Opportunistic Encryption for Robust Wireless Security

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Wireless Link Properties

- Wireless link states randomly vary with time and space.
  - Bursty errors (e.g., fading channels).
  - Random errors.
- Effect of link errors.
  - Bit flips.
  - Bit deletion and bit insertion
Overview of Encryption Schemes

• Block cipher.
  - Fixed blocks of plaintext encrypted to same blocks of ciphertext using an encryption key.

• Stream cipher.
  - Block size equal to one bit.

• Symmetric key encryption.
  - Same key for encryption and decryption.

• Public key encryption.
  - Encryption and decryption keys are different.

Avalanche Criterion

• Block ciphers satisfy avalanche criterion.
  - One input bit change causes (on an average) one half of output bits to be in error.
  - Removes statistical correlation between input and output bits.
  - Cryptanalysis is made harder (security increases).

• One bit error in received ciphertext block due to wireless link state implies:
  - Multiple bit errors in decrypted plaintext block (throughput decreases).

Trade-off in security vs. wireless throughput
Popular Symmetric Block Cipher Modes

- **Electronic codebook mode (ECB).**
  - Every plaintext block is encrypted independently: \( c_i = E(K, p_i) \)
  - One bit error in ciphertext block causes error propagation in decrypted plaintext block.
  - No loss of synchronization if integer multiples of blocks are lost.
    - Explicit re-synchronization is needed otherwise.
  - Identical plaintext is mapped to identical ciphertext

- **Cipher block chaining (CBC).**
  - Before encryption plaintext block is XORed with previous ciphertext block:
    \[
    c_i = E(K, p_i \oplus c_{i-1})
    \]
  - An erroneous ciphertext block results in two plaintext blocks in error.
  - If an integer number of blocks are in error then one additional plaintext block is in error before re-synchronization.
    - Explicit re-sync is needed otherwise.
  - Identical plaintext is encrypted to non-identical ciphertext.
• **Cipher feedback mode (CFB).**
  - Operates on blocks of size \( j < b \) (plaintext block size).
  - Plaintext is XORed with output of an encryption algorithm.
  - Feedback to encryption algo. are previous ciphertexts.
  - An erroneous ciphertext block distorts corresponding decrypted plaintext block and following \( \lceil \frac{b}{j} \rceil \) blocks.
  - If number of lost bits is an integer multiple of \( j \) then \( \lceil \frac{b}{j} \rceil \) additional plaintext block are distorted before re-synchronization.
    - Explicit re-sync is needed otherwise.

• **Encryption is performed more often.**
  - Hardware throughput is low.
  - Higher power consumption—not desirable for low power mobile devices.

• **Output feedback mode (OFB).**
  - Operates on blocks of size \( j < b \) (plaintext block size).
  - Plaintext is XORed with output of an encryption algorithm.
  - Feedback to encryption algo. are previous outputs.
  - One bit error in ciphertext causes single bit error in decrypted plaintext; no error propagation.
  - If some bits are lost explicit re-synchronization needed.

• **Encryption is performed more often.**
  - Hardware throughput is low.
  - Higher power consumption—not desirable for low power mobile devices.
Trade-offs...

- High (hardware) throughput/low power consumption modes (ECB, CBC)
  - Bit error propagation after decryption.
  - Reduce network throughput.
- Low (hardware) throughput/high power consumption modes (1-CFB, OFB)
  - No bit error propagation.
  - Higher network throughput.

No single encryption mode is the clear winner.

Error Propagation vs. Encryption Block Length

- $P_b$ : wireless link bit error rate
- $P_{b, post}$ : post decryption bit error rate
- $N$ : encryption block length

\[ P_{b, post} \approx \frac{N}{2} P_b \]
**Throughput vs. Security Trade-off**

- Throughput, \( D = R(1-NP_b) \) bits/sec;
- \( R \): Transmission rate.
- Security level against brute force attack is \( \sim 2^N \)

\[ P_b = 10^{-2} \]

BPSK modulation, Rayleigh distributed, flat fading channel.

**Approaches to Error Control**

- **Forward error control (FEC) code.**
  - FEC may fail due to error propagation effect.
- Reduce diffusion in encryption.
  - Reduced error propagation.
  - Reduced security.
- **Interleaving.**
  - Causes delay depending on interleaving depth.
- **ARQ protocols.**
  - Overhead, high delay bandwidth product...
- **Opportunistic encryption.**
  - Optimize encryption block size based on security and wireless link state conditions.
  - Optimally trade-off security for throughput.
Scenarios

- Case 1: Exact wireless channel signal to noise ratio (SNR) known.

