IEEE Int. Conf. on Acoust., Speech, Signal Processing (ICASSP 2002), May 13-17, 2002, Orlando, FL, USA

DESIGN OF ALLPASS-BASED NON-UNIFORM OVERSAMPLED DFT FILTER BANKS

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ABSTRACT

In this paper we address design and properties of an oversampled non-uniform DFT filter bank derived by an allpass frequency transform from its uniform version. The novel synthesis bank utilizes only stable FIR filters, which can be designed via closed-form expressions. The overall analysis-synthesis system leads to a near-perfect-reconstruction solution, where the phase compensation error can be made arbitrarily small at the expense of additional system delay. Furthermore, we also address the case of different subsampling factors in the subbands. The filter bank design is carried out by utilizing a lifting factorization for the prototypes, which has the advantage that the overall system delay can be controlled in an efficient way.

1. INTRODUCTION

For many applications a non-uniform time-frequency representation of a signal is more suitable, where examples are the approximation of the critical bands in the human auditory system with non-uniform filter banks or subband-based noise reduction approaches. A non-uniform frequency resolution can be obtained by applying a frequency transformation to the classical uniform DFT polyphase filter bank, where all delay elements in the DFT filter bank are replaced with allpass filters [1, 2]. This, however, leads to unstable synthesis filters when an overall perfect reconstruction (PR) system is desired. One solution of this problem may be obtained by anticausal filtering combined with a double buffering scheme for the processing of infinite-length signals as in [3, 4], where segmentation and time-reversal of the subband signals is required. The disadvantage of this method is the request for extra information which grows with the order of the allpass filters in the analysis, since initial conditions for the synthesis allpasses have to be additionally transmitted. Furthermore, the system delay is increased due to the buffering steps.

In this paper we present a near-perfect-reconstruction (near-PR) approach for designing the synthesis filters in the allpass-based non-uniform DFT filter bank, where the proposed synthesis bank only employs FIR subband filters. By factorizing analysis and synthesis prototypes into lifting steps the system delay can be efficiently controlled. In extension to the work in [5] we also address the case of different subsampling factors for the subbands and derive general design conditions for the near-PR oversampled allpass-transformed DFT filter bank. Furthermore, it is shown that all distortions in the reconstructed signal can be made arbitrarily small at the expense of additional system delay and vice versa.

This work was supported in part by the German Research Foundation "Deutsche Forschungsgemeinschaft" (DFG Graduate College No. 357).

2. ANALYSIS FILTER BANK BASED ON ALLPASS-TRANSFORMS

In Fig. 1 a generalized M-band DFT filter bank in polyphase representation is depicted, where all delay blocks are replaced by allpass transfer functions $A(z)$. The matrix $\mathbf{W}_M$ represents the

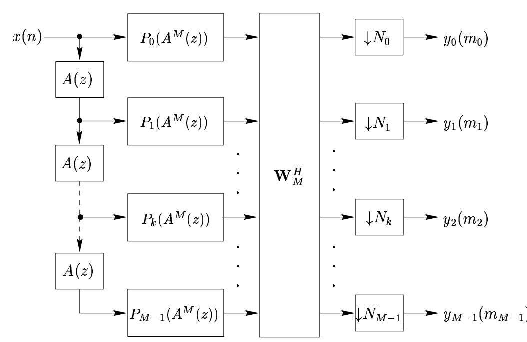

Fig. 1. Generalized analysis polyphase DFT filter bank

$M \times M$ DFT matrix with elements $[\mathbf{W}_M]_{ki} = W_M^{ki} = e^{-j\frac{2\pi}{M}ki}$, $k, i = 0, \ldots, M-1$, and N_k denotes the subsampling factor for the k-th subband. In the following we address stable and causal first-order allpass filters, which are defined as

$$A(z) = \frac{z^{-1} + a^*}{1 + a\, z^{-1}}, \quad a \in \mathbb{C}, \quad |a| < 1, \tag{1}$$

with the frequency response $A(e^{j\omega}) = e^{j\phi(\omega)}$. The phase response can be written as

$$\phi(\omega) = -\omega + 2\arctan\left(\frac{R\sin(\omega-\theta)}{R\cos(\omega-\theta)-1}\right) \tag{2}$$

with radius R, angle θ and $a = -R\, e^{j\theta}$. For the sake of simplicity we restrict ourselves to a real-valued parameter a.

