On the Performance of Joint and Separate Channel and Network Coding in Wireless Fading Networks

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***Abstract*—For wireless fading networks where M source nodes wish to multicast their information to N destination nodes via a common relay, it has been shown that network coding at the relay can reduce the number of required transmissions, if the destination nodes overhear $M-1$ source nodes transmitting to the relay. Nested coding is a recently proposed alternative where, unlike network coding, physical and network layers are jointly designed. We find that for a given network throughput ideal nested coding in many situations can lead to a significant increase in transmission reliability compared to network coding. We also consider the case of feedback and retransmissions when received packets fail to be decoded. For the case of two source nodes it is shown that by using nested codes the expected number of relay transmissions is reduced. However, for a larger number of source nodes the gain depends on the employed nested coding strategy at the relay.**

I. Introduction

Various works, e.g., [1], [2], [3], [4], show that network coding can offer significant benefits for wireless networks, in terms of throughput or energy. Many works consider separate channel and network coding. Since channel and network coding do not separate in general for wireless networks [5], it is natural to consider whether practical gains can be achieved with joint channel and network coding.

Nested codes have been originally proposed in [6] for the generalized broadcast problem with *a priori* information at the receivers, and a related concept was used in [7] in the context of two-way relaying. The idea is that instead of information words, codewords of different subcodes are algebraically superimposed via a bitwise XOR. Since any combination of the subcodes is intended to form a good channel code, the effective code rate depends on the available *a priori* information at the destination node.

In this paper we compare the performance of network coding and nested codes at a relay in simple wireless fading networks, where M source nodes multicast their information via a common relay to N destination nodes. Such a setup is a typical building block for network coding in wireless ad-hoc networks [3]. Our goal is to investigate whether nested codes can lead to performance improvements. We first look at the reliability of the overall transmission to the destination nodes as a function of rate and throughput for both schemes. Then, we consider the case of feedback and retransmissions on links, and compare the expected number of retransmissions for network and nested coding.

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II. Wireless network model

Consider the scenario in Fig. 1 where M source nodes $\mathcal{S}_\ell$,

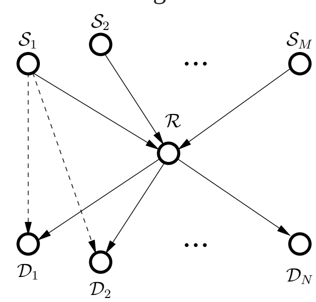

Fig. 1. Joint relaying of data from multiple source nodes $\mathcal{S}_\ell$, $1 \le \ell \le M$, via a relay $\mathcal{R}$, where each destination node $\mathcal{D}_i$, $1 \le i \le N$, intends to receive the information from all source nodes.

$1 \le \ell \le M$, intend to jointly broadcast their data via the relay $\mathcal{R}$ to N destination nodes $\mathcal{D}_i$, $1 \le i \le N$. Since the wireless medium is a shared medium several destination nodes may also overhear the direct transmission from $\mathcal{S}_\ell$, where an example for $\mathcal{S}_1$ is given by the dashed lines in Fig. 1. If the destination nodes are able to decode the overheard message from the source nodes, they can use this information as *a priori* knowledge in order to increase the achievable rate region for the relay-to-destination broadcast channel.

A. Nested codes

In particular, a joint channel-network coding approach has been proposed in [6] to address this problem. In this approach all data at the source nodes is broadcasted to the destination nodes in $M+1$ time slots. Each of the M uplink transmissions consists of encoding a length-k_ℓ information word $\mathbf{i}_\ell$ with an rate $R_\ell = k_\ell/n$ channel code with the generator matrix $\mathbf{G}_\ell$. At the relay, decoding the received codewords $\mathbf{i}_\ell^T\,\mathbf{G}_\ell$ leads to M information vectors $\hat{\mathbf{i}}_\ell$. For the downlink a new codeword $\mathbf{c}$ is now generated according to

$$\mathbf{c}^T = \hat{\mathbf{i}}_1^T\,\mathbf{G}_1 \oplus \hat{\mathbf{i}}_2^T\,\mathbf{G}_2 \oplus \cdots \oplus \hat{\mathbf{i}}_M^T\,\mathbf{G}_M \qquad (1)$$

and broadcasted to the destination nodes, where $\oplus$ represents a bitwise XOR. The overall coderate of the resulting nested code with generator matrix $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_M]^T$ is $R = 1/n \cdot \sum_{\ell=1}^{M} k_\ell$ for all destination nodes.

