen from Figure 3, the vacancy in a Poisson topology is about 2% of the deployment region and is depicted in the Figure 5(b). This shows that coverage in random deployments of nodes are accurately modeled by PCPP rather than the Poisson model.

Figure 6 shows the number of clumps minus voids (m-redundancy) in a region with PCPP nodes, where $m<k$ denotes the redundancy that can be increased to a level k. As the intensity of nodes increases, the m-redundancy increases in the region.

Figure 7 shows the variation of the m-redundancy with the clump factor. As the clump factor increases, m-redundancy has an almost linearly increasing relationship with the clump factor.

Figure 8 shows the number of neighbors of a node in a WSN with PCPP process of node distribution. As expected, the number of neighbors increases with the intensity of the PCPP nodes. These results draw attention to the need for realistic simulation of the placement process of nodes in WSNs by considering the natural tendency of clustering in a random deployment process in the deployment region.

6. CONCLUSION

We have given an introduction to coverage properties in clustered networks of wireless sensor nodes. We looked at coverage in terms of the expected vacancy, m-coverage and k-coverage, $(k < m)$ where coverage in m-redundancy indicates the coverage that can be decreased to that achieved by k-redundancy by power management. Having built up the theory for clustered nodes in a deployment region, we analyze the properties of coverage in a realistic scenario with varying intensity of clustering in the deployment region. These results started with an initial guess to the properties of coverage in clustered networks, where clustered networks have larger vacancy in the deployment region that has been verified by simulation results. Although this article studies clustered networks of wireless sensor nodes, it can easily be extended to include a structured analytical model for ad hoc networks of mobile nodes. In general, this analysis replaces the often-used notion of coverage in a Poisson deployment of nodes. Our future work incorporates exploiting the coverage properties of clustered networks for adaptive density through power management schemes for WSNs.

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Received April 2009; revised November 2009 and January 2010; accepted January 2010