Physics 121 Practice Problem Solutions 06
Capacitance

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PROBLEM 26-3Q: For each circuit in Fig. 26-20, are the capacitors connected in series, in parallel, or in neither mode?

(a) Series
(b) Parallel
(c) Parallel
PROBLEM 26-4Q: (a) In Fig. 26-21a, are capacitors $C_1$ and $C_3$ in series? (b) In the same figure, are capacitors $C_1$ and $C_2$ in parallel? (c) Rank the equivalent capacitances of the four circuits shown in Fig. 26-21, greatest first.

(a) No. $C_{eq}$ = \frac{(C_1+C_2)C_3}{C_1+C_2+C_3}$

(b) Can reform to load like (a)

$C_{eq}$ = Same as (a)

(c) Same as above

(d) Same as above
PROBLEM 26-6Q: You are to connect capacitances $C_1$ and $C_2$, with $C_1 > C_2$, to a battery, first individually, then in series, and then in parallel. Rank those arrangements according to the amount of charge stored, greatest first.

For same $V$, $Q = CV$ so $Q$ is in $LC$.

Need $C_{eq}$ for series or parallel.

Parallel: $C_{para} = C_1 + C_2 > C_1 > C_2$

Series: $C_2 = C_2 \left( \frac{C_1 + C_2}{C_1 + C_2} \right) = \frac{C_2 C_1}{C_1 + C_2} + \frac{C_2}{C_1 + C_2}$

Note $C_{series} = \frac{C_2 C_1}{C_1 + C_2}$

As $C_2 > C_1$ series overall order is:

$C_{para} > C_1 > C_2 > C_{series}$
PROBLEM 26-3P: The capacitor in Fig. 26-25 has a capacitance of 25 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

Charge flows counterclockwise across capacitor: 

Q = CV = 25 \times 10^{-6} \times 120 

Q = 3 \times 10^{-3} \text{ Coul}

Fall 2012
PROB. 121P06 – 5P

PROBLEM 26-5P*: A parallel-plate capacitor has circular plates of 8.2 cm radius and 1.3 mm separation. (a) Calculate the capacitance. (b) What charge will appear on the plates if a potential difference of 120 V is applied?

For a parallel plate capacitor:

\[ C = \frac{\varepsilon A}{d} \]

\[ = \frac{8.85 \times 10^{-12} \times \pi \times (8.2 \times 10^{-2})^2}{1.3 \times 10^{-3}} \]

\[ C = 1.44 \times 10^{-10} \text{ F} \]

\[ C = 144 \text{ pF} = 1.44 \times 10^{-9} \mu\text{F} \]

\[ A = \pi r^2 \]

\[ r = 8.2 \text{ cm} = 8.2 \times 10^{-2} \text{ m} \]

\[ d = 1.3 \text{ mm} = 1.3 \times 10^{-3} \text{ m} \]

\[ \varepsilon = \varepsilon_0 = 1 \times \varepsilon_0 \]
PROBLEM 26-7P: A spherical drop of mercury of radius $R$ has a capacitance given by $C = 4\pi \varepsilon_0 R$. If two such drops combine to form a single larger drop, what is its capacitance?

The new drop was twice the volume of the old added drop: $V = \frac{4}{3} \pi R^3$ (new drop) $V' = \frac{4}{3} \pi R'^3$.

$$\frac{V'}{V} = 2 = \frac{\frac{4}{3} \pi R'^3}{\frac{4}{3} \pi R^3} \Rightarrow R' = 2^{1/3} R$$

New capacitance $C' = 4\pi \varepsilon_0 R'$.

$$C' = 4\pi \varepsilon_0 \cdot 2^{1/3} R.$$ 

In terms of old $C$: $\frac{C'}{C} = 2^{1/3}$
PROB. 121P06 – 10P

PROBLEM 26-10P*: In Fig. 26-26, find the equivalent capacitance of the combination. Assume that \( C_1 = 10.0 \mu F \), \( C_2 = 5.00 \mu F \), and \( C_3 = 4.00 \mu F \).