The construction of the overall codeword $\mathbf{c}$ in (1) can be seen as a form of joint network-channel coding where the individual codewords associated with each source node are XORed. In contrast, a separation of network and channel coding implies XORing the individual information words $\hat{\mathbf{i}}_\ell$ prior to channel encoding [2], [3]. This can be achieved by choosing identical subcodes in (1), i.e., $\mathbf{G}_\ell = \mathbf{G}'$ with $k_\ell = k$ for all $1 \le \ell \le M$ [6].

Let us now denote $\mathcal{K}_i$ as the set containing the indices of those information words $\hat{\mathbf{i}}_\ell$ which are *a priori* known at the i-th destination node, $1 \le i \le N$, by overhearing the uplink transmission from some of the source nodes. The corresponding codeword contributions $\hat{\mathbf{i}}_\ell^T\,\mathbf{G}_\ell$ can then be removed from (1), which leads to a *lower* overall coderate of $R_i = 1/n \cdot \sum_{\ell \notin \mathcal{K}_i} k_\ell$ compared to the one for $\mathbf{G}$ at the i-th destination node.

Proposition 1. *Let the set $\mathcal{K}_i$ contain the indices of the known information words at the i-th destination node, and let the channel between the relay and the i-th destination node have capacity C_i. For joint network-channel encoding according to* (1) *the rate vector $\mathbf{R} := [R_1, \ldots, R_i, \ldots, R_N]$ is achievable if and only if*

$$\sum_{\ell \notin \mathcal{K}_i} R_\ell \le C_i, \quad 1 \le i \le N.$$

Proposition 1 represents a special case of the generalized broadcast problem in [8] for a degraded broadcast channel. It can be proved analogously to the considerations in [8].

As a consequence of Proposition 1 it is easy to see that for separate network and channel coding the rate vector $\mathbf{R}$ is achievable if and only if for all i $R_\ell \le C_i$ for a single index $\ell \notin \mathcal{K}_i$, $|\mathcal{K}_i| = M-1$, and $R_\ell = 0$ otherwise. Hence, a reliable transmission to the i-th destination node is only possible if $M-1$ source nodes are overheard at the destination node. In contrast, in the joint network-channel coding case for $|\mathcal{K}_i| < M-1$ the destination nodes are only penalized by a higher coderate.

Nested codes for $M = 2$ have been designed in [6] based on convolutional codes and in [9] based on low-density generator matrix codes. However, for $M > 2$ the code design becomes difficult since $\sum_{\ell=1}^{M} \binom{M}{\ell}$ different good codes need to be obtained from any combination of the subcodes. As a workaround, so called partially multiplexed (PMP) codes based on irregular repeat accumulate (IRA) codes [10] are proposed in [11]. However, these codes have the drawback that they suffer from a rate loss compared to nested codes which increases with the rate and the number of subcodes.

B. Channel model

Each channel in the network in Fig. 1 is modeled as an ergodic Rayleigh fading channel with capacity

$$C = \log\left(1 + \frac{\gamma}{d^\rho}\right), \quad \gamma = \frac{f^2\,P}{\sigma^2} \tag{2}$$

where γ/d^ρ is the instantaneous SNR, P is the average transmission power, σ^2 the noise power, d the distance between transmitter and receiver, and ρ the path loss exponent. The fading is modeled as an iid Rayleigh random variable f^2 with $E\{f^2\} = \mu$, leading to the following pdf for γ:

$$p_\gamma(x) = \frac{1}{\Gamma}\exp\left(-\frac{x}{\Gamma}\right) \tag{3}$$

where $\Gamma = \mu\,P/\sigma^2$. For a given rate R on the channel the outage probability is defined as $\mathrm{Pr}_{\mathrm{Out}} = \Pr(C < R) = \Pr(\gamma < (2^R - 1)\,d^\rho)$. By defining the link reliability as $\Lambda = 1 - \mathrm{Pr}_{\mathrm{Out}}$ we obtain

$$\Lambda(d, R) := 1 - \int_0^{(2^R-1)d^\rho} p_\gamma(x)\,dx = \exp\left(-\frac{d^\rho(2^R-1)}{\Gamma}\right) \tag{4}$$

where Γ/d^ρ denotes the average SNR.

