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On the Design of Near-Perfect-Reconstruction IIR QMF Banks Using FIR Phase-Compensation Filters

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Abstract

In this paper we describe a novel approach for the design of near-perfect-reconstruction mixed FIR-/allpass-based quadrature mirror filter banks. The design is carried out in the polyphase domain, where FIR filters, obtained via simple closed-form expressions, are employed for compensating the non-linear phase introduced by the allpass filters. Starting from a generalized two-band structure, we introduce three different types of analysis-synthesis banks based on the same design principle. In all systems the remaining amplitude and phase distortions can be made arbitrarily small at the expense of additional system delay. Simultaneously, aliasing can be minimized, or completely canceled if further delay can be tolerated.

1. Introduction

Maximally decimated quadrature mirror filter banks (QMF banks) are utilized in a variety of applications, for example in audio and image compression, where most design techniques are dealing with FIR analysis and synthesis filters. However, employing IIR filters leads to very efficient allpass-based realizations [4, 8, 9], which can be designed such that they are free from aliasing and amplitude distortion, but phase distortion usually remains in the reconstructed signal.

One solution for canceling the phase distortion would be using anticausal filtering [2] and employing a double buffering scheme as in [1, 7] for the processing of infinite-length signals. However, this method leads to a computationally costly synthesis filter bank, which requires segmentation and time-reversal of the subband signals. Therefore, more recent publications propose the use of stable FIR- or mixed FIR-/IIR-based synthesis filter banks, where in most cases the synthesis filters are optimized such that the overall system satisfies the near-PR property (see for example [5, 6, 10]).

In this paper we present a novel low-complexity near-PR design technique for mixed FIR-/allpass-based QMF filter banks, which uses FIR filters for phase compensation in the polyphase domain. In contrast to many other approaches in the literature (e.g. [5, 6, 10]) the FIR compensation filters are designed via simple closed-form expressions. This approach has the interesting property that an increase of the reconstruction error at the output of the synthesis filter bank can be traded for a reduction of the overall system delay and vice versa.

2. Two-Band QMF Banks

In this section we review the basic relations for the two-band critically subsampled filter bank structure depicted in Fig. 1, where $H_k(z)$ denotes the analysis subband filters and $G_k(z)$ the synthesis subband filters for $k = 0, 1$. The input-output relation for this system can be given as

$$\hat{X}(z) = X(z)\, T_{\text{lin}}(z) + X(-z)\, T_{\text{alias}}(z) \qquad (1)$$

with the linear distortion transfer function

$$T_{\text{lin}}(z) = \frac{1}{2}\,[H_0(z)\, G_0(z) + H_1(z)\, G_1(z)]. \qquad (2)$$

If $T_{\text{lin}}(z) \stackrel{!}{=} c \cdot z^{-D}$ with $c \in \mathbb{R}$ and D denoting the overall system delay, the filter bank has no amplitude and phase

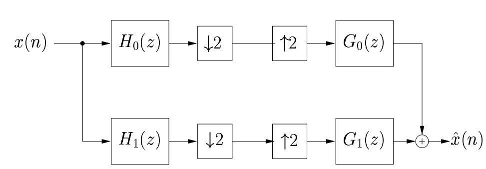

Figure 1. Two-band filter bank structure

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distortion. The transfer function $T_{\text{alias}}(z)$ in (1) corresponds to the aliasing distortion and can be written as follows:

$$T_{\text{alias}}(z) = \frac{1}{2}\left[H_0(-z)\,G_0(z) + H_1(-z)\,G_1(z)\right]. \quad (3)$$

By choosing the synthesis filters such that

$$G_0(z) = H_1(-z) \quad \text{and} \quad G_1(z) = -H_0(-z) \quad (4)$$

we can completely cancel aliasing. Furthermore, when the analysis filters are related as $H_0(z) = H_1(-z)$ they are referred to as quadrature mirror filters. A very efficient way of representing the analysis filter bank can be obtained by using polyphase components.

