Joint Channel/Network Coding for Star Networks

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Abstract—Channel coding alone is not sufficient to reliably transmit a message of finite length from a source to one or more destinations as in, e.g., file transfer. To ensure that no data is lost, it must be combined with rateless erasure correcting schemes on a higher layer, such as a time-division multiple access (TDMA) system paired with automatic repeat request (ARQ) or random linear network coding (RLNC). We consider binary channel coding on a binary symmetric channel (BSC) and q-ary RLNC for erasure correction in a star network, where Y sources send messages to each other with the help of a central relay. We focus on finite block lengths and compare the expected throughputs of RLNC and TDMA. For a total message length of K bits, which can be subdivided into blocks of smaller size prior to channel coding, we obtain the channel coding rate and the number of blocks that maximize the expected throughput of both RLNC and TDMA, and we find that TDMA is more throughput-efficient for small K and small q.

I. INTRODUCTION

Random linear network coding (RLNC) has recently been shown to improve network performance in several broadcast and multicast scenarios. For example, considering packet erasure channels on the link layer, RLNC is known to improve throughput and reduce delay for wireless broadcast [1]–[4]. In contrast, we consider the joint design of channel and network coding. We assume that the size of a block is not predetermined and, for a finite message length, the sources in a network may choose the number of data blocks so that the throughput of the overall system is maximized. Joint error and erasure correcting coding for finite message lengths was analyzed in [5]–[7]. In [5] the authors bound the performance of random coding on the physical and link layer using error exponents to trade off system throughput and delay. In [6] the combination of RLNC and continuous-time orthogonal waveform channels was investigated. Both papers aim to maximize throughput given a maximum delay constraint. By contrast, in this paper we do not enforce a maximum delay constraint, but focus instead on the expected throughput for reliable communication, assuming the senders continue to transmit until the receivers have correctly received the entire message as in, e.g., file transfer. By maximizing the expected throughput, we also minimize the expected delay.

In this paper, we extend the results of [7] which considered broadcast from one source to multiple destinations to a star network as depicted in Fig. 1. With the help of a central relay,
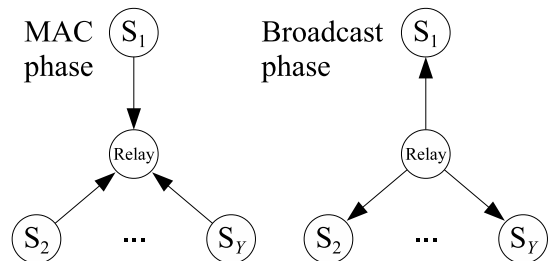
Fig. 1. Star network in which Y sources communicate over noisy BSCs with the help of a central relay.

Y sources, $S_1, \ldots, S_Y$, communicate with each other over noisy binary symmetric channels (BSCs). We assume there is no direct path between any of the sources, i.e., they are only connected to the central relay, which receives transmissions from all sources and can broadcast to all sources. We consider the case where each source S_i has a message of finite length K bits that is intended for all the other $Y - 1$ sources S_j, $j = 1, \ldots, Y$, $j \neq i$. In this setting, channel coding alone is not sufficient to guarantee reliable communication and it must be combined with rateless erasure correcting schemes, such as a time-division multiple access (TDMA) system paired with automatic repeat request (ARQ) [8] or q-ary RLNC.

We define the time that it takes to transmit one bit as a time unit and, when maximizing the expected throughput, we minimize the expected number of time units it takes to successfully transmit messages from Y sources to the other $Y - 1$ sources. Our goal is to jointly find the number of blocks m and the channel coding rate R that maximize system throughput. Choosing a star network as a model allows us to combine several prominent features of more general networks. In the RLNC case, the star network model includes a multiple-access channel (MAC) phase followed by a broadcast (BC) phase. We first analyze these two phases separately before combining them to maximize the throughput of the star network.

