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# CLIPPING ERROR RESILIENCE FOR PEAK POWER-CONSTRAINED DMT TRANSMISSION VIA IMPLICIT FREQUENCY DOMAIN REDUNDANCY

*Jörg Kliewer*
University of Kiel
Institute for Circuits and Systems Theory
24143 Kiel, Germany
Email: jkl@tf.uni-kiel.de

*Tanja Karp*
Texas Tech University
Dept. of ECE
Lubbock, TX 79409-3102, USA
Email: tanja.karp@ttu.edu

**ABSTRACT**

In this paper we present a new error correction approach for a simple peak-to-average power ratio (PAR) reduction scheme for discrete multi-tone (DMT) systems based on coefficient clipping. The proposed method utilizes existing or intentionally placed redundancy in the frequency domain in form of unused subcarriers. Compared to existing PAR reduction algorithms, all signal processing is performed at the receiver after the transmit signal has been simply clipped to a desired maximum amplitude at the front end of the transmitter. The introduced frequency-domain redundancy allows to consider each output vector of the IDFT in the DMT transmitter as codeword of a Bose-Chaudhuri-Hocquenghem (BCH) block code over the field of real numbers. The decoding operation is carried out by a low-complexity linear reconstruction. Simulation results for noisy transmission show that by using the proposed method clipping does not lead to significant errors in the received data if a reasonable amount of redundancy is provided.

## 1. INTRODUCTION

The basic block structure of a discrete multi-tone (DMT) system as used for high data rate transmission via telephone lines is shown in Fig. 1. $M$ complex quadrature amplitude modulation (QAM) symbols $u_\ell(k)$, $\ell = 0,\dots,M-1$, are modulated on equidistant subcarriers by using the Inverse Discrete Fourier Transform (IDFT) and transmitted in parallel. The insertion of a guard interval (GI) of $L$ samples allows an easy equalization at the receiver: as long as the channel impulse response is shorter than $L+2$ samples, the distortion inserted by the channel can be ideally equalized in the frequency domain (i.e. after the DFT at the receiver side) by one complex multiplication per subcarrier.

If the $M$ QAM symbols $u_\ell(k)$, $\ell = 0,\dots,M-1$, at the transceiver add together constructively in the IDFT the peak envelope power of the transmit symbols $v_\ell(k)$ in Fig. 1 may be as much as $M$ times the mean power. This results in the need for high resolution analog-to-digital converters and costly power amplifiers with a large linear range. Simple clipping of high amplitudes at the transmitter causes an unacceptable increase of the bit-error rate (BER) since it affects all subcarriers after modulation into the frequency domain through the receiver DFT.

Several approaches have been presented in literature in order to reduce the peak-to-average power ratio (PAR) (see e.g. [1–4]), where a subset is given by those methods which exploit unused subcarriers for PAR reduction [3, 4]. Common to the methods in [3, 4] is the fact that the PAR reduction is performed at the transmitter. Whenever a transmit amplitude exceeds the clipping level, symbols in unused subcarriers are introduced at the transmitter in such a way that they reduce the maximum amplitude of the transmit symbol. At the receiver, no additional signal processing is needed. It has been shown though, that this method works best if the unused subcarriers are spread randomly over the available frequency range and are not all located in high frequency ranges, as it is often the case in DMT scenarios [4].

In this paper we present a simple algorithm for reducing the bit errors after DMT transmission due to clipping the DMT-encoded data symbols prior to transmission. The proposed approach utilizes frequency domain redundancy in form of subcarriers, which are allocated as unused or being intentionally left unallocated. Since unused subcarriers correspond to vectors with zero entries at the IDFT input in a DMT transmitter, the resulting IDFT output vectors can be interpreted as codewords of a Bose-Chaudhuri-Hocquenghem (BCH) block channel code over the field of real numbers [5, 6]. Since the positions of the symbol errors due to clipping can be retrieved at the receiver, only a low-complexity matrix-vector multiplication is required to perform the calculation of the corrected samples.