The analysis subband filters are derived from the prototype impulse response $p(n)$ for the modified DFT bank in Fig. 1 by an allpass transformation and a subsequent complex modulation. We assume in the following that the prototype length is restricted to integer multiples of M, i.e. $L_p = m \cdot M$, $m \in \mathbb{N}$. The transfer function of the k-th subband filter $H_k(z)$ can be expressed as

$$H_k(z) = \sum_{\rho=0}^{M-1} P_\rho(A^M(z))\, A^\rho(z)\, W_M^{-k\rho}, \qquad (3)$$

by using the (type 1) allpass-transformed polyphase components for $\rho = 0, 1, \ldots, M-1$

$$P_\rho(A^M(z)) = \sum_{\lambda=0}^{m-1} p(\lambda M + \rho)\, A^{\lambda M}(z). \qquad (4)$$

As an example the nonlinear transformation of the frequency scale is shown in Fig. 2 for $M = 8$ and $a = -0.5$, where the choice of the parameter a in (1) determines the nonlinearity of the frequency transformation. Note that the classical uniform DFT filter bank

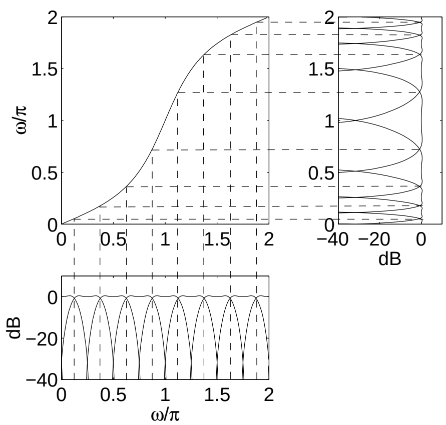

Fig. 2. Nonlinear transformation of the frequency scale for the parameters $M = 8$ and $a = -0.5$

can be derived as a special case from the structure in Fig. 1 if we choose the parameter a to be equal zero.

3. DESIGN OF THE SYNTHESIS FILTER BANK

In order to design a near-PR analysis-synthesis system we generally require stable analysis and synthesis filters which excludes the use of allpasses having inverse transfer functions in the synthesis filter bank. In the following we present a new synthesis structure where only stable FIR subband filters are utilized. These filters can be designed such that the phase distortion of the allpasses in the analysis bank is almost compensated, which leads to near-PR for the oversampled analysis-synthesis system.

3.1. Compensation of the phase distortion

We propose a closed-form expression for FIR filters which approximately compensate the phase distortion caused by a first-order allpass up to a certain error [5]. The main idea is that by employing the polynomial factorization relation

$$z^{-d} - (-1)^d \cdot a^d = (z^{-1} + a) \sum_{i=0}^{d-1} (-1)^i\, a^i\, z^{-(d-1-i)} \qquad (5)$$

with $d \in \mathbb{N}$, a phase compensation FIR filter $F(z)$ can be stated according to

$$F(z) = (1 + a\, z^{-1}) \sum_{i=0}^{d-1} (-1)^i\, a^i\, z^{-(d-1-i)}. \qquad (6)$$

It can now easily be verified from (1), (5) and (6) that

$$\begin{aligned} A(z)\, F(z) &= z^{-d} - (-1)^d\, a^d = \\ &= z^{-d} - (-1)^d\, \varepsilon(a,d), \quad d \in \mathbb{N}. \end{aligned} \qquad (7)$$

Since $-1 < a < 1$ the compensation error $\varepsilon(a,d)$ can be made arbitrarily small at the expense of additional delay by appropriate choice of the filter order d. Then (7) can be approximated as $A(z)\, F(z) \approx z^{-d}$.