II. SYSTEM MODEL

A. Star Network Setup

As shown in Fig. 2, a source S_i, $i = 1, \ldots, Y$, splits its message of length K bits into m binary data blocks D_{ij}, $j = 1, \ldots, m$, of length K/m bits, or equivalently m q-ary data blocks $\tilde{D}_{ij}$, $j = 1, \ldots, m$, of length $K/(ml)$ q-ary symbols. We assume that q is a power of two, i.e., $q = 2^l$ and that K
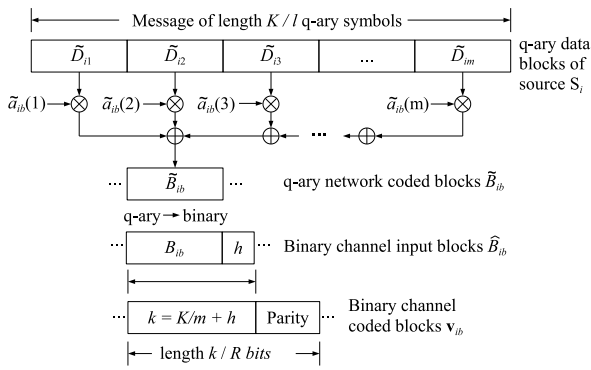
Fig. 2.   Combined channel and network coding at source S_i.

is divisible by ml. A source S_i then performs RLNC on its m data blocks to create a network coded block $\tilde{B}_{ib}$ by choosing a vector $\tilde{\mathbf{a}}_{ib}$ of length m of coefficients from GF(q), where the index b does not have a fixed range, since as many blocks are created as are necessary to achieve reliable communication. The coded block $\tilde{B}_{ib}$ is then the linear combination of the m data blocks multiplied by the corresponding components of the coefficient vector $\tilde{\mathbf{a}}_{ib}$, i.e., $\tilde{B}_{ib} = \sum_{j=1}^{m} \tilde{a}_{ib}(j)\tilde{D}_{ij}$, which can also be represented as a binary block of length K/m bits using the notation B_{ib}. A header of constant size h bits is then appended to each coded block B_{ib} to form a channel input block $\hat{B}_{ib}$ of length $k = K/m + h$ bits. The header can, for example, contain a cyclic redundancy check (CRC) to detect decoding failures. Finally, each channel input block $\hat{B}_{ib}$ is protected by a binary channel code of rate R, forming the channel coded block $\mathbf{v}_{ib}$.

a) MAC phase: During the MAC phase, all sources transmit to the relay simultaneously. We model the channel from the sources to the relay as a binary adder channel [9], [10], so that the relay receives a value equal to the (real) sum of the bits sent by the sources plus a noise term. The relay then quantizes each received value to the nearest integer and makes a hard decision. If the quantized value is even, it decides a received zero, and if the quantized value is odd, it decides a received one, so that the resulting received bit can be modeled as the modulo-2 superposition of the bits sent by all the sources plus a noise bit. Equivalently, the received superimposed vector at the relay is given by

$$\mathbf{r}_b = \mathbf{v}_b \oplus \mathbf{e} = \mathbf{v}_{1b} \oplus \mathbf{v}_{2b} \oplus \ldots \oplus \mathbf{v}_{Yb} \oplus \mathbf{e}, \qquad (1)$$

where $\oplus$ symbolizes modulo-2 addition and $\mathbf{e}$ is a binary vector whose elements are Bernoulli i.i.d. random variables that are one with probability p_{MAC}. If the relay is able to decode $\mathbf{v}_b$, it broadcasts $\mathbf{v}_b$ to the sources. Should the relay not be able to decode, it does not transmit. We assume the sources can sense the channel, so if the relay fails to decode and does not transmit, the sources immediately transmit another channel coded block and we have another MAC phase.

b) BC phase: During the BC phase we assume that the relay is connected to each of the destinations via independent BSCs. We also assume that the sources are at about the same distance from the relay, and thus experience the same path loss, so that they share a common channel crossover probability p_{BC}.

Each channel coded block $\mathbf{v}_b$ sent by the relay during the BC phase is a linear combination of Ym data blocks, multiplied by a corresponding set of Ym network coding coefficients. We assume that the sources and the receivers use Y synchronized pseudo-random number generators, each source with a different seed, to generate the sequences for $\tilde{\mathbf{a}}_b$, so that any source knows the network coding coefficients of all sources.