- Case 2: Only current average SNR and probability distribution of randomly time-varying SNR also known.

- Case 3: A Markov channel model is known for channel/link states.

Security and Adversary Models

- $Q_N$: Set of available encryption block lengths

\[ S_i(N_i) = \frac{\log_2 N_i}{S_{\text{max}}} \quad \text{and} \quad S_{\text{max}} = \log_2 \left( \max_{N_i \in Q_N} N_i \right) \]

Average Security: \[ \bar{S} = \frac{1}{n} \sum_{i=1}^{n} S_i(N_i) \]

$\Phi_i(N_i) = \Pr(\alpha \geq N_i)$ where $\alpha$ is the "attacker success prob."

Average vulnerability: \[ \Phi = \frac{1}{n} \sum_{i=1}^{n} \Pr(\alpha \geq N_i) \]
Case 1: Exact Channel SNR Known

- Channel SNR at ith time slot: $\gamma_i$

Throughput (of ith time slot): $D_i(\gamma_i, N_i) = R_i \left(1 - N_i P_b(\gamma_i)\right)$

Required Security: $S_{req} = \frac{1}{n} \sum_{i=1}^{n} S_i(N_i)$

The Lagrangian of the problem:

$$C^{(a)} = \frac{1}{n R_{max}} \sum_{i=1}^{n} D_i(\gamma_i, N_i) + \frac{\lambda}{n} \sum_{i=1}^{n} S_i(N_i)$$

Optimum encryption block length:

$$N_i = \left(\prod_{i=1}^{n} \left[R_i P_b(\gamma_i)\right]\right)^{1/2} e^{\left(S_{max} S_{req}\right) \log_2}$$

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**Goodput Gain**

Performance with fixed rate (BPSK)
Case 1: Exact Channel SNR Known (II)

\[ \Phi_i(N_i) = \Pr(\alpha \geq N_i) \] where \( \alpha \) is the "attacker success prob."

Allowable vulnerability level of the message, \( \Phi = \frac{1}{n} \sum_{i=1}^{n} \Pr(\alpha \geq N_i) \)

\[ \Phi_{\text{max}} > \Phi_i > \Phi_{\text{min}}, \quad \text{for all } i \]

1. Exponential
   \[ \Pr(\alpha \geq N_i) = e^{-kN_i} \]
   Optimum solution resembles
   “Water-filling” algorithm

2. Uniform
   \[ \Pr(\alpha \geq N_i) = \frac{N_{\text{max}} - N_i}{N_{\text{max}} - N_{\text{min}}} \]
   Optimum solution is found
   Using fractional knapsack algorithm

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Exponential distribution: \( \Pr(\alpha \geq N_i) = e^{-kN_i} \)

Throughput (of \( i \)th time slot): \( D_i(y_i, N_i) = R(1 - N_iP_i(y_i)) \)

The Lagrangian of the problem:
\[
C = R \sum_{i=1}^{N} \left( 1 + \frac{1}{k} \ln \Phi_i \right) + \nu \left( \sum_{i=1}^{N} \Pr(\alpha \geq N_i) - \Phi \right) + \sum_{i=1}^{N} \lambda_i (\Phi_i - \Phi_{\text{max}}) + \sum_{i=1}^{N} \mu_i (\Phi_{\text{max}} - \Phi_i)
\]

Karush Kuhn Tucker Conditions:
\[
\frac{\partial C}{\partial \Phi_i} = 0 \Rightarrow \Phi_i = -\frac{RP}{k(\nu + \lambda_i - \mu_i)}
\]
\[
\lambda_i (\Phi_i - \Phi_{\text{max}}) = 0, \mu_i (\Phi_{\text{max}} - \Phi_i) = 0 \quad \text{(complementary slackness)}
\]
Case I: $\lambda_0 = 0, \mu_0 = 0 \Rightarrow \Phi_{\text{max}} > \Phi_i = -\frac{RP}{k^v} > \Phi_{\text{min}}$