- Equivalent capacitance for the series pair

\[
\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{10 \times 5}{10 + 5} \mu F
\]

\( C_{\text{series}} = 3.33 \mu F \)

- Use parallel capacitor rule for \( C_{\text{series}} + C_3 \)

\[
C_{\text{eq}} = C_{\text{series}} + C_3 = 3.33 + 4
\]

\( C_{\text{eq}} = 7.33 \mu F \)
PROBLEM 26-11P*: How many 1.00 \( \mu F \) capacitors must be connected in parallel to store a charge of 1.00 C with a potential of 110 V across the capacitors?

Capacitors add in parallel.

(1) \( Q = C_{eq}V \) \[ C_{eq} = \frac{Q}{V} = \frac{1}{110} = 9.09 \times 10^{-3} \text{ F} \]

\# of 1 \( \mu F \) capacitors

\[ \frac{C_{eq}}{1 \mu F} = 9.09 \]
PROBLEM 26-13P: In Fig. 26-28 find the equivalent capacitance of the combination. Assume that $C_1 = 10.0 \mu F$, $C_2 = 5.00 \mu F$, and $C_3 = 4.00 \mu F$.

- $C_1$ and $C_2$ are in parallel: $C_{11} = C_1 + C_2$.
- The pair is in series with $C_3$.

\[
\begin{align*}
C_{eq} &= \frac{C_{11}C_3}{C_{11} + C_3} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} \\
C_{eq} &= \frac{(10.0 \times 4.0)}{10.0 + 5.0 + 4.0} = 3.16 \mu F
\end{align*}
\]
PROBLEM 26-25P: Two capacitors, of 2.0 and 4.0 μF capacitance, are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.

\[ U = \frac{1}{2} CV^2 \text{ for a capacitor} \]

\[ C_{\text{Total}} = C_1 + C_2 \]
\[ = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 \]
\[ = \frac{1}{2} (C_1 + C_2) V^2 \]
\[ = \frac{1}{2} (2 + 4) \times 10^{-6} \times (300)^2 \]

\[ C_{\text{Total}} = 0.27 \text{ Joules} \]
PROBLEM 26-29P: A parallel-plate capacitor has plates of area $A$ and separation $d$ and is charged to a potential difference $V$. The charging battery is then disconnected, and the plates are pulled apart until their separation is $2d$. Derive expressions in terms of $A$, $d$, and $V$ for (a) the new potential difference; (b) the initial and final stored energies, $U_i$ and $U_f$; and (c) the work required to separate the plates.

Initially:

$$C = \varepsilon_0 \frac{A}{d}$$

$$\varepsilon = CV$$

After:

$$\varepsilon' = \varepsilon_0 \frac{A}{(2d)} = \frac{C}{2}$$

Work is done to pull them apart

$$Q = C'V' = CV \Rightarrow V' = \frac{C'}{C} V = \frac{1}{2} V$$

a) $V' = \frac{1}{2} V$

Energy Stored:

$$U_i = \frac{1}{2} CV$$

$$U_f = \frac{1}{2} C'V' = \frac{1}{2} \frac{C}{2} (2V)^2 = 2U_i$$

$$U_f = 2U_i$$

$$C_f = 2C_i = 2 \times \frac{1}{2} V \varepsilon_0 A$$

$$C_f = \frac{\varepsilon_0 A V^2}{d}$$

c) $\Delta W = C_f - C_i = C_i = \frac{1}{2} C_f = \frac{1}{2} \frac{\varepsilon_0 A V^2}{d}$

$$\Delta W = \frac{\varepsilon_0 A V^2}{2d}$$

Fall 2002
PROBLEM 26-34P: An air-filled parallel-plate capacitor has a capacitance of 1.3 pF. The separation of the plates is doubled and wax is inserted between them. The new capacitance is 2.6 pF. Find the dielectric constant of the wax.

With \( K \approx 1 \),

Initially \( C = \frac{\varepsilon_0 A}{d} = 1.3 \text{ pF} \).

Double \( d \) \( C' = \frac{\varepsilon_0 A}{2d} = \frac{1}{2} C = 0.65 \text{ pF} \).

Add wax: \( C'' = KE' = \frac{K C}{2} = 2.6 \text{ pF} \).

\[
K = \frac{2.6 \text{ pF} \times 2}{1.3 \times 2} = 4.
\]