The column vector of Ym network coding coefficients $\tilde{\mathbf{a}}_b = [\mathbf{a}_{1b}, \ldots, \mathbf{a}_{Yb}]'$ corresponding to a block b is the bth column in the generator matrix $\mathbf{G}$ employed by the RLNC in the star network, and $\tilde{B}_b = \sum_{i=1}^{Y} \tilde{B}_{ib}$, the superposition of the network coded blocks, can be viewed as a code symbol of the RLNC. When a source S_i receives a $\mathbf{v}_b$ from the relay, it first decodes the binary channel code to obtain $\hat{B}_b = \sum_{i=1}^{Y} \hat{B}_{ib}$. If decoding is successful, the header is removed and, after binary to q-ary conversion, we obtain $\tilde{B}_b$. Source S_i then subtracts its own contribution to $\tilde{B}_b$, which is $\tilde{B}_{ib}$, and stores the superposition of the other $Y - 1$ network coded blocks $\tilde{B}_{jb}, j = 1, \ldots, Y, j \neq i$, as an element in a vector of received RLNC symbols. It also stores the subset of $(Y-1)m$ network coding coefficients in $\tilde{\mathbf{a}}_b$ involved in creating the superposition $\tilde{B}_{jb}$ as a column in its coefficient matrix $\mathbf{G}_i$, the perceived generator matrix of the RLNC from the point of view of source S_i. After the end of the BC phase, another MAC phase begins.

Once a source S_i has received enough blocks from the relay to form a matrix $\mathbf{G}_i$ with $(Y - 1)m$ linearly independent columns, it can recover the $(Y - 1)m$ data blocks from the other sources by inverting the matrix $\mathbf{G}_i$ and multiplying it by its vector of received RLNC symbols. Then it sends a single acknowledgment (ACK) to the relay. Once the relay has collected Y ACKs from the Y sources, it broadcasts an ACK to the sources, terminating transmission. All sources continue to transmit until they receive an ACK from the relay. We assume that the transmission of an ACK is instantaneous and reliable, i.e., it does not consume any resources and it is never received erroneously.

As a reference scheme we consider TDMA transmission of the sources, paired with ARQ. We also assume a source splits its message into m data blocks, but no network coding is used. The MAC phase in Fig. 1 is replaced by a TDMA phase, where only one source transmits to the relay at a given time and the individual data blocks are again protected by a binary channel code of rate R. The transmitting source S_i repeats the transmission of a channel coded block as many times as is necessary for the relay to receive the data block correctly, at which point the relay transmits an ACK. After the relay has received the data block correctly it broadcasts it to all sources. When a source receives the data block correctly, it sends an ACK to the relay. The relay repeats the BC transmission as many times as is necessary until all $Y - 1$ sources S_j, $i = 1, \ldots, Y$, $j \neq i$, receive the data block correctly. After the steps described above have been successfully completed for source S_i, it is the turn of the next source to transmit a data block to the relay, and the sources are scheduled in a round robin fashion with m rounds. After each source has successfully transmitted m data blocks, the transmission ends.

B. Channel Coding

The block error probability ϵ of random coding on the BSC with code rate R can be bounded above as a function of the random coding error exponent $E(R)$. Using the union bound, we have $E(R) = R_0 - R$, where R_0, the cutoff rate of the channel, depends on the crossover probability p of the BSC [11]. Then we obtain

$$\epsilon \le 2^{-n\,(R_0-R)}, \tag{2}$$

where $n = k/R$ is the block length of the code and $k = K/m + h$ bits. Using the union bound to bound channel coding performance allows us to obtain analytical expressions for the optimum channel coding rate and optimum number of data blocks in Sections III–V.