III. Transmission reliability and throughput

In the following we compare different transmission strategies at the relay based on routing, network coding, and nested codes. Let us start with an example, the butterfly network in Fig. 2, which is a special case of Fig. 1 for $N = M = 2$. The information at the sources $\mathcal{S}_1$ and $\mathcal{S}_2$ is multicasted to both the destination nodes $\mathcal{D}_1$ and $\mathcal{D}_2$ via the relay $\mathcal{R}$, where $\mathcal{D}_1/\mathcal{D}_2$ is able to overhear $\mathcal{S}_1/\mathcal{S}_2$. The distances of the direct links between source and destination nodes and the relay, resp., are denoted as $d_{\ell i}$, $d_{\ell\mathcal{R}}$, and $d_{\mathcal{R}i}$ for $\ell, i \in \{1, 2\}$. We compare

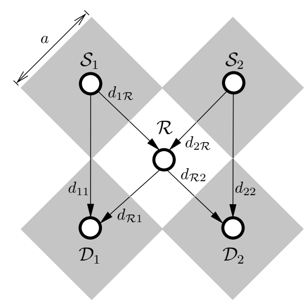

Fig. 2. Simple network example where the information at sources $\mathcal{S}_1$ and $\mathcal{S}_2$ is multicasted to both the destination nodes $\mathcal{D}_1$ and $\mathcal{D}_2$.

the following relay transmission strategies:

1) *Network coding:* We employ binary network coding of the decoded information words at the relay before physical layer error correction is applied. This corresponds to a separation of channel and network coding. The transmission is carried out in three time slots: in the first time slot, $\mathcal{S}_1$ broadcasts to $\mathcal{D}_1$ and $\mathcal{R}$, in the second time slot, $\mathcal{S}_2$ broadcasts to $\mathcal{D}_2$ and $\mathcal{R}$, and in the third time slot, $\mathcal{R}$ broadcasts to $\mathcal{D}_1$ and $\mathcal{D}_2$. For a transmission rate of R on each link the throughput in bits / time slot is given as $T = 2R/3$.
2) *Nested coding:* Here, the same network coding strategy as in case 1) is applied with the difference that joint network-channel coding by using nested codes (see (1)) with $M = 2$ is employed at the relay.
3) *Routing:* In this case the transmission is carried out in four time slots. The first two time slots are as in 1), in the third time slot $\mathcal{R}$ transmits the information word from $\mathcal{S}_1$ to $\mathcal{D}_1$ and $\mathcal{D}_2$, and in the fourth time slot the same is done for the information word from $\mathcal{S}_2$. The throughput is given as $T = 2R$.

Consider now the general case for arbitrary M and N in Fig. 1. For network and nested coding we require $M+1$ time slots for transmitting information from the sources to the receivers where M time slots are used for the uplink and one for the broadcast to the destination nodes. Likewise, for routing $2M$ time slots are required. We now state general expressions for the reliability of the different coding schemes. Let $M' \leq M-1$ represent the maximal number of source nodes which can be overheard at any destination node. Let S_λ denote the set of all overheard source nodes at the λ-th receiver, $|S_\lambda| = M'$, and $S_\lambda(i)$ the i-th element from this set. Then, the reliability is given for ideal network coding as

$$\Lambda_{\text{nec}} = \prod_{\ell=1}^{M} \Lambda(d_{\ell\mathcal{R}}, R) \prod_{\lambda=1}^{N} \sum_{k=0}^{2^{M'}-1} \Lambda(d_{\mathcal{R}\lambda}, (w([\mathbf{k}]_2)+1)\,R)\cdot \prod_{i=1}^{M'} (1-\Lambda(d_{S_\lambda(i),\lambda}, R))^{k_i} \Lambda(d_{S_\lambda(i),\lambda}, R)^{\bar{k}_i} \quad (5)$$

