Physics 121 Practice Problem Solutions 08B
RC Circuits

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28-44 A capacitor with initial charge $Q_0$ is discharged through a resistor.

In terms of $D = RC$, the time constant $\tau = D$.

How long does it take for $C$ to charge:

(a) One third of its charge
(b) Two thirds of its charge

(a) $Q(t) = Q_0 e^{-t/\tau}$

$Q = \frac{2}{3} (\text{remaining}) = e^{-t/\tau}$

$\ln \left( \frac{2}{3} \right) = -t/\tau \quad \Rightarrow \quad t = -\tau \ln \frac{2}{3}$

$\frac{t}{\tau} = -\ln \frac{2}{3} = 0.41$

(b) Similarly

$\frac{t}{\tau} = -\ln \frac{1}{3} = 1.1$
PROBLEM 121P08 - 46P*: In an RC series circuit, \( E = 12.0 \text{ V} \), \( R = 1.40 \text{ M}\Omega \), and \( C = 1.80 \mu\text{F} \). (a) Calculate the time constant. (b) Find the maximum charge that will appear on the capacitor during charging. (c) How long does it take for the charge to build up to 16.0 \( \mu\text{C} \)?

\( T = \text{time constant} = RC = 1.4 \times 10^6 \times 1.8 \times 10^{-6} \mu\text{sec} \)

\[ T = 2.52 \mu\text{sec} \]

\( Q_{\infty} = E V_{\infty} \)

\[ V_{\infty} = E \cos(t/T) \]

\[ Q_{\infty} = 1.8 \times 10^{-6} \times 12 \]

\[ Q_{\infty} = 21.6 \mu\text{C} \]

\( Q(t) = 16 \mu\text{C} = Q_{\infty} (1 - e^{-t/T}) \)

\[ Q(t) = 16 \]

\[ Q_{\infty} = 21.6 \]

\[ 1 - \frac{16}{21.6} = 0.259 = e^{-t/2.52} \]

\[ t = -2.52 \times \ln(0.259) \]

\[ t = 3.85 \]
PROBLEM 121P08 - 50: An initially uncharged capacitor $C$ is fully charged by a device of constant emf $E$ connected in series with a resistor $R$. (a) Show that the final energy stored in the capacitor is half the energy supplied by the emf device. (b) By direct integration of $I^2R$ over the charging time, show that the thermal energy dissipated by the resistor is also half the energy supplied by the emf device.
PROBLEM 121P08 - 51P*: A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10.0$ s, the potential difference across the capacitor is 1.00 V. (a) What is the time constant of the circuit? (b) What is the potential difference across the capacitor at $t = 17.0$ s?

$$V(t) = V_0 e^{-t/\tau}$$

$$V(t=10) = 0.01 = e^{-10/\tau}$$

$$V_0 \ln(0.1) = -4.61 = -\frac{10}{\tau}$$

$$\tau = \frac{10}{4.61} = 2.17 \text{ s}$$

$$V(t=17) = 100 e^{-17/2.17}$$

$$V = 3.98 \times 10^{-2} \text{ Volts}$$
PROBLEM 121P08 - 52P: The figure shows the circuit of a flashing lamp, like those attached to barrels at highway construction sites. The fluorescent lamp L (of negligible capacitance) is connected in parallel across the capacitor C of an RC circuit. There is a current through the lamp only when the potential difference across it reaches the breakdown voltage $V_L$; in this event, the capacitor discharges completely through the lamp and the lamp flashes briefly. Suppose that two flashes per second are needed. For a lamp with breakdown voltage $V_L = 72.0$ V, wired to a 95.0 V ideal battery and a 0.150 $\mu$F capacitor, what should be the resistance $R$?

Need to charge C to $V_L$ twice per second. Discharge determined instantaneously.

$$V_L = \frac{E}{2}(1 - e^{-t/RC})$$

$$e^{-t/RC} = \frac{E-V_L}{E}$$

$$\frac{t}{RC} = -\ln\left(\frac{E-V_L}{E}\right)$$

$$t = 0.5 \text{ sec}$$

$$R = -\frac{t}{C \ln\left(\frac{E-V_L}{E}\right)} = -\frac{0.5}{0.15 \times 10^{-6} \ln\left(\frac{95}{72}\right)} = -\frac{0.5}{0.15 \times 10^{-6} \times (1.92)}$$

$$R = 2.35 \times 10^5 \Omega$$
PROBLEM 121P08 - 53P*: A 1.0 μF capacitor with an initial stored energy of 0.50 J is discharged through a 1.0 MΩ resistor. (a) What is the initial charge on the capacitor? (b) What is the current through the resistor when the discharge starts? (c) Determine $V_C$, the potential difference across the capacitor, and $V_R$, the potential difference across the resistor, as functions of time. (d) Express the production rate of thermal energy in the resistor as a function of time.