In the following we consider QMF banks where (4) is satisfied and where the polyphase components consist of allpass transfer functions $A_k(z)$, $k = 0, 1$, thus leading to IIR subband filters. The analysis and synthesis filters have the power-complementarity property and can be given as follows [9]:

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} A_0(z^2) \\ z^{-1}A_1(z^2) \end{bmatrix}, \quad (5)$$

$$\begin{bmatrix} G_0(z)\,G_1(z) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} z^{-1}A_1(z^2)\,A_0(z^2) \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (6)$$

The resulting analysis-synthesis structure is depicted in Fig. 2. By inserting (5) and (6) into (2) we obtain the linear distortion function

$$T_{\text{lin}}(z) = \frac{1}{2}\,z^{-1}\,A_0(z^2)\,A_1(z^2),$$

which is now an allpass transfer function. Thus, amplitude distortion is eliminated and aliasing distortion is canceled since (4) is satisfied, but phase distortion persists and depends on the phase responses of the allpass filters $A_0(z)$ and $A_1(z)$.

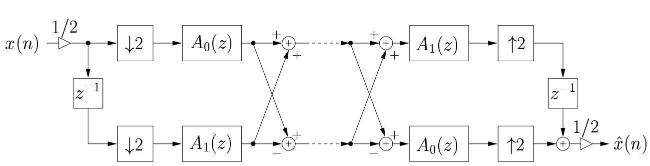

Figure 2. Two-band power complementary allpass filter bank

3. Proposed Filter Bank Design

3.1. Compensation of the phase distortion

It has been shown in [3] that an FIR filter of order d can be constructed such that the phase distortion of an allpass filter is approximately compensated up to a certain error. We assume in the following that $A_i(z)$, $i = 0, 1$, is a stable and causal allpass filter of first order according to

$$A_i(z) = \frac{z^{-1} + a_i}{1 + a_i z^{-1}}, \quad 0 < a_i < 1, \quad a_i \in \mathbb{R}. \quad (7)$$

The design of the FIR phase compensation filter is based on the polynomial factorization relation

$$z^{-d} - (-1)^d \cdot a^d = (z^{-1} + a)\sum_{k=0}^{d-1}(-1)^k\,a^k\,z^{-(d-1-k)} \quad (8)$$

with $d \in \mathbb{N}$. Exploiting this relation, the FIR filters $F_i(z)$, $i = 0, 1$, can now be defined according to

$$\begin{aligned} F_i(z) &= (1 + a_i\,z^{-1})\sum_{k=0}^{d_i-1}(-1)^k\,a_i^k\,z^{-(d_i-1-k)}, \quad (9) \\ &= (-1)^{d_i-1}\,a_i^{d_i-1} + \\ &+ \sum_{k=1}^{d_i-1}(-1)^{d_i-1-k}\,a_i^{d_i-1-k}\,(1 - a_i^2)\,z^{-k} + a_i\,z^{-d_i}. \quad (10) \end{aligned}$$

It is now easy to show from (7), (8) and (9) that

$$A_i(z)\,F_i(z) = z^{-d_i} - (-1)^{d_i}\,a_i^{d_i} = z^{-d_i} - (-1)^{d_i}\,\epsilon(a_i, d_i). \quad (11)$$

Since $0 < a_i < 1$, we can select the order d_i of the filter $F_i(z)$ such that the error $\epsilon(a_i, d_i)$ can be made arbitrarily small. Then (11) can be approximated with the delay $A_i(z)\,F_i(z) \approx z^{-d_i}$.

3.2. Generalized two-band IIR / FIR filter bank

Let us now consider the generalized two-band filter bank depicted in Fig. 3, where the $A_{a/s,k}(z)$, $k = 0, 1$, denote allpass filters and the $F_{a/s,k}(z)$ FIR filters in the analysis and synthesis bank, respectively. A general expression for the linear distortion function $T_{\text{lin}}(z)$ can be given as

$$\begin{aligned} T_{\text{lin}}(z) = \frac{1}{4}\,z^{-1}\big(&A_{a,0}(z^2)\,F_{a,0}(z^2)\,A_{s,0}(z^2)\,F_{s,0}(z^2) + \\ &+ A_{a,1}(z^2)\,F_{a,1}(z^2)\,A_{s,1}(z^2)\,F_{s,1}(z^2)\big), \quad (12) \end{aligned}$$

and for the aliasing function $T_{\text{alias}}(z)$ we have

$$\begin{aligned} T_{\text{alias}}(z) = \frac{1}{4}\,z^{-1}\big(&A_{a,0}(z^2)\,F_{a,0}(z^2)\,A_{s,0}(z^2)\,F_{s,0}(z^2) - \\ &- A_{a,1}(z^2)\,F_{a,1}(z^2)\,A_{s,1}(z^2)\,F_{s,1}(z^2)\big). \quad (13) \end{aligned}$$

In the following we discuss three special cases arising from Fig. 3.