with $R = T(M+1)/M$. The vector $[\mathbf{k}]_2 = [k_1 k_2 \ldots k_{M'}]$ represents the binary notation of the index k with $k_i \in \{0,1\}$, $\bar{k}_i = 1-k_i$, and $w(\cdot)$ denotes the Hamming distance. For network coding, a similar relation can be obtained as

$$\Lambda_{\text{nc}} = \prod_{\ell=1}^{M} \Lambda(d_{\ell\mathcal{R}}, R) \prod_{\lambda=1}^{N} \Lambda(d_{\mathcal{R}\lambda}, R) \prod_{i=1}^{M-1} \Lambda(d_{S_\lambda(i),\lambda}, R). \quad (6)$$

Proposition 2. *For any $M' \leq M-1$ it follows that $\Lambda_{nec} \geq \Lambda_{nc}$. In other words, the reliability of ideal nested coding is always larger or equal than the one for network coding.*

Proof: For $M' < M-1$ the result is trivial since in this case the achievable rates on the relay-to-destination channels are zero for network coding. Let us consider $M' = M-1$. By combining (5) and (6) we can express Λ_{nec} as

$$\begin{aligned}\Lambda_{\text{nec}} &= \Lambda_{\text{nc}} + \prod_{\ell=1}^{M} \Lambda(d_{\ell\mathcal{R}}, R) \prod_{\lambda=1}^{N} \sum_{k=1}^{2^{M'}-1} \Lambda(d_{\mathcal{R}\lambda}, (w([\mathbf{k}]_2)+1)\,R)\cdot \\ &\quad \prod_{i=1}^{M'} (1-\Lambda(d_{S_\lambda(i),\lambda}, R))^{k_i} \Lambda(d_{S_\lambda(i),\lambda}, R)^{\bar{k}_i} \\ &= \Lambda_{\text{nc}} + c(N, M', R)\end{aligned}$$

where $c(N, M', R)$ represents a reliability which is bounded by $0 \leq c(N, M', R) \leq 1$. Hence, $\Lambda_{\text{nec}} \geq \Lambda_{\text{nc}}$. ■

We now present examples for the performance of the different transmission schemes. We first consider the network in Fig. 2 with $M = N = 2$. In order to provide a more realistic scenario (e.g., for ad-hoc wireless networks) we assume that the source and destination nodes are distributed uniformly at random within each square region of area a^2 with $a = 1$ as shown in Fig. 2; the relay is kept at a fixed location. The average SNR for $d^\rho = 1$ is chosen as 20 dB, and the path loss exponent is $\rho = 2$.

The reliability versus throughput relation for different transmission strategies is shown in Fig. 3, averaged over 10,000 random placements of the source and destination nodes. We can observe that, as predicted by Proposition 2, ideal nested coding and also PMP codes (with a design rate of $R = 0.2$) outperforms network coding. Also, routing is seen to be always inferior to all other schemes.

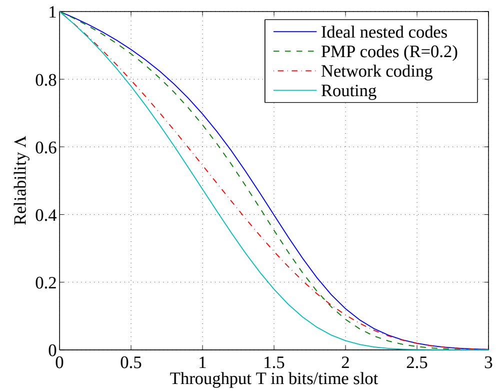

Fig. 3. Reliability versus throughput for different transmission strategies averaged over 10,000 uniformly at random placements of the source and destination nodes in Fig. 2 ($M = N = 2$, SNR= 20 dB, ergodic fading channels, $a = 1$, $\rho = 2$).