3.2. Near-PR conditions in the oversampled case

The phase compensation FIR filters in (6) can now be used to design a near-PR synthesis filter bank when both the analysis and synthesis prototype of length L_p are designed appropriately, which will be discussed below. The resulting synthesis bank is depicted in Fig. 3, where the "modified" (type 1) polyphase components are defined with $\rho = 0, 1 \ldots, M-1$ as

$$\hat{Q}_\rho(z) = \sum_{\lambda=0}^{m-1} q(\lambda M + \rho)\, F^{(M(m-\lambda)-\rho-1)}(z)\, z^{-Md\lambda}. \qquad (8)$$

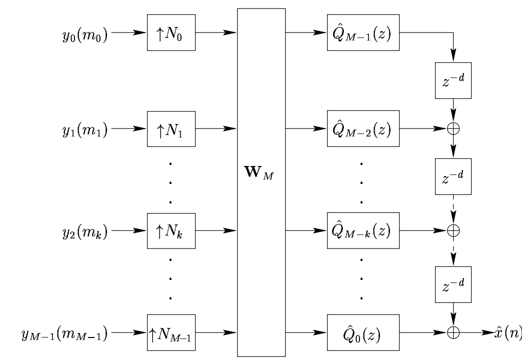

Fig. 3. Modified DFT synthesis filter bank

Non-subsampled case. In this case with $N_k = 1$ for all k we have only linear distortions in the reconstructed signal at the output of the filter bank. Thus, the linear distortion transfer function $T(z)$, which should at least approximately correspond to a delay, can be written with appropriately constructed analysis and synthesis polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ [5] as

$$T(z) = \mathbf{e}_d^T(z)\, \mathbf{R}(z)\, \mathbf{E}(z)\, \mathbf{a}(z) \overset{!}{\approx} z^{-D}, \qquad (9)$$

where D denotes the delay of the overall analysis-synthesis system, $\mathbf{a}(z) = [1, A(z), \ldots, A^{\frac{M}{2}-1}(z)]^T$ and $\mathbf{e}_d(z) = [z^{-(\frac{M}{2}-1)d}, z^{-(\frac{M}{2}-2)d}, \ldots, 1]^T$. In order to fulfill the condition (9) it is obvious that by employing the diagonal matrix $\mathbf{F}(z) = \mathrm{diag}\,[F^0(z), F^1(z), \ldots, F^{\frac{M}{2}-1}(z)]$ the relation

$$\mathbf{R}(z)\, \mathbf{E}(z) \overset{!}{=} \frac{2}{M}\, z^{-s}\, \mathbf{F}(z) \quad \text{with} \quad s \in \mathbb{N} \qquad (10)$$

has to be satisfied for the polyphase matrices when we want the phase compensation error $\varepsilon(a,d)$ to be the only distortion in the reconstructed signal. It can now be shown [5] that by rewriting (10) we may obtain the following conditions for the analysis and synthesis polyphase components:

$$\begin{aligned} z^{-\frac{M\,d}{2}}\, \hat{Q}_{M-1-k}(z)\, P_k(A^M(z)) + A^{\frac{M}{2}}(z)\, \hat{Q}_{\frac{M}{2}-1-k}(z) \cdot \\ \cdot\ P_{\frac{M}{2}+k}(A^M(z)) \overset{!}{=} \frac{2}{M^2}\, z^{-s}\, F^k(z) \end{aligned} \qquad (11)$$

where $k = 0, 1, \ldots, \frac{M}{2} - 1$. Furthermore, the parameter s in (11) has to be chosen as $s = q\,M\,d + \frac{M}{2}\,d,\ q \in \mathbb{N}$ in order to achieve an overall system delay of $D = q\,M\,d + (M-1)\,d$ samples. Note that when we have a linear-phase prototype $p(n) = q(n) = p(L_p - 1 - n)$ satisfying (11) it can be shown that its type 1 polyphase components also satisfy the PR-conditions for the uniform twofold oversampled DFT filter bank except for an amplification factor. Thus, a PR prototype designed for the oversampled uniform case [6] can be used also in the non-uniform case and only causes linear distortions due to the nonideal phase compensation.