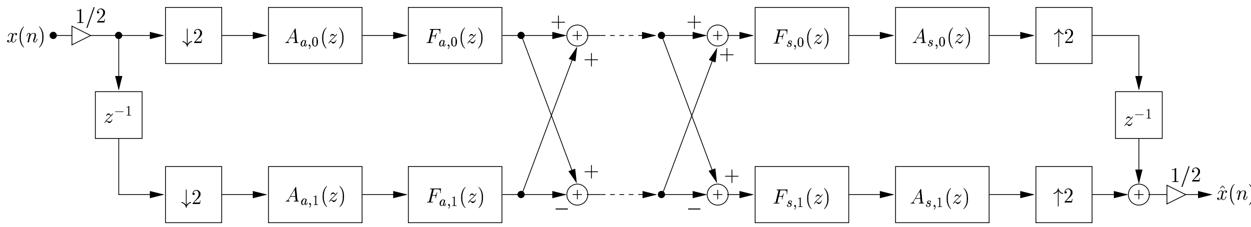

Figure 3. Generalized two-band analysis-synthesis IIR / FIR filter bank

3.2.1 Case (i): Minimization of amplitude, phase and aliasing distortion

By choosing

$$F_{a,0}(z)=1, A_{a,0}(z)=A_0(z), F_{a,1}(z)=1, A_{a,1}(z)=A_1(z), \tag{14}$$

we obtain the classical power-complementary analysis filter bank with allpass polyphase components from Section 2. A simple synthesis filter bank that employs only FIR subband filters and ensures near-PR can be constructed with the following choice of the synthesis filters in Fig. 3:

$$\begin{aligned} F_{s,0}(z) &= 2\,F_0(z), \quad A_{s,0}(z) = z^{-(d_1-d_0)}, \\ F_{s,1}(z) &= 2\,F_1(z), \quad A_{s,1}(z) = 1, \end{aligned} \tag{15}$$

where $F_0(z)$ and $F_1(z)$ are designed according to (10). Without loss of generality we can assume that $d_1 > d_0$. In the following we show that by using (15) in the synthesis bank amplitude, phase and aliasing distortions can be made arbitrarily small, depending on the delay of the filter bank.

The linear distortion transfer function can be obtained by inserting (14) and (15) into (12) and applying (11) as

$$\begin{aligned} T_{\text{lin}}(z) &= z^{-2\,d_1-1}+ \\ &+ \underbrace{\frac{1}{2}\big((-1)^{d_0+1}\,a_0^{d_0}\,z^{-2(d_1-d_0)-1} + (-1)^{d_1+1}\,a_1^{d_1}\,z^{-1}\big)}_{=:\,E_{\text{lin}}(z)}, \end{aligned}$$

where $E_{\text{lin}}(z)$ denotes the remaining linear distortion error. If the parameters d_i with $d_0 < d_1$ are selected such that $\epsilon(a_i, d_i) \approx 0$ in (11), we also have $E_{\text{lin}}(z) \approx 0$. Then $T_{\text{lin}}(z)$ has approximately linear phase and we have $|T_{\text{lin}}(e^{j\omega})| \approx 1$ for all ω. Furthermore, the overall system delay of the filter bank is fixed to the value $D = 2\,d_1 + 1$.

Likewise, by combining (14), (15), (13) and (11) the aliasing distortion transfer function can be written according to

$$T_{\text{alias}}(z) = \frac{1}{2}\,(-1)^{d_1}\,a_1^{d_1} z^{-1} - \frac{1}{2}\,(-1)^{d_0}\,a_0^{d_0} z^{-2(d_1-d_0)-1}.$$

Again, if the parameters a_0, a_1, d_0 and d_1 are chosen appropriately, the aliasing distortion transfer function tends to zero at all frequencies.