The next example considers a network with hexagon cells and $M = N = 3$ source and destination nodes, which are placed uniformly at random within the hexagons. The results for an averaging over 10,000 random placements are shown in Fig. 4. The gain of ideal nested coding is smaller than in the previous example since the hexagonal cell topology seems to provide a sufficient level of overhearing at the destination nodes. Also, PMP codes lead to an inferior performance due to the increased rate loss for larger M.

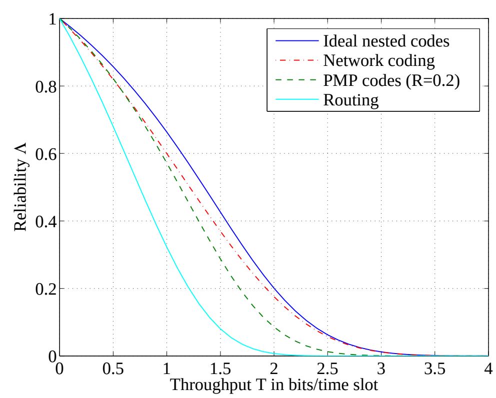

Fig. 4. Reliability versus throughput for a network with hexagon cells and $M = 3$ source and $N = 3$ destination nodes for different transmission strategies ($M' = M-1$). The results are averaged over 10,000 uniformly at random placements of source and destination nodes within the hexagons (SNR= 20 dB, ergodic fading channels, $\rho = 2$).

IV. Networks with Retransmissions

We now incorporate feedback on the network links and look at the case of retransmissions when transmitted packets fail to be received by the intended next hop nodes. In particular, we analyze how network and nested coding at the relay affect the average number of retransmissions in these networks.

Consider multicast of one packet from each of $\mathcal{S}_1, \mathcal{S}_2$ to both $\mathcal{D}_1$ and $\mathcal{D}_2$ in the example network of Figure 2. Suppose that first $\mathcal{S}_1, \mathcal{S}_2$ transmit until both packets are received by relay node $\mathcal{R}$ (during which time $\mathcal{D}_1$ and $\mathcal{D}_2$ may receive a packet from $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively), and then $\mathcal{R}$ transmits until both packets are received by $\mathcal{D}_1$ and $\mathcal{D}_2$.

The classical network coding scenario occurs when $\mathcal{D}_1$ and $\mathcal{D}_2$ each receive a packet directly from $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively, whereupon $\mathcal{R}$ broadcasts a network coded combination of the two packets. However, in the case where only one of the destinations, say $\mathcal{D}_1$, receives a packet directly from $\mathcal{S}_1$, nested coding does better than network coding. The reason is as follows. If the channel between $\mathcal{R}$ and $\mathcal{D}_2$ is sufficiently good, $\mathcal{D}_2$ can receive both packets with a single nested transmission. Otherwise, nested coding is equivalent to network coding in terms of the sequences of channel conditions under which transmissions are successfully completed, since if $\mathcal{D}_2$ receives a nested coded transmission at rate R, the relay can then switch to sending the packet from $\mathcal{S}_2$ directly, i.e., without network or nested coding.

We derive analytical expressions for the expected number of relay transmissions in the latter case. For simplicity we let all links have distance 1, the same SNR, and independent Rayleigh fading. Let Λ_R denote again the link reliability, i.e., the probability that the channel is able to support a transmission rate R. Let $t_i, i = 1, 2$ denote the number of relay transmissions needed for $\mathcal{D}_i$ to have both packets. For network coding and nested coding, the expected total number of transmissions is given by

$$\begin{aligned} E(Tr) &= \sum_{m=1}^{\infty} \Pr(\max(t_1, t_2) \geq m) \\ &= \sum_{m=1}^{\infty} \left[1 - (1 - \Pr(t_1 \geq m))(1 - \Pr(t_2 \geq m))\right]. \end{aligned} \tag{7}$$

For network coding we have

$$\begin{aligned} \Pr(t_1 \geq m) &= (1 - \Lambda_R)^{m-1} \\ \Pr(t_2 \geq m) &= (1 - \Lambda_R)^{m-1} + (m-1)\Lambda_R(1 - \Lambda_R)^{m-2} \end{aligned}$$