Combining the new PAR reduction algorithm with any one of [1–4, 7] allows the performance of all additional signal processing for bidirectional data transmission at one side

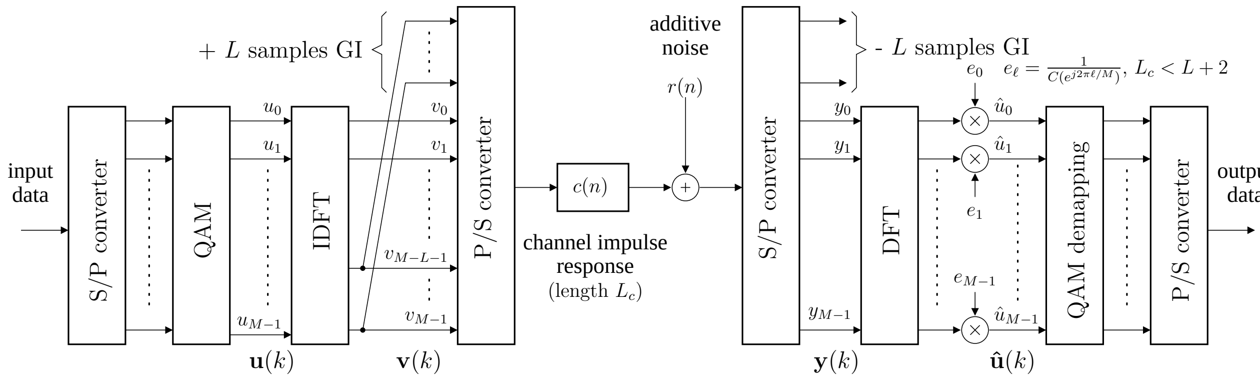


Figure 1: DMT transmission scheme

of the transceiver. This is preferable in applications where the available computational power at the transceiver terminals is asymmetrically distributed.

## 2. ESTIMATION OF THE CLIPPED SAMPLES

In this section we propose a reconstruction technique for the clipped samples at the receiver by exploiting the redundancy introduced by the unused subcarriers. In the following clipping of the vector elements in $\mathbf{v}(k) = [v_0(k), v_1(k), \ldots, v_{M-1}(k)]^{\mathrm{T}}$ is carried out according to

$$v'_\ell(k) = \operatorname{sgn}(v_\ell(k)) \cdot \min(T_{\text{clip}}, |v_\ell(k)|), \quad \ell = 0, \ldots, M-1,$$

at the transmitter front end, where $T_{\text{clip}}$ denotes a given clipping threshold and $\operatorname{sgn}(\cdot)$ the signum function. For the sake of simplicity we restrict ourselves to an even number $M$ of subcarriers, however, an extension to odd $M$ can be carried out in a straightforward way. The presented method uses the framework of real-valued BCH codes [5,6,8], where the transmit vector $\mathbf{v}(k) \in \mathrm{I\!R}^M$ in Fig. 1 is used as the codeword. We here exploit the fact that for BCH codes the DFT of the codeword $\mathrm{DFT}\{\mathbf{v}(k)\} = \mathbf{W}_M \mathbf{v}(k) = \mathbf{u}(k)/M$ with $[\mathbf{W}_M]_{i,\ell} = e^{j2\pi i\ell/M}$ exhibits a certain number of zeros due to unused subcarriers.

Note that in the noiseless case (i.e. for $r(n) = 0$) the number and the positions of the inserted zero values are preserved by the DMT transmission system. In case of additive channel noise, the positions of the unused subcarriers contain values depending on the channel noise power instead of zero values.

### 2.1. Correction of the clipping errors

**(a) Ideal channel impulse response.** In order to derive the reconstruction technique for the clipped samples first the case of an ideal flat transmission channel with $c(n) = \delta(n)$ is considered. We assume that $E$ consecutive subcarriers are unused in the lower part of the DMT spectrum up to the Nyquist frequency. Since real-valued transmit data is required, the input vector $\mathbf{u}(k)$ should have conjugate complex symmetry. Then, $\mathbf{u}(k)$ in total contains $2E$ unallocated channels, due to the symmetry condition

$$u_\ell(k) = u^*_{M-\ell}(k), \quad u_0(k), u_{M/2}(k) \in \mathrm{I\!R},$$

$\ell = 0, \ldots, M-1$, for an even number $M$ of subchannels. Considering just the $2E$ unused channels in the transmitter IDFT $\mathbf{v}(k) = \mathbf{W}^*_M/M \cdot \mathbf{u}(k)$ at the receiver leads to the condition

$$\mathbf{W}_{2E\times M} \cdot \mathbf{y}(k) \stackrel{!}{=} \mathbf{0}_{2E} \tag{1}$$

with $\mathbf{y}(k) = [y_0(k), y_1(k), \ldots, y_{M-1}(k)]^{\mathrm{T}}$ at the DMT receiver input, where the term $\mathbf{W}_{2E\times M}\,\mathbf{y}(k)$ is called syndrome. $\mathbf{W}_{2E\times M}$ denotes the $2E \times M$ submatrix of the DFT matrix $\mathbf{W}_M$ which only contains the rows corresponding to the positions of the unused subcarriers.