C. The Expected Overhead of RLNC

Considering a single source on its own and RLNC over GF(q), the probability that $m+x$ independently created column vectors of network coding coefficients $\tilde{\mathbf{a}}$ form an $m \times (m+x)$ matrix of rank m, i.e., the probability that $m + x$ network coded blocks are sufficient to decode the RLNC of that source is given by $P_{\text{success}}(m, x, q) = \prod_{i=1}^{m} \left(1 - q^{-x-i}\right)$ [12]. In the star network, a block broadcast by the relay is a linear combination of Ym data blocks and, since the network coding coefficients are chosen independently at all sources, the probability that all Y sources can construct an invertible matrix of rank $(Y-1)m$ from $(Y-1)m+x$ correctly received blocks is given by $P_{\text{success}}^*(m, x, q, Y) = \left(P_{\text{success}}((Y-1)m, x, q)\right)^Y$. We can now make use of a result from [12] to derive the following upper and lower bounds on the expected overhead $X^*(q, Y)$ of RLNC in the star network (see [13] for details):

$$\sum_{j=1}^{Y} \binom{Y}{j}(-1)^{j+1}\frac{(q^2-q)^j - q^j}{(q-1)^j(q^j-1)^2} <$$
$$X^*(q, Y) < \sum_{j=1}^{Y} \binom{Y}{j}(-1)^{j+1}\frac{q^{2j} - (q-1)^j}{(q-1)^j(q^j-1)^2}, \tag{3}$$

where both bounds are independent of the number of data blocks m and tend to zero as q gets large.

Modeling the expected coding overhead of RLNC as a constant fractional number of blocks leads to opposing optimization criteria for channel coding and RLNC when a message of finite size K bits is divided into m data blocks:

- More data blocks, and thus shorter channel coded blocks, lead to a smaller coding overhead of RLNC in bits.
- Longer channel coded blocks, and thus fewer data blocks, lead to more powerful channel codes.

III. THE MAC PHASE

To optimize throughput for the MAC phase, we assume that the channels from the relay to the sources are error-free, i.e., $p_{\text{BC}} = 0$, so that the relay does not need a channel code, and that the relay removes the h header bits prior to broadcasting. Modeling the expected coding overhead $X^*(q, Y)$ of RLNC as a constant fractional number of blocks, on average each source must collect $(Y-1)m + X^*(q, Y)$ network coded blocks to

be able to decode. Using (2) and letting $n = k/R = (K/m + h)/R$, we obtain (see [13])

$$N_{\text{RLNC}}^{\text{MAC}} \approx \frac{k\left((Y-1)m + X^*(q, Y)\right)}{R\left(1 - 2^{-k(R_0/R-1)}\right)} \tag{4}$$

for the expected number of bits that must be sent by the sources. For TDMA, a total of Ym blocks must be transmitted to the relay by the Y sources, and we obtain (see [13])

$$N_{\text{TDMA}}^{\text{MAC}} \le \frac{Y(K + mh)}{R\left(1 - 2^{-\left(\frac{K}{m}+h\right)\left(\frac{R_0}{R}-1\right)}\right)} \tag{5}$$

for the expected total number of transmitted bits. (4) and (5) depend on the BSC crossover probability p_{MAC} through the channel cutoff rate R_0.

A. The Optimum Channel Coding Rate

Taking the partial derivative of (4) with respect to R and setting it to zero, we obtain (see [13])

$$\frac{R}{R_0} = \frac{-\ln(2)k}{W_{-1}\left(-e^{-(\ln(2)k+1)}\right) + 1} \tag{6}$$

for the optimum channel coding rate as a fraction of the cutoff rate of the channel, where $W_{-1}(x)$ represents the lower branch of the Lambert-W function. From (6) we see that the optimum channel coding rate ratio R/R_0 is only a function of the channel input block length k and is independent of the expected overhead $X^*(q, Y)$ of RLNC and the number of sources Y. It is thus also the optimum channel coding rate for a scheme employing TDMA.

