Contents:

121P03 - 1Q, 4P, 6P, 8P, 13P, 21P, 23P, 39P

- Recap & Definition of Electric Field
- Electric Field Lines
- Charges in External Electric Fields
- Field due to a Point Charge
- Field Lines for Superpositions of Charges
- Field of an Electric Dipole
- Electric Dipole in an External Field: Torque and Potential Energy
- Method for Finding Field due to Charge Distributions
  - Arc of Charge
  - Ring of Charge
  - Disc of Charge
  - Infinite line of Charge
- Summary
PROBLEM 121P03 - 1Q: The figure shows three electric field lines. What is the direction of the electrostatic force on a positive test charge placed at (a) point A and (b) point B? (c) At which point, A or B, will the acceleration of the test charge be greater if the charge is released?

* Field line by definition shows direction of force on + charge.
  9q/m for test charge.
  (a) \( F = qE \) tangent to field at A
  (b) \( F = qE \) at B

\[ F = qE = ma \]
\[ a = \frac{qE}{m} \]
where \( E \) is strongest, \( a \) is greatest.

\( E \) is strongest where lines are dense.

\( \Rightarrow \) greatest \( a \) at A.
PROBLEM 121P03 - 4P: What is the magnitude of a point charge that would create an electric field of 1.00 N/C at points 1.00 m away?

\[ E_1 = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \]

\[ q = 4\pi \varepsilon_0 E_1 r^2 \]

\[ = \frac{4}{3} \times 10^{-9} \times 1 \times 1 \]

\[ = 1.3 \times 10^{-9} \text{ coulomb} = 1.3 \times 10^{-4} \text{ microcoulomb} \]
PROBLEM 121P03 - 6P: Two particles with equal charge magnitudes $2.0 \times 10^{-7}$ C but opposite signs are held 15 cm apart. What are the magnitude and direction of the Electric Field $E$ at the point midway between the charges?

By symmetry, $E_1 \text{ at } P = E_2 \text{ at } P$.

Magnitude:

$E_{\text{net}} \text{ at } P = 2 \times |E_1| (-\hat{i})$

$|E_1| = \frac{9 \times 10^9 \cdot 9}{2 \times \left(\frac{0.075}{2}\right)^2}$

$|E_1| = 3.2 \times 10^5 \text{ N/C}$

Direction:

$E_{\text{net}} = -6.4 \times 10^5 \hat{i}$

(To the Left)
PROBLEM 121P03 - 8P*: In the figure, two fixed point charges $q_1 = +1.0 \times 10^{-6}$ C and $q_2 = +3.0 \times 10^{-6}$ C are separated by a distance $d = 10$ cm. Plot their net electric field $E(x)$ as a function of $x$ for both positive and negative values of $x$, taking $E$ to be positive when the vector $E$ points to the right and negative when $E$ points to the left.
PROBLEM 121P03 - 13P: What are the magnitude and direction of the electric field at the center of the square in the figure, if \( q = 1.0 \times 10^{-8} \) C and \( a = 5.0 \) cm?

Choose x-y axes as shown. 
- \( d \) is distance from charge to point P at origin.

\[ E_p = \frac{1}{2} \sqrt{2a} = \frac{a}{\sqrt{2}}. \]

\[ E_p = E_{p1} + E_{p2} + E_{p3} + E_{p4}. \]

Let \( E_0 = \frac{kq}{d^2} = \frac{9 \times 10^9 \times 10^{-8}}{25} = 7.2 \) N/C.

\[ E_p = E_0 (\mathbf{i}) + E_0 \times 2 \mathbf{j} + E_0 (\mathbf{1} - 2 \mathbf{0} \mathbf{)} \]

\[ E_p = E_0 (\mathbf{i} + \mathbf{j}) \]

\[ E = \begin{cases} \begin{align*} 7.2 \text{ N/C} \mathbf{i} + 7.2 \text{ N/C} \mathbf{j} \end{align*} \end{cases} \]

or

\[ 7.2 \sqrt{2} = 10.2 \text{ N/C}. \]

\( \phi + \Phi = +45^\circ \)
**PROBLEM 121P03 - 21P**: A thin glass rod is bent into a semicircle of radius $r$. A charge $+q$ is uniformly distributed along the upper half, and a charge $-q$ is uniformly distributed along the lower half, as shown in the figure. Find the magnitude and direction of the electric field $\mathbf{E}$ at $P$, the center of the semicircle.