Subsampled case. The input-output relation for the analysis-synthesis filter bank in the subsampled case can be given with

$$\hat{X}(z) = \sum_{k=0}^{M-1} \frac{1}{N_k} \sum_{\ell=0}^{N_k-1} X(z\,W_{N_k}^{\ell})\,H_k(z\,W_{N_k}^{\ell})\,G_k(z) \qquad (12)$$

where the $G_k(z)$ denote the subband synthesis filters. In this case the aliasing components should be suppressed by a sufficiently high stopband attenuation of the prototype filters designed for the non-subsampled case. Therefore, the subsampling factor N_k in the k-th subband has to be chosen such that the aliasing components in each subband do not overlap with the subband spectrum of the input signal. However, critical subsampling without strong aliasing distortions in the reconstructed signal is not possible here (exactly as for the uniform case with $L_p > M$ [7]).

In the following we state an expression for the individual aliasing transfer functions. Let N denote the least common multiple of all subsampling factors $N_k,\ k = 0, \ldots, M-1$. Then, with $L_k = N/N_k$ (12) can also be written as

$$\hat{X}(z) = \frac{1}{N} \sum_{\ell=0}^{N-1} X(z\,W_N^{\ell}) \sum_{k=0}^{M-1} H_k(z\,W_N^{\ell})\,G_k(z) \sum_{\lambda=0}^{L_k-1} W_N^{\lambda \ell N_k} \qquad (13)$$

where $\sum_{\lambda=0}^{L_k-1} W_N^{\lambda \ell N_k} = \begin{cases} L_k & \text{for } \ell \in \{0, L_k, \ldots, L_k(N_k - 1)\}, \\ 0 & \text{elsewhere.} \end{cases}$

The aliasing transfer functions $U_\ell(z),\ \ell = 1, \ldots, N{-}1$ for the ℓ-th aliasing component $X(zW_N^{\ell})$ can be obtained from (13) as

$$U_\ell(z) = \frac{1}{N} \sum_{k=0}^{M-1} H_k(z\,W_N^{\ell})\,G_k(z) \sum_{\lambda=0}^{L_k-1} W_N^{\lambda \ell N_k}. \qquad (14)$$

Thus, in the case of unequal subsampling factors in the subbands we generally have $N - 1$ aliasing components. Note that some of these components may be zero, which depends on the choice of the factors N_k. As an example an allpass-transformed DFT bank is designed for $M = 8$ subbands with the set of subsampling factors chosen as $\mathbf{n} = [N_0, \ldots, N_k, \ldots, N_{M-1}] = [4, 3, 3, 2, 2, 2, 3, 3]$. The resulting magnitude bifrequency system function [8] in Fig. 4 reveals that only five different (attenuated) aliasing components are present in the reconstructed signal.

3.3. Design via lifting factorizations

For longer prototypes with higher stopband attenuation the system delay may increase drastically due to the longer allpass-transformed analysis polyphase components, which also require higher order compensation filters. As a possible solution we utilize a lifting factorization [9] for the analysis and synthesis prototype. This method has the advantage that by applying lifting and dual lifting steps with different amounts of additional delay to the polyphase components of the prototype filters, the length of the filters can be increased while constraining the overall delay to a desired value. In a first step the PR-conditions in (11) for every k can be written as

$$\left[P_k(A^M(z)) \quad A^{\frac{M}{2}}(z) P_{k+\frac{M}{2}}(A^M(z))\right] \mathbf{I} \begin{bmatrix} z^{-\frac{M\,d}{2}} \hat{Q}_{M-1-k}(z) \\ \hat{Q}_{\frac{M}{2}-1-k}(z) \end{bmatrix} \stackrel{!}{=} \frac{2}{M^2}\,z^{-s}\,F^k(z).$$