B. The Optimum Number of Blocks

Now taking the partial derivative of (4) with respect to m and setting it to zero, we obtain

$$2^{z\left(\frac{K}{m}+h\right)} = \left(1 + \frac{\ln(2)zK\left(\frac{K}{m}+h\right)\left(X^*(q, Y)+m(Y-1)\right)}{K\,X^*(q, Y) - h\,m^2(Y-1)}\right), \tag{7}$$

where $z = (R_0/R) - 1$. In general, a closed form solution of (7) cannot be found. However, for $h = 0$ and $Y = 2$ we can again use the Lambert-W function to solve for m, and the optimum number of blocks m, given a constant R/R_0 and the message length K, is

$$m = \frac{-\ln(2)zK}{1 + \ln(2)\frac{zK}{X(q,2)} + W_{-1}\left(-e^{-\left(1+\ln(2)\frac{zK}{X(q,2)}\right)}\right)}. \tag{8}$$

For other values of h and Y we solve (7) and (6) jointly using numerical methods to obtain the optimum number of blocks m that minimizes the expected number of transmissions and maximizes the throughput. For $h = 16$, Fig. 3 shows the optimum number of blocks m given the total message length K, the number of sources Y, and RLNC over GF(q). As K increases, we observe that the maximum throughput is achieved for a larger number of blocks m. Since the expected coding overhead $X^*(q, Y)$ in blocks increases with the number of sources in the star network, the optimum number of blocks m increases with Y for a fixed message length K. On the other
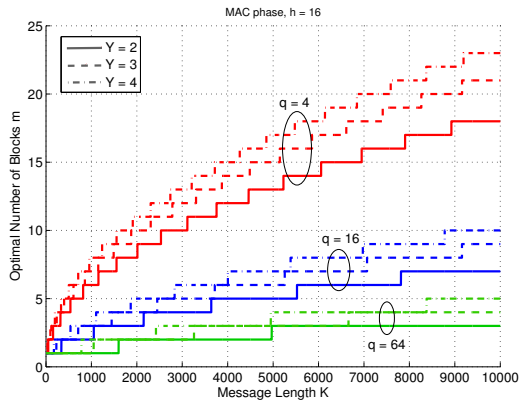
Fig. 3. Optimum number of blocks m given the message length K, the number of sources Y, and header size $h = 16$ for RLNC over different Galois field sizes q.
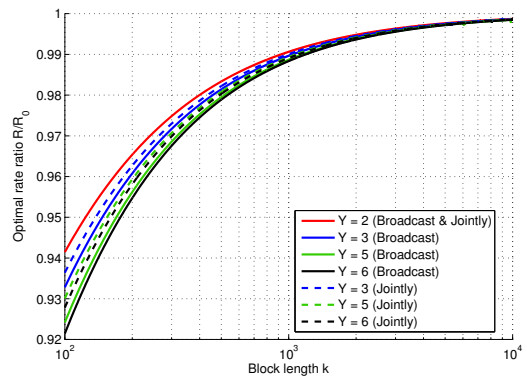

Fig. 4. Optimum channel coding rate R for BC (solid lines) and considering transmission for the MAC and BC phase jointly (dashed lines) as a fraction of the cutoff rate R_0 for different channel input block lengths k when TDMA is employed.
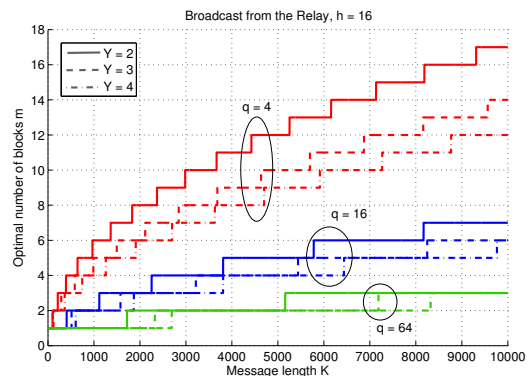
hand, since the expected coding overhead $X^*(q, Y)$ decreases with increasing Galois field size, the optimum number of blocks decreases with q.

IV. THE BC PHASE

To optimize throughput for the BC phase, we assume that the channels to the relay are error free, i.e., $p_{\mathrm{MAC}} = 0$. Further, since $p_{\mathrm{MAC}} = 0$, we assume that during the MAC phase the sources transmit to the relay uncoded, i.e., $R = 1$, and that no header is used. A header of length h is then appended to each block at the relay, and the relay uses a channel code of rate $R < 1$ to protect the blocks.