(a) $U_C = \frac{1}{2} \frac{Q^2}{C}$ $\Rightarrow$ $Q = \sqrt{2EC_0} = \sqrt{2 \times 10^{-8} \times 0.5}$

(b) $i(t) = $ Io $e^{-\frac{t}{\tau}}$

$$P = i(t)R = \frac{Q}{t} e^{-\frac{t}{\tau}}$$

(c) $V_C = V_{C0} e^{-\frac{t}{\tau}}$ $V_{C0} = \frac{Q}{C} = \frac{10^{-3}}{10^{-6}} = 10^3$ Volts.

$$V_C(t) = 10^3 e^{-\frac{t}{10^3}}$$

$$V_R(t) = i(t)R = 10^{-3} A \times 10^6 \Omega \times e^{-\frac{t}{10}}$$

(d) $P = i^2 R = (\frac{Q}{t})^2 R e^{-\frac{t}{\tau}} = 10^{-6} \times 10^6 e^{-\frac{t}{10}}$

$$P = 1.0 e^{-\frac{t}{10}} \text{ Watts}$$
PROBLEM 121P08 - 55: In the circuit of the figure \( \mathcal{E} = 1.2 \text{ kV} \), \( C = 6.5 \mu \text{F} \), \( R_1 = R_2 = R_3 = 0.73 \text{ M}\Omega \). With \( C \) completely uncharged, switch S is suddenly closed (at \( t = 0 \)). (a) Determine the current through each resistor at \( t = 0 \) and as \( t \to \infty \). (b) Draw qualitatively a graph of the potential difference \( V_2 \) across \( R_2 \) from \( t = 0 \) to \( t = \infty \). (c) What are the numerical values of \( V_2 \) at \( t = 0 \) and as \( t \to \infty \)? (d) What is the physical meaning of “\( t \to \infty \)” in this case?

\[ \text{Junction Rule: } V_1 = V_2 + V_3 \]

\[ \text{Left Loop: } \mathcal{E} - V_1 R_1 - V_2 R_2 = 0 \]

\[ \text{Right Loop: } V_2 R_2 + V_3 + V_3 R_3 = 0 \]

\[ \text{for all } R \text{ 's } = R (= R_1 = R_2 = R_3) \]

\[ \text{Loop Eqns: } \begin{align*}
\mathcal{E} &= (V_1 + V_2)R \\
(V_2 - V_3)R &= V_c.
\end{align*} \]

\( a) \) \( t = 0 \) \( V_c = 0 \) \( \Rightarrow V_2 = V_3 \), \( V_1 = 2V_2 \), \( \mathcal{E} = 3V_2R \)

\( V_2 = \frac{\mathcal{E}}{3R} = 5.5 \times 10^{-6} \text{A} \)

\( V_1 = 2V_2 = 1.1 \times 10^{-5} \text{A} \)

\( b) \) \( t \to \infty \) \( V_3 = 0 \) \( \Rightarrow V_1 = V_2 \)

\( \mathcal{E} = 2V_2R \)

\( V_1 = \frac{\mathcal{E}}{2R} = 8.22 \times 10^{-6} \text{A} \)

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Path: Need to solve loop equations to get \( i_1, i_2, i_3 \)

- Find \( V_{Ba} = i_2 R \)

- Eliminate \( i_1 \) in (2) using (1)
  \[ E = (2i_2 + i_3)R \Rightarrow i_2 = \frac{E - i_3 R}{2R} \]

- Eliminate \( i_2 \) in (5)
  \[ \frac{E}{2} - \frac{6 i_2}{2} R - i_3 R = V_c = \frac{Q}{C} \]
  \[ i_3 = \frac{dQ}{dt} \]

\[ \frac{dQ}{dt} + \frac{2Q}{3R} - \frac{E}{3R} = 0 \quad \text{Differential Equation} \]

\[ \frac{3RC}{2} \text{ is a time constant } T' \]

\[ \frac{Q}{3R} = \frac{C_0}{Q_0} = \frac{Q_0}{T'} \]

\[ \frac{Q(t)}{Q_0} = \frac{CE}{2} \left( 1 - e^{-2t/T} \right) \]

\( Q \) is the charge on the capacitor.
\[
\frac{d\Phi_3(t)}{dt} = \Phi_3 = \frac{E}{3R} \left( -X \right) e^{-\frac{t}{\tau}} \]

\[
\Phi_3 = \frac{E}{3R} e^{-\frac{t}{\tau}} \]

\[
I_2 = \frac{E}{3R} e^{-\frac{t}{\tau}} + V_c \\
I_2 = \frac{E}{3R} e^{-\frac{t}{\tau}} + \frac{E}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \\
I_2 = \frac{E}{3R} e^{-\frac{t}{\tau}} + \frac{E}{2} - \frac{E}{2} e^{-\frac{t}{\tau}} = \frac{E}{6R} \left( 3 - e^{-\frac{t}{\tau}} \right) \\
V_2 = I_2 R = \frac{E}{6} \left( 3 - e^{-\frac{t}{\tau}} \right) \quad \tau = \frac{3RC}{2}
\]