Axes as shown.

By symmetry $E_+ = 1E_+ = E_0$.

$E_p = E_+ + E_-$ (along $-y$ direction).

Use result for arc of charge.

For intermediate $E_0 = \frac{2}{\pi r} \frac{q}{\epsilon_0}$.\[ \theta = \frac{\pi}{2} - \frac{\pi}{2} = \frac{\pi}{4} \]

$x = \frac{\sqrt{2}}{2} = \sqrt{\frac{1}{2}}$.

$E_0 = \frac{2}{\pi r} \frac{q}{\epsilon_0} = \frac{2q}{\pi r \epsilon_0}$.

Unit vectors

$E_p = E_0 \hat{j}$

$E_p = E_0 (\cos \theta - \hat{j} \sin \theta)$

$E_p = \frac{1}{\sqrt{2}} E_0 \hat{j}$

$E_p = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right) \hat{j}$

$E_p = \frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} \right) \hat{j}$

$E_p = \frac{1}{2} \hat{j}$

$E_p = \frac{1}{2} \hat{j}$

Down
PROBLEM 121P03-23: In Fig. 23-35, a nonconducting rod of length L has charge -q uniformly distributed along its length. (a) What is the linear charge density of the rod? (b) What is the electric field at point P, a distance a from the end of the rod? (c) If P were very far from the rod compared to L, the rod would look like a point charge. Show that your answer to (b) reduces to the electric field of a point charge for a >> L.

\[
dq = \lambda \, dx
\]

\[
dE_P = \frac{k dq}{r^2} \hat{i} \quad \text{along } x\text{-axis by symmetry} \quad r = a + L - x
\]

\[
\lambda = \text{linear charge density} = -\frac{q}{L} \quad dq = \lambda \, dx
\]

\[
E_P = \int_0^L \frac{k \lambda \, dx}{(a + L - x)^2} \hat{i} = k \lambda \hat{i} \int_0^L \frac{dx}{(a + L - x)^2} = k \lambda \hat{i} \frac{1}{(a + L - x)} \bigg|_0^L
\]

\[
E_P = \frac{kq}{L} \hat{i} \left[ \frac{1}{a + L} - \frac{1}{a} \right] = \frac{1}{4\pi\varepsilon_0 L} \frac{q \hat{i}}{a(a + L)} \left[ \frac{L}{a(a + L)} \right]
\]

\[
\therefore E_P = \frac{q}{4\pi\varepsilon_0} \frac{1}{a^2} \left[ \frac{1}{1 + L/a} \right] \hat{i}
\]

Note

\[
\frac{d}{dx} \left[ \frac{1}{a + L - x} \right] = \frac{(-1)(-1)}{(a + L - x)^2}
\]

For L << a, the result approaches the point charge formula.
PROBLEM 121P03 - 39P: A uniform electric field exists in a region between two oppositely charged plates. An electron is released from rest at the surface of the negatively charged plate and strikes the surface of the opposite plate, 2.0 cm away, in a time $1.5 \times 10^{-8}$ s. (a) What is the speed of the electron as it strikes the second plate? (b) What is the magnitude of the electric field $E$?

\[ \frac{V_f}{t} = \frac{V_0 t}{t} = \frac{1}{2} at^2 \]
\[ V_f^2 = (2 \frac{V_0}{t}) \]
\[ V_f = \frac{2d}{t} = \frac{2 \times 2 \times 10^{-2}}{1.5 \times 10^{-8}} \]
\[ t = 2.67 \times 10^{-6} \text{ m/s} \]

Find $E$
\[ E = q \frac{E}{m} = \frac{101 \text{ E}}{m} \]
\[ d = \frac{1}{2} at^2 = \frac{1}{2} \frac{101 \text{ E}}{m} \]
\[ E = \frac{2md}{101^2} = \frac{2 \times 9.11 \times 10^{-3} \times 2 \times 10^{-2}}{1.6 \times 10^{-19}(1.5 \times 10^{-8})^2} \]
\[ E = 1.01 \times 10^3 \text{ N/C} \]

\[ V = E \]
\[ J = 8.85 \times 10^{-9} \text{ C/m}^2 \]

Fall 2002