A. TDMA BC Paired With ARQ

Consider the TDMA scheme, where the expected number of blocks that the relay must broadcast, $M_{\mathrm{TDMA}}^{\mathrm{BC}}$, is given in [3]. Using (2) and $N_{\mathrm{TDMA}}^{\mathrm{BC}} = k M_{\mathrm{TDMA}}^{\mathrm{BC}}/R$, we obtain for the expected number of bit transmissions by the relay (see [13])

$$N_{\mathrm{TDMA}}^{\mathrm{BC}} = \frac{Y(K+mh)}{R} \sum_{i=0}^{\infty} 1 - \left(1 - 2^{-i\left(\frac{K}{m}+h\right)\left(\frac{R_0}{R}-1\right)}\right)^{Y-1}, \quad (9)$$

where R_0 is the cutoff rate of a BSC with crossover probability p_{BC}. For any fixed coding rate R, the factor $Y(K+mh)/R$ in (9) as well as the BC channel block error probability ϵ_{BC} are strictly increasing with increasing m. So the throughput for the TDMA system paired with ARQ is maximized for $m = 1$ and a channel input block of size $k = K + h$.

To obtain the channel coding rate that maximizes throughput, we transform (9) into a finite sum and use the partial derivative w.r.t. R to obtain (see [13])

$$\sum_{i=1}^{Y} (-1)^i \binom{Y}{i} \frac{1 - 2^{-izk} - ik\ln(2)\frac{R_0}{R}2^{-izk}}{\left(1 - 2^{-izk}\right)^2} = 0, \quad (10)$$

where $z = (R_0/R) - 1$.

For TDMA and $Y = 2$, the channel coding rate that maximizes throughput (10) in the BC phase is the same as the rate that maximizes throughput for transmission to the relay (6), obtained in Section III. In both cases, messages are transmitted from one sender to one intended receiver. For

broadcast to a larger number of sources Y, we can numerically find the solution of (10), and the optimum rate ratios R/R_0 for different numbers are destinations are shown as the solid lines in Fig. 4. We see that, while (6) does not depend on the number of sources transmitting to the relay, during the BC phase the optimum channel coding rate R for TDMA as a fraction of the cutoff rate R_0 decreases as the number of BC destinations increases and, for $Y > 2$, is smaller than (6). The optimum number of blocks for the TDMA scheme, however, is $m = 1$ for both transmission to the relay, considered in Section III, and the BC phase.

B. BC Using RLNC

Using RLNC, the expected number of network coded blocks $M_{\mathrm{RLNC}}^{\mathrm{BC}}$ that the relay must broadcast is given in [3], and the expected number of bits that the relay must transmit is given by $N_{\mathrm{RLNC}}^{\mathrm{BC}} = k M_{\mathrm{RLNC}}^{\mathrm{BC}}/R$. We solve the resulting multidimensional optimization problem using numerical methods. For the BC scenario using RLNC, Fig. 5 shows the optimum number of data blocks m for $h = 16$. Comparing the optimum number of blocks in Fig. 5 to the MAC phase displayed in Fig. 3, the number of blocks that maximizes throughput is generally smaller for the BC phase. The most prominent difference between Figs. 5 and 3 is that, while for the MAC phase the optimum number of data blocks increases with the number of sources, for the BC phase the optimum number of blocks decreases with an increase in the number of BC destinations
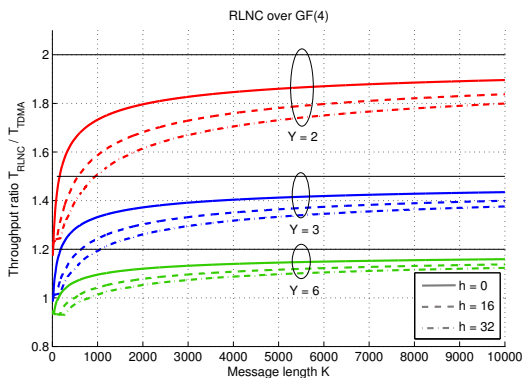

Fig. 5. Optimum number of blocks for BC from the relay for $h = 16$.