The identity matrix is then replaced by $\mathbf{I} = \mathbf{C}\,\mathbf{B}\,(\mathbf{C}\,\mathbf{B})^{-1}$, where in a zero delay lifting step the matrices $\mathbf{B}$ and $\mathbf{C}$ are defined according to

$$\mathbf{CC}^{-1} = \begin{bmatrix} 1 & 0 \\ c\,A^{\frac{M}{2}}(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -c\,A^{\frac{M}{2}}(z) & 1 \end{bmatrix} = \mathbf{I},$$

$$\mathbf{BB}^{-1} = \begin{bmatrix} 1 & b\,A^{\frac{M}{2}}(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -b\,A^{\frac{M}{2}}(z) \\ 0 & 1 \end{bmatrix} = \mathbf{I}.$$

The parameters b and c represent free parameters, which can be optimized to improve the stopband attenuation of the prototype filter. Likewise, we can use a single lifting step approach, which increases the delay by $2\,M\,d$ samples. The overall system delay is given as $D = q_2\,M\,d + (M-1)\,d$ with $q_2 = (m_i - 1) + r_s$, where $L_{p_i} = m_i\,M$ denotes the length of the initial prototype and r_s the number of the single delay lifting steps.

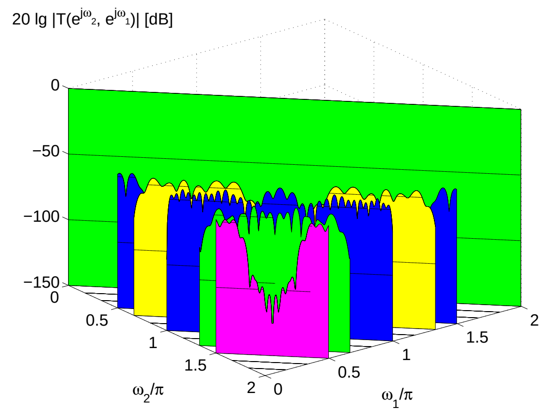

Fig. 4. Magnitude bifrequency system function for the parameters $M = 8$ and $\mathbf{n} = [4, 3, 3, 2, 2, 2, 3, 3]$

4. DESIGN EXAMPLES

4.1. Prototype design

In order to design the lowpass prototype we utilize the lifting factorization from Section 3.3 for the uniform case with $a = 0$, where a rectangular window of length L_{p_i} is used as initial filter. We select zero lifting and single lifting steps in such a way that an overall system delay of $D = (3M - 1)d$ samples is obtained for the analysis-synthesis system, where the free parameters in the lifting factorization are fixed by nonlinear optimization under minimization of the stopband energy. The magnitude frequency responses for two design examples with the design parameters (a) $L_p = 64$, $L_{p_i} = 8$, $M = 8$, $D = 23\,d$ and (b) $L_p = 192$, $L_{p_i} = 24$, $M = 24$, $D = 71\,d$ are displayed in Fig. 5.

4.2. Filter bank design examples

Case $M = 8$**.** In this example we apply the prototype from Fig. 5(a) to both the analysis and synthesis bank of the allpass-

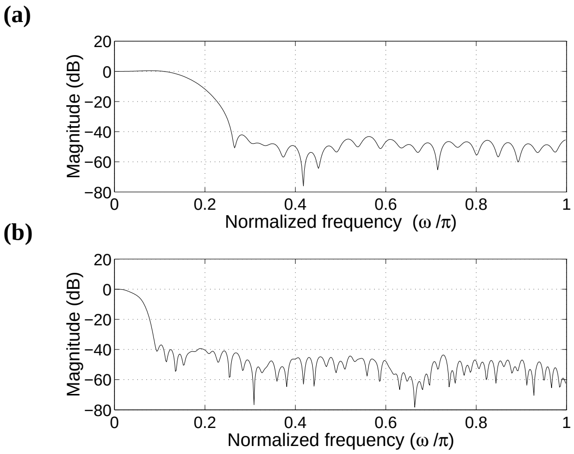

Fig. 5. Magnitude frequency responses for two prototype design examples: (a) $M = 8$, $L_p = 64$, $L_{p_i} = 8$, $D = 23\,d$; (b) $M = 24$, $L_p = 192$, $L_{p_i} = 24$, $D = 71\,d$.