Case II: $\lambda_i = 0, \mu_i \neq 0 \Rightarrow \Phi_i^2 = \Phi_{\text{max}} = -\frac{RP}{k(v + \lambda_i)} \Rightarrow \lambda_i = -\frac{RP}{k\Phi_{\text{max}}} - v$

Case III: $\lambda_i \neq 0, \mu_i = 0 \Rightarrow \Phi_i = \Phi_{\text{min}} = -\frac{RP}{k(v + \mu_i)} \Rightarrow \mu_i = -\frac{RP}{k\Phi_{\text{min}}} - v$

The solution should satisfy:

$$\sum_{i=1}^{n} \max(\Phi_{\text{min}}, \alpha P_i) \land \min(\Phi_{\text{max}}, \alpha P_i) = \Phi$$

where $\alpha = -\frac{R}{k^v}$

The solution involves iterations to identify the best set of channels associated with each of the above three cases. In each iteration $\alpha$ satisfies

$$n_1\Phi_{\text{min}} + n_2\Phi_{\text{max}} + \sum_{i=n_1}^{n_2} \alpha P_i = s\Phi$$

$s$ is the set of time slots with $\Phi_{\text{min}} < \Phi_i < \Phi_{\text{max}}$
• **Uniform distribution:** 
  \[ Pr(\alpha \geq N_i) = \frac{N_{\text{max}} - N_i}{N_{\text{max}} - N_{\text{min}}} \]

• Throughput maximization subject to vulnerability constraint.
  - Re-formulate as fractional knapsack problem.

• Outline of optimal solution:
  - Sort the channel SNR’s in decreasing order.
  - Allocate minimum block lengths so that \( \Phi_{\text{min}} \) is achieved.
  - Allocate block lengths to the ordered channels corresponding to \( (\Phi_{\text{max}} - \Phi_{\text{min}}) \)
  - Stop when it cannot be done anymore.

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**Case 2: Average Channel SNR Known**

• Define cost function: 
  \[ C(\gamma, N) = (1 - \lambda)D(\gamma, N) + \lambda S(N) \]

• Rijndael cipher; \( N = \{128, 160, 192, 224, 246\} \)

• Average SNR = 7dB.

• Min. sec. = 0.875 (~128bit key)

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[Graph showing average throughput and cost as functions of \( \lambda \)]
Fixed block encryption -- 128 bit AES

$S_{req} = 0.875$
Case 3: Finite State Markov Channel Model

- Channel SNR is quantized to finite set of states.
- Channel jumps from one state to another as a Markov process.
- Each state corresponds to a range of bit error rates.
- State transition probabilities are functions of various physical layer parameters.

A Markov Decision Process (MDP) Model (I)

- Define state of the MDP as: \( \{(c_s, b_t), s = 1...r; t = 1...q\} \)
- \( r \)-number of channel states.
- \( q \)-capacity of receiver buffer.
- Assume ACK/NAK sent to transmitter.
- \( Q_n \): set of available encryption block lengths (action set). \( |Q_n| = k \).
- Buffer occupancy: \( b_t = \sum_{a=1}^{m_a} N_a \) where \( m_a \) blocks of lengths \( N_a \) were successfully transmitted.
A Markov Decision Process (MDP) Model (II)

- MDP state transition probability:
  \[ P_j^y(a) = P(c(n + 1) = c_j, b(n + 1) = b_{t_j} | c(n) = c_{s_j}, b(n) = b_t, a) \]

- Reward function of MDP:
  \[ r(i, a) = b_i + N_a (1 - P_{bl,a}(c_{s_j})) \]

- Bellman’s equation (dynamic program):
  \[
  v_{a,T}(i) = \max_a \{ r(i, a) + \alpha \sum_j P_j^y(a) v_{a,T-1}(j) \}
  \]
  \[ v_{a,T}(.): \text{Optimum function value T steps into future} \]
  \[ 0 < \alpha < 1: \text{discount factor} \]

Throughput Gain

- \(N_c=\{128, 160, 192, 224, 256\}\)
- \(r = 8\) (no. of channel states)
- \(T=1000; \\alpha = 0.5\)
- Fixed encryption uses 224 bit block

\[ S_{\text{avg}} = 0.875 \]
Conclusions

• Link state adaptive encryption (opportunistic encryption) results in significant throughput increase for a wide range of channel SNR.

• Opportunistic encryption performs well with varying degrees of side information about the channel conditions.

• Provides a framework to model security vs. throughput trade-off.