Note that (1) corresponds to an error-free transmission (i.e. no clipping has occurred at all) in the noiseless channel case. Analog to the decoding of BCH codes the idea is now to determine the erroneous values (i.e. the clipped values for negligible channel error probabilities) in $\mathbf{y}(k)$ from the correct ones such that the condition (1) is approximated as close as possible in a certain error sense. In the following we assume that the clipping threshold $T_{\text{clip}}$ is already known as a-priori information at the receiver. Then, the position of the clipped values can be obtained by searching for values in the time-domain receive signal $\mathbf{y}(k)$ that are close to the clipping threshold, where especially in the noiseless case a clipped value is received unchanged. Thus, we are able to partition $\mathbf{y}(k)$ into two subvectors $\mathbf{y}_e(k) \in \mathrm{I\!R}^{M_e}$ and $\mathbf{y}_c(k) \in \mathrm{I\!R}^{M_c}$, resp., where the former vector contains all erroneous entries affected by clipping and the latter all unmodified elements of $\mathbf{y}(k)$. The parameter $M_e$ denotes the number of errors created by clipping with $M = M_c + M_e$. From (1) we obtain by decomposition of the DFT submatrix $\mathbf{W}_{2E\times M}$

$$\mathbf{W}_{2E\times M_c} \cdot \mathbf{y}_c(k) + \mathbf{W}_{2E\times M_e} \cdot \mathbf{y}_e(k) \stackrel{!}{=} \mathbf{0}_{2E}. \tag{2}$$

An approximation $\hat{\mathbf{y}}_e(k)$ for the clipped coefficients $\mathbf{y}_e(k)$ can now be obtained by using the linear reconstruction

$$\hat{\mathbf{y}}_e(k) = -[\mathbf{W}_{2E\times M_e}]^\dagger\, \mathbf{W}_{2E\times M_c}\, \mathbf{y}_c(k), \tag{3}$$

with $[\mathbf{W}_{2E\times M_e}]^\dagger$ denoting the pseudo-inverse of $\mathbf{W}_{2E\times M_e}$. It can be shown that $\mathbf{W}_{2E\times M_e}$ has the rank $R = \min(2E, M_e)$, where the proof is omitted here due to space limitations.

Since $\mathbf{W}_{2E\times M_e}$ has maximal rank the solution in (3) satisfies the condition [9]

$$\hat{\mathbf{y}}_e(k) = \underset{\mathbf{y}_e(k)}{\operatorname{argmin}} \,||\underbrace{\mathbf{W}_{2E\times M_c}\cdot\mathbf{y}_c(k)}_{=:\mathbf{b}(k)} - \mathbf{W}_{2E\times M_e}\cdot\mathbf{y}_e(k)||^2. \quad (4)$$

Herein, the vector $\mathbf{W}_{2E\times M_e}\mathbf{y}_e(k)$ is an element of a $M_e$-dimensional subspace of $\mathbb{R}^{2E}$ spanned by the columns of $\mathbf{W}_{2E\times M_e}$. Since the vector $\mathbf{b}(k) = \mathbf{W}_{2E\times M_c}\mathbf{y}_c(k)$ is an element of $\mathbb{R}^{2E}$ we can observe from (4) that (3) projects the vector $\mathbf{b}(k)$ to a subspace $\mathbf{W}_{2E\times M_e}\cdot\mathbb{R}^{M_e}$ by choosing the proper vector $\mathbf{y}_e(k)$, which minimizes the resulting error in the least-squares error sense. If $M_e \leq 2E$ the obtained solution for $\hat{\mathbf{y}}_e(k)$ is unique, however, for $M_e > 2E$ the vector $\mathbf{b}(k)$ is "projected" to a larger vector space, where we get $M_e - 2E$ additional degrees of freedom for the solution. In the latter case (3) then yields the solution with minimal norm $||\hat{\mathbf{y}}_e(k)||$. This corresponds to the results for BCH codes over finite fields, where a code with minimum distance $D = 2E+1$ is able to correct up to $2E$ erasures.