For nested coding we have

$$\begin{aligned} \Pr(t_1 \geq m) &= (1 - \Lambda_R)^{m-1} \\ \Pr(t_2 \geq m) &= (1-\Lambda_R)^{m-1} + 1(m-1)(\Lambda_R - \Lambda_{2R})(1-\Lambda_R)^{m-2} \end{aligned}$$

The expected number of relay transmissions, normalized by the rate R, is plotted in Figure 5 for SNR= 20 dB and SNR= 0 dB. The advantage of nested coding over network coding in this scenario is larger for lower SNR. For comparison we include the plot for routing, where

$$E(Tr) = 1/\Lambda_R + \sum_{m=1}^{\infty} \left[1 - (1 - \Pr(t_1 \geq m))^2\right]. \tag{8}$$

We can similarly derive expressions for the expected number of relay transmissions for the network in Fig. 6, where there are three source nodes and three destination nodes communicating via a central relay node. $\mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3$ have received packets directly from the sets of source nodes $\{\mathcal{S}_1, \mathcal{S}_2\}, \{\mathcal{S}_2\}, \{\}$,

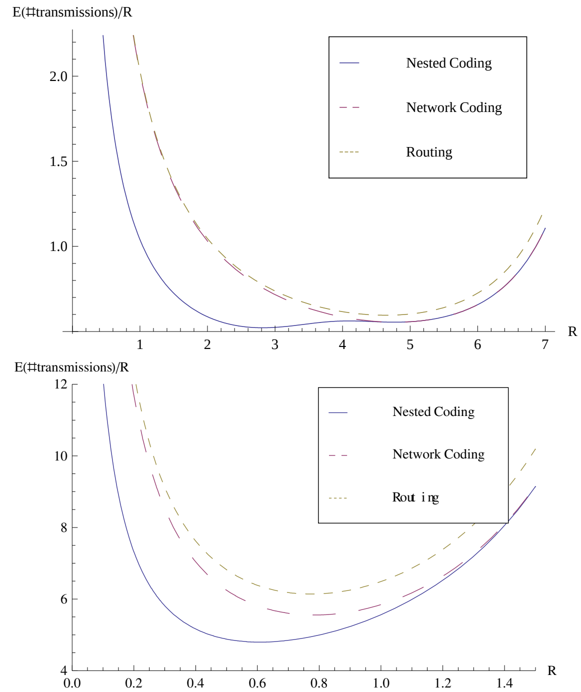

Fig. 5. Plots of the expected number of relay transmissions divided by the rate per transmission for the network of Fig. 2 in the case where $\mathcal{D}_1$ but not $\mathcal{D}_2$ receives a packet on the direct source-destination link. Top graph: SNR= 20 dB, bottom graph: SNR= 0 dB (ergodic fading channels).

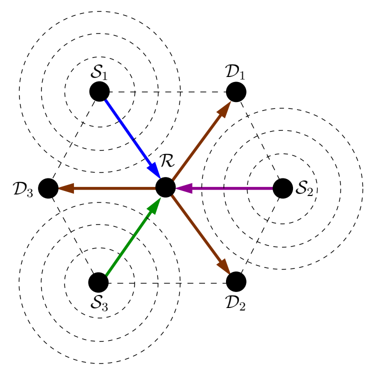

Fig. 6. Network with three source and destination nodes communicating via a central relay.

respectively. For network coding we have

$$\begin{aligned} \Pr(t_1 \geq m) &= (1 - \Lambda_R)^{m-1} \\ \Pr(t_2 \geq m) &= (1 - \Lambda_R)^{m-1} + (m-1)\Lambda_R(1 - \Lambda_R)^{m-2} \\ \Pr(t_3 \geq m) &= (1 - \Lambda_R)^{m-1} + (m-1)\Lambda_R(1 - \Lambda_R)^{m-2} \\ &\quad + \binom{m-1}{2}\Lambda_R^2(1 - \Lambda_R)^{m-3}. \end{aligned}$$