3.2.2 Case (ii): Minimization of amplitude and phase distortion, cancelation of aliasing distortion

In this case, the filters in the analysis bank are again chosen as in (14). For the synthesis filter bank we first define $Q_i(z) = A_i(z)\,F_i(z) = z^{-d_i} - (-1)^{d_i}\,a_i^{d_i}$ as FIR filter of order d_i. The synthesis polyphase components in Fig. 3 are then specified as

$$\begin{aligned} F_{s,0}(z) &= 2\,F_0(z)\,Q_1(z), \quad A_{s,0}(z) = 1, \\ F_{s,1}(z) &= 2\,F_1(z)\,Q_0(z), \quad A_{s,1}(z) = 1. \end{aligned} \tag{16}$$

It is easy to verify that aliasing is completely canceled, i.e. $T_{\text{alias}}(z) = 0$. The linear distortion transfer function $T_{\text{lin}}(z)$ of the near-PR analysis-synthesis system can be stated from (11), (12), (14) and (16) as

$$\begin{aligned} T_{\text{lin}}(z) &= z^{-(2\,d_1+2\,d_0+1)} + E_{\text{lin}}(z) \quad \text{with} \\ E_{\text{lin}}(z) &= (-1)^{d_0+d_1}\,a_0^{d_0}\,a_1^{d_1}\,z^{-1} - \\ &- (-1)^{d_0}\,a_0^{d_0}\,z^{-(2\,d_1+1)} - (-1)^{d_1}\,a_1^{d_1}\,z^{-(2\,d_0+1)}. \end{aligned}$$

Clearly, the linear distortion error $E_{\text{lin}}(z)$ represents the only distortion in the reconstructed signal. The aliasing cancelation approach leads to an increased system delay of $D = 2\,d_1 + 2\,d_0 + 1$ samples compared to case (i).

3.2.3 Case (iii): Almost linear-phase analysis and synthesis filters

Two-band QMF filter banks with linear-phase analysis and synthesis filters are desired in some applications. One simple way to reduce the group delay of the analysis and synthesis filters and to obtain approximately linear-phase is based on a simple modification of the structure from case (ii) above. In the analysis filter bank of Fig. 3 the polyphase componenents are chosen according to

$$\begin{aligned} A_{a,0}(z) &= 1, \quad F_{a,0}(z) = Q_0(z), \\ A_{a,1}(z) &= A_1(z), \quad F_{a,1}(z) = F_0(z). \end{aligned} \tag{17}$$

Likewise we have

$$\begin{aligned} A_{s,0}(z) &= 1, \quad F_{s,0}(z) = 2\,Q_1(z), \\ A_{s,1}(z) &= A_0(z), \quad F_{s,1}(z) = 2\,F_1(z), \end{aligned} \tag{18}$$

in the synthesis filter bank. Now, the modified analysis filters can be given as

$$H_0'(z) = \frac{1}{2}\left(Q_0(z^2) + z^{-1}A_1(z^2)F_0(z^2)\right) = H_0(z)\,F_0(z^2),$$
$$H_1'(z) = \frac{1}{2}\left(Q_0(z^2) - z^{-1}A_1(z^2)F_0(z^2)\right) = H_1(z)\,F_0(z^2). \tag{19}$$

These filters have a smaller deviation from a linear phase response compared to the pure allpass-based QMF analysis bank, since the multiplication of $A_0(z)$ and $A_1(z)$, resp., with $F_0(z)$ also ensures a partial phase compensation for $A_1(z)$. Similar considerations hold for the synthesis filters.

Note that the magnitude frequency responses $|H_0'(e^{j\omega})|$ and $|H_1'(e^{j\omega})|$ of the filters in (19) are identical to those corresponding to the pure allpass-based analysis filter bank $|H_0(e^{j\omega})|$ and $|H_1(e^{j\omega})|$ from (5) since the compensation filter $F_0(z)$ has approximate allpass behavior:

$$|H_i'(e^{j\omega})| = |F_0(e^{j2\omega})|\,|H_i(e^{j\omega})| \approx |H_i(e^{j\omega})|, \quad i = 0, 1.$$

Furthermore, it is straightforward to verify that the same delay and reconstruction errors as in case (ii) hold here.