**(b) Nonideal channel impulse response ($L_c < L+2$).** For an arbitrary impulse response $c(n)$ of length $L_c$ it is not possible to determine the positions of the clipped samples directly from $\mathbf{y}(k)$. Due to convolution with the channel impulse response all elements of $\mathbf{y}(k)$ are affected when even only one sample is clipped in the transmitted vector $\mathbf{v}(k)$. However, a solution is to apply the above reconstruction technique to the *equalized* receive vector

$$\mathbf{y}'(k) = \mathbf{W}_M^*\,\hat{\mathbf{u}}(k) = \mathbf{W}_M^*\,\mathbf{E}\,\mathbf{W}_M\,\mathbf{y}(k) \quad (5)$$

with $\hat{\mathbf{u}}(k) = \mathbf{W}_M\,\mathbf{y}(k)$ (cmp. Fig. 1) in order to eliminate the influence of the non-flat transmission channel. The matrix $\mathbf{E}$ in (5) represents the block equalizer matrix $\mathbf{E} = \operatorname{diag}[e_0, e_1, \ldots, e_{M-1}]$ with $e_i = 1/C(e^{j2\pi i/M})$ for $i = 0, \ldots, M-1$, where $C(e^{j\omega})$ denotes the frequency response of the transmission channel. Then, the clipped samples with amplitude $T_{\text{clip}}$ can be identified in the equalized receive vector $\mathbf{y}'(k)$, and an approximation of the clipped coefficients is obtained analog to (3) as

$$\hat{\mathbf{y}}'_e(k) = -[\mathbf{W}_{2E\times M_e}]^\dagger\,\mathbf{W}_{2E\times M_c}\,\mathbf{y}'_c(k). \quad (6)$$

Herein, similar to (3) the subvectors $\mathbf{y}'_c(k)$ and $\mathbf{y}'_e(k)$ correspond to the unmodified and clipped entries of $\mathbf{y}'(k)$, respectively.

### 2.2. Overall reconstruction algorithm

We can now state the overall reconstruction algorithm at the receiver as follows:

1. Calculate an equalized version $\mathbf{y}'(k)$ of the receive vector $\mathbf{y}(k)$ from Fig. 1 according to (5).
2. Estimate the position of the clipped samples in $\mathbf{y}'(k)$: If $|y'_\ell(k)| \in [T_{\text{clip}} - \sigma_r, T_{\text{clip}} + \sigma_r]$, the sample $y'_\ell(k)$, $\ell = 0, \ldots, M-1$, is assumed to be modified by clipping at the transmitter, where for the case of a noisy transmission $\sigma_r^2$ denotes the power of an additive white Gaussian channel noise (AWGN channel).
3. Obtain an approximation for the clipped coefficients from (6).
4. Calculate the corrected vector $\hat{\mathbf{u}}'(k)$ at the output of the DMT receiver via $\hat{\mathbf{u}}'(k) = \mathbf{W}_M\,\hat{\mathbf{y}}'(k)$ where $\hat{\mathbf{y}}'(k)$ is obtained from $\mathbf{y}'(k)$ by replacing the clipped elements with the corrected ones from (6).

## 3. SIMULATION RESULTS

In order to assess the performance of the proposed reconstruction technique, simulations are carried out for the DMT system from Fig. 1. The inputs $u_i(k)$, $i = 0, \ldots, M-1$, of all $M = 64$ subchannels contain complex 16-QAM symbols normalized to an average power of one. We assume an AWGN transmission channel and a lowpass channel impulse response of $c(n) = (5\,\delta(n) + 4\,\delta(n-1) + 3\,\delta(n-2) + 2\,\delta(n-3) + \delta(n-4))/15$.

Fig. 2 shows the results for a channel SNR of 30 dB and $E \in \{2, 4, 8\}$ consecutively unused subcarriers located in the "stopband" region of the channel frequency response, where the DFT indices of the unused subcarriers up to the Nyquist frequency are given in the vector $\boldsymbol{\mu}_c$. The reconstruction approach from Section 2 is marked with "(+ rec.)" in the following. In Fig. 2(a) the mean-squares error (MSE) $||\mathbf{u}(k) - \hat{\mathbf{u}}'(k)||^2$ averaged over 500 simulated transmissions of 100 QAM symbols in each subchannel is displayed versus the clipping threshold $T_{\text{clip}}$. Clearly, it can be observed that by increasing the number of unused subchannels $2E$ it is possible to further reduce the clipping threshold without a significant increase in MSE. For example, for $E = 8$ and $T_{\text{clip}} = 2$ we gain a reduction in MSE over three orders of magnitude compared to using the (unmodified) receiver without employing the above reconstruction technique. Note that the remaining non-zero MSE on the right-hand side of the "waterfall" region is due to the noisy transmission channel. In this case the average number of erasures is smaller than or equal to the number of unallocated subcarriers (i.e. $M_e \leq 2E$ in average), and the estimation of the clipped values yields an optimal result in the least-squares error sense (see Section 2). However, a further decrease of the clipping threshold towards the "waterfall" region leads to a drastic increase in MSE which for small clipping thresholds also exceeds that for the unmodified receiver. Here, it becomes more likely that $M_e > 2E$, and (6) in this case may only yield the suboptimal minimum-norm solution, which could be worse compared to the solution obtained from the unmodified receiver.