Fig. 6. Average throughput ratio $T_{\mathrm{RLNC}}/T_{\mathrm{TDMA}}$ for GF(4).

Y, thus putting more emphasis on the channel coding being able to provide more reliable individual blocks.

V. JOINT OPTIMIZATION FOR THE STAR NETWORK

From Sections III and IV, we see that the number of data blocks and the channel coding rate that maximize throughput differ for transmission from the sources to the relay and for BC from the relay. In a practical system, however, it would be desirable to have the same channel coding rate and the same block size for transmission to and from the relay so we now jointly optimize the throughput of the star network, keeping m and R constant. We refer to the time it takes to transmit one block as a time slot.

For the RLNC scheme, the expected number of time slots that are occupied by transmissions in the star network is given by $M^*_{\mathrm{RLNC}} = M^{\mathrm{BC}}_{\mathrm{RLNC}}(1 + 1/(1 - \epsilon_{\mathrm{MAC}}))$, which relies on the fact that, for every block that the relay broadcasts, on average $1/(1 - \epsilon_{\mathbf{MAC}})$ transmissions from the sources to the relay are necessary, for MAC block error probability $\epsilon_{\mathbf{MAC}}$.

Similarly, for the TDMA scheme, the expected number of time slots that are occupied by transmissions is given by $M^*_{\mathrm{TDMA}} = M^{\mathrm{MAC}}_{\mathrm{TDMA}} + M^{\mathrm{BC}}_{\mathrm{TDMA}}$. In this case, since the throughput for both the transmission phase to the relay and the BC phase from the relay is maximized for $m = 1$, one channel input block of length $k = K + h$ bits for each source S_i is also optimum when considering both phases jointly.

In the following, we consider the symmetric case, where $\epsilon_{\mathrm{MAC}} = \epsilon_{\mathrm{BC}}$. In this case, the channel coding rate that maximizes the throughput for TDMA can be obtained by taking the derivative of $N^*_{\mathrm{TDMA}} = kM^*_{\mathrm{TDMA}}$ w.r.t. the channel coding rate R (see [13] for details), and the optimum channel coding rate that jointly maximizes throughput for the TDMA scheme is also depicted in Fig. 4. For $Y > 2$, the optimum channel coding rate for the star network decreases with the number of sources, similar to the TDMA BC case. However, comparing the optimum rate for the BC phase alone to the jointly optimum rate for the same number of sources Y, we find that the channel coding rate that jointly maximizes throughput for the star network is higher than the one that gives the maximum throughput for the BC phase alone.

Fig. 6 shows the average throughput ratio $T_{\mathrm{RLNC}}/T_{\mathrm{TDMA}} = M^*_{\mathrm{TDMA}}/M^*_{\mathrm{RLNC}}$ of RLNC over GF(4) to TDMA and the asymptotic throughput ratios are plotted as horizontal black lines. For small message lengths K, we

see that the average throughput ratio rises steeply before the curves flatten out and slowly approach their asymptotic value given by $T_{\mathrm{RLNC}}/T_{\mathrm{TDMA}} = Y/(Y-1)$. As the header size h increases, the average throughput ratio decreases, and a larger message length K is needed to obtain a given average throughput ratio. For small message lengths K and large header sizes h, TDMA is more throughput-efficient. For example, for $h = 32$ and $Y = 6$ sources, we require $K > 900$ bits for RLNC to be more throughput-efficient than TDMA.

VI. CONCLUSIONS

We analyzed the joint design of channel coding on the physical layer and random linear network coding on the link layer for a star network where Y outer sources send fixed length messages to each other with the help of a central relay. For RLNC over a finite Galois field of size q and a message of total length K at each source, we obtain the number of data blocks and the channel coding rate that should be used to maximize the throughput of the star network using RLNC, assuming binary symmetric channels between the sources and relay and a binary adder channel model at the relay. We also obtain the optimum number of blocks and the optimum rate for a reference TDMA system and compare the throughputs of the two transmission schemes. We find that, for small message lengths K and RLNC over small Galois fields q, TDMA is more throughput-efficient than RLNC, while RLNC is more throughput-efficient when the message length K gets large.

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