transformed DFT filter bank, where the design parameters are chosen as $a = -0.3$ and $d = 8$ leading to an overall system delay of $D = 184$ samples. In Fig. 6(a) the resulting magnitude frequency responses of the analysis subband filters are depicted. For the subband subsampling factors $\mathbf{n} = [6, 6, 4, 2, 2, 2, 4, 6]$ we obtain the peak aliasing distortion $E_a(\omega)$ according to Fig. 6(b), which is calculated as the root mean square (RMS) error over all non-zero aliasing transfer functions $U_\ell(z)$ in (14). We can see that the aliasing distortion has the same order of magnitude as the stopband attenuation of the prototype in Fig. 5(a).

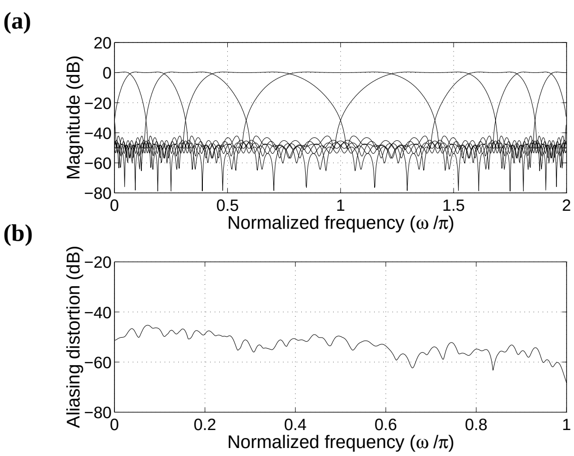

Fig. 6. Overall analysis-synthesis system ($M = 8$, $a = -0.3$, $d = 8$, prototype from Fig. 5(a)): (a) Subband magnitude frequency responses, (b) RMS aliasing distortion for $\mathbf{n} = [6, 6, 4, 2, 2, 2, 4, 6]$.

Case $M = 24$. In this case we apply the prototype filter from the design example in Fig. 5(b) to an analysis-synthesis system with $M = 24$ and $a = -0.3$. With $d = 8$ an overall system delay of $D = 568$ is obtained. Fig. 7 shows the resulting magnitude frequency responses for the subband filters and the group delay.

5. CONCLUSION

This paper generalizes a near-PR design approach for a stable FIR-based synthesis filter bank corresponding to an allpass-transformed oversampled DFT analysis bank. As a new result we have shown that the overall analysis-synthesis system satisfies the near-PR property also for different subsampling factors in the subbands. Furthermore, the proposed system has the interesting property that an increased system delay may be traded for a lower reconstruction error and vice versa. By using allpass-transformed lifting steps for representing the polyphase components of the prototypes the overall system delay can be constrained to a desired value independently of the prototype lengths.

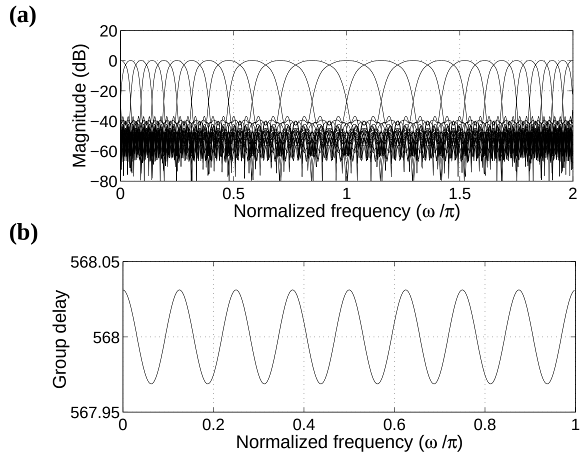

Fig. 7. Overall analysis-synthesis system ($M = 24$, $a = -0.3$, $d = 8$, prototype from Fig. 5(b)): (a) Magnitude frequency responses for the subband filters, (b) group delay $D \approx 71\,d = 568$.

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