4. Design examples

Analysis filter bank. In order to design the IIR analysis filters in (5) we use a direct optimization approach, where the coefficients a_0 and a_1 of the first-order allpass filters are obtained via nonlinear optimization under minimization of the stopband energy [9]. The resulting magnitude frequency responses for the analysis filters with the values $a_0 = 0.2031$ and $a_1 = 0.6783$ are depicted in Fig. 4, where the stopband edge frequency for the lowpass filter was chosen as $\omega_s = 0.61\,\pi$. This design example will be used as corresponding analysis bank in all three following cases.

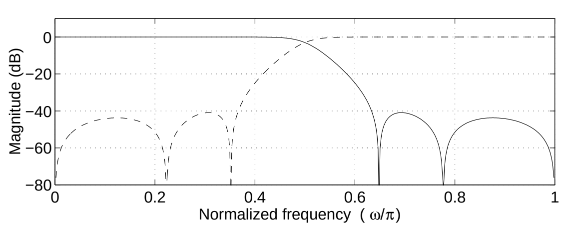

Figure 4. Magnitude frequency responses in the allpass-based IIR QMF analysis bank ($|H_0(e^{j\omega})|$: solid line, $|H_1(e^{j\omega})|$: dashed line)

Case (i). Here we choose the synthesis polyphase components according to (15), where the filters $F_i(z)$ are designed via (10). By using the above design example for the analysis bank, we obtain the amplitude distortion depicted in Fig. 5(a) for the overall analysis-synthesis system. The solid line corresponds to $d_0 = 8$, $d_1 = 30$, which results in an overall system delay of $D = 61$ samples, whereas the dashed line refers to $d_0 = 6$, $d_1 = 24$, leading to a delay of $D = 49$ samples. The magnitude aliasing distortion $|T_{\text{alias}}(e^{j\omega})|$ for both parameter sets is shown in Fig. 5(b), and the group delay for the analysis-synthesis system is given in Fig. 5(c). We can see that the overall distortion function has approximately linear phase. Note that the choice of d_0 and d_1 is arbitrary and reflects a compromise between low complexity and low delay and the minimization of aliasing and linear distortion.

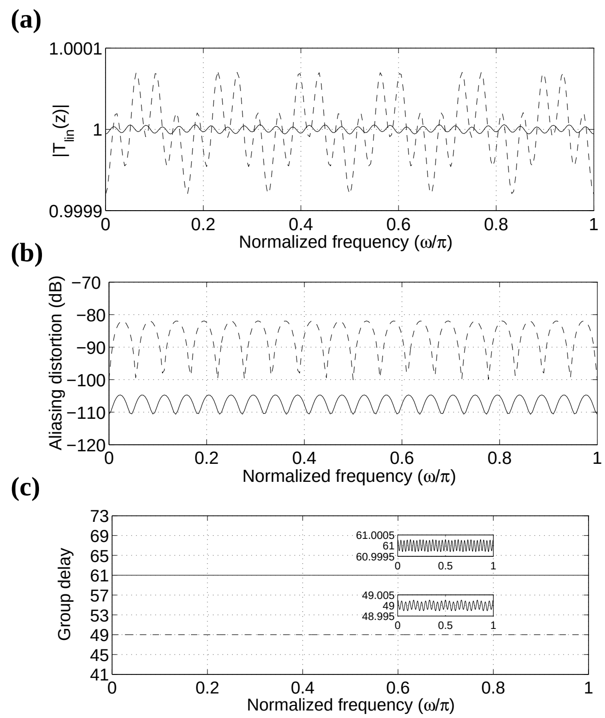

Figure 5. Overall analysis-synthesis system for case (i) (dashed: $d_0 = 6$, $d_1 = 24$; solid: $d_0 = 8$, $d_1 = 30$): (a) Amplitude distortion, (b) aliasing distortion, (c) group delay.

Case (ii). In this case the synthesis polyphase components are given in (16). This choice ensures that the aliasing distortion is exactly zero at all frequencies. Fig. 6 shows the amplitude distortion and the group delay for the above analysis bank example, using the delay parameters $d_0 = 6$ and $d_1 = 24$.

Case (iii). Here, all distortions are the same as in case (ii) (see Fig. 6), and the delay parameters are chosen again as $d_0 = 6$ and $d_1 = 24$. The magnitude frequency responses and group delays are shown in Fig. 7 for the analysis filters, and in Fig. 8 for the synthesis filters, respectively. It can be observed that the analysis and the synthesis filters have approximately linear-phase both in the passband and the stopband.

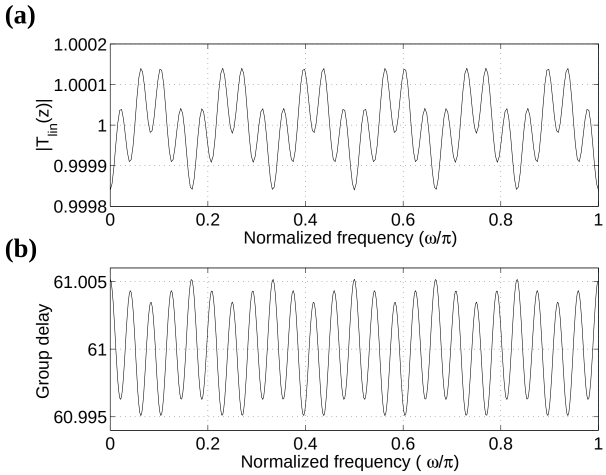

Figure 6. Overall analysis-synthesis systems for cases (ii) / (iii) ($d_0 = 6$, $d_1 = 24$): (a) Amplitude distortion, (b) group delay.

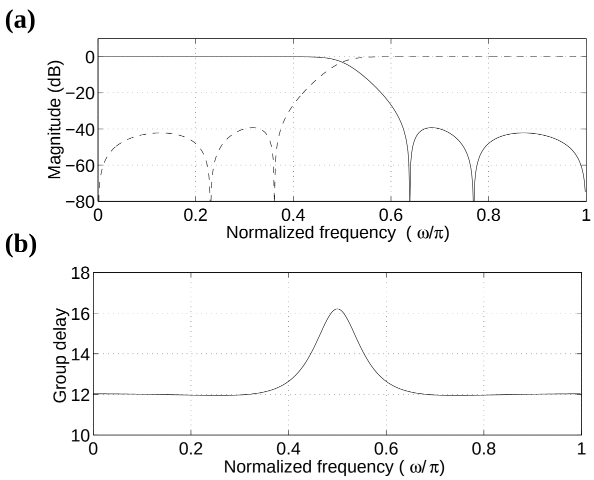

Figure 7. Analysis filters, case (iii): (a) Magnitude frequency responses; (b) group delays.

5. Conclusion

We have proposed a new method for the near-PR design of a two-band IIR power complementary QMF filter bank, where FIR filters are employed for phase compensation in the polyphase domain. These phase compensation filters can be easily designed via closed-form expressions. We have shown that all distortions at the output of the synthesis filter bank can be made arbitrarily small at the expense of additional system delay. Especially, it is also possible to cancel the aliasing components completely, and to allow subband filters with approximate linear phase if further delay can be tolerated.

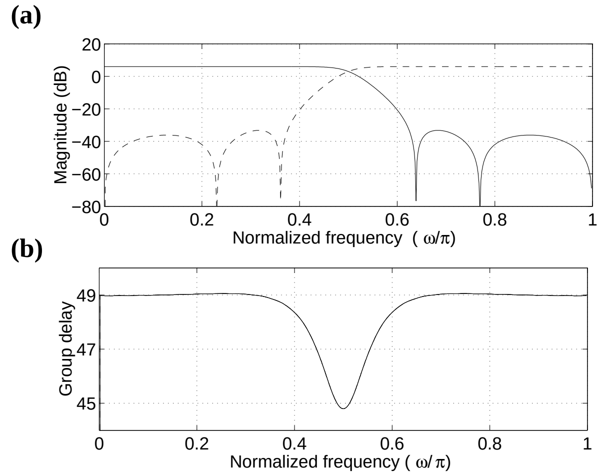

Figure 8. Synthesis filters, case (iii): (a) Magnitude frequency responses; (b) group delays

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