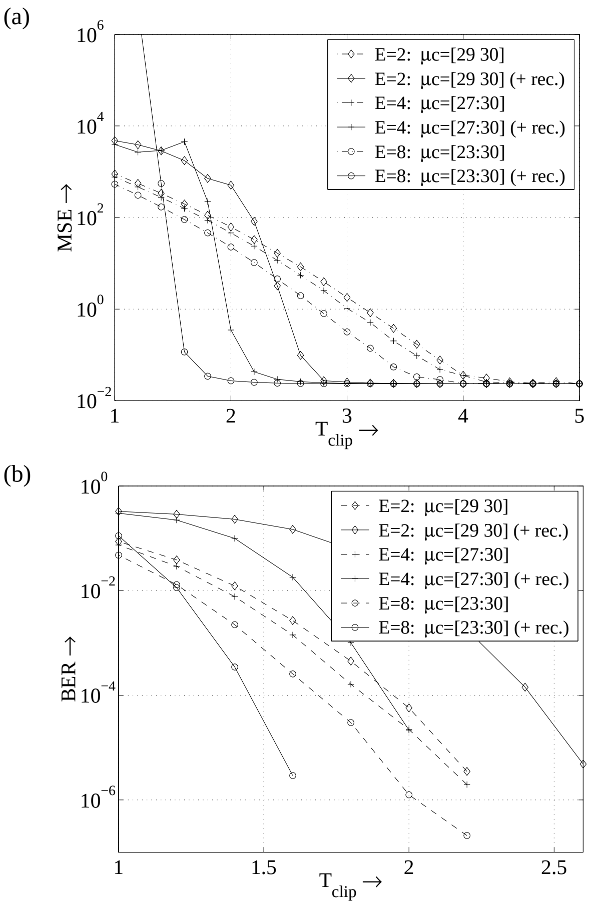


Figure 2: Simulation results for the proposed estimation approach (AWGN channel with 30 dB SNR): (a) Mean-squares error (MSE) $||\mathbf{u}(k) - \hat{\mathbf{u}}'(k)||^2$ vs. clipping threshold $T_{\text{clip}}$, (b) bit-error rate (BER) vs. $T_{\text{clip}}$, Gray mapping for the 16-QAM signal points.

Fig. 2(b) depicts the BER versus the clipping threshold after mapping the reconstructed vector $\hat{\mathbf{u}}'(k)$ back to the binary QAM information word. Only results are shown for those clipping thresholds where at least three bit errors have occurred for all $5 \cdot 10^4$ DMT input vectors used for averaging. In order to reduce the number of erroneous bits for small reconstruction errors a Gray mapping is used for the QAM symbols. We can see from Fig. 2(b) that for $E = 2$ the added redundancy is not sufficient to outperform the unmodified case. Despite Fig. 2(a) shows a smaller MSE for $T_{\text{clip}} > 2.3$ compared to the unmodified case this does not lead to a lower BER automatically: since the proposed reconstruction technique exhibits a strong MSE increase if $M_e > 2E = 4$ (cmp. Fig. 2(a)), it is more likely that $|u_\ell(k) - \hat{u}'_\ell(k)|$, $\ell = 0, \ldots, M-1$, is larger than half the distance between adjacent QAM signal points, which then leads to a QAM symbol error (and thus to at least one bit error). However, for both $E = 4$ and $E = 8$ a performance gain can be observed: for example for $E = 8$ and $T_{\text{clip}} = 1.6$ the BER is reduced by two orders of magnitude compared to the unmodified case.

## 4. CONCLUSION

We have shown as a new result that unused subcarriers in a DMT transmission system can be utilized for reducing the effects of a simple peak-to-average power ratio reduction based on clipping large values prior to transmission. This can be obtained by interpreting the redundancy in the