It is more complicated to design an optimal nested coding strategy in this case. For instance, suppose initially the relay broadcasts a nested coded combination of all three packets, and suppose $\mathcal{D}_2$ receives the first transmission at rate R while the other two destination nodes have not received anything. If the relay continues to broadcast a nested coded combination of all three packets, and $\mathcal{D}_2$ receives the next transmission at rate R, it remains unable to decode. On the other hand, if the relay switches to sending either of the packets from $\mathcal{S}_1$ or $\mathcal{S}_3$ directly (without network and nested coding), then $\mathcal{D}_2$ can decode if it receives the next transmission at rate R, but $\mathcal{D}_3$ then loses the possibility of decoding all three packets if it receives at rate $3R$. For simplicity, we analyze the simple time-invariant strategy where the relay continues to broadcast a nested combination of all three packets, which gives

$$\Pr(t_i \geq m) = (1 - \Lambda_{iR})^{m-1} \quad i = 1, 2, 3.$$

The corresponding plot is given in Fig. 7, where we have used

$$E(Tr) = \sum_{m=1}^{\infty} \left[1 - \prod_{i=1}^{3} (1 - \Pr(t_i \geq m)) \right]. \tag{9}$$

This represents a baseline for nested coding which would be exceeded by adaptive strategies taking into account packet receptions by destination nodes.

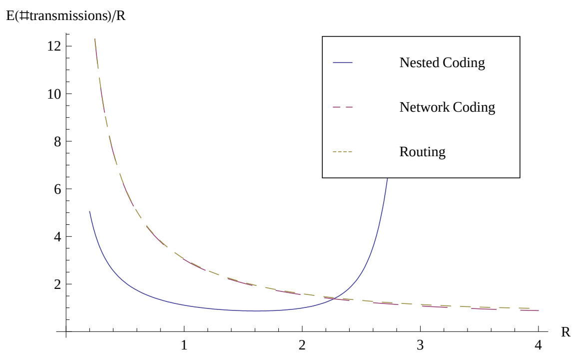

Fig. 7. Expected number of relay transmissions divided by the rate per transmission for the network of Figure 6 (SNR= 20 dB, ergodic fading channels).

In the following we present some preliminary results for the multi-hop case with backpressure routing. We consider the nine node network in Fig. 8 with two unicast sessions. The top

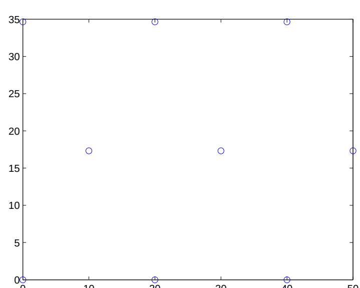

Fig. 8. A nine node grid network.

left node transmits to the bottom right node, and the bottom left node transmits to the top right node. For each link we assume Rayleigh fading, SNR 0 dB, $\rho = 2$, log normal shadowing. We also assume orthogonal modulation, equal node transmission rates, and log utility-based rate control at the sources. We employ FIFO queuing and simple backpressure-style strategies: at each transmission opportunity, a node makes routing and coding decisions, given feedback on overheard packets as in [2], to maximize total expected link rate weighted by queue length difference.

For $R = 0.76$, which is roughly optimal for network coding, the experimental results (throughput, transmissions per packet) are: No nested/network coding: 0.2813, 6.2816, network coding: 0.2936, 5.896, nested+network coding: 0.3118, 5.3515. This corresponds to a gain of about 6% from incorporating nested coding, over network coding alone.

V. Concluding remarks

We have discussed the benefit of network and nested coding at relay nodes in simple wireless networks with fading channels. In particular, networks with two or three source nodes communicating via a single relay have been considered. We have shown that for a given throughput ideal nested coding in many situations leads to a larger transmission reliability than network coding. If in the case of a decoding failure retransmissions are allowed, nested coding is also seen to be advantageous in terms of fewer retransmissions compared to network coding for a two-source network. For three sources several nested coding strategies are possible. As an example, a simple retransmission scheme at the relay seems to be beneficial specifically for low channel SNRs in the network. However, despite these benefits, nested coding has the drawback that for $M > 2$ practical codes are either increasingly hard to design or suffer from a performance loss as the recently proposed PMP codes. Therefore, finding good nested code constructions for larger M might be a rewarding direction for future research. Another interesting research direction is the combination of nested coding with backpressure routing, where we have some promising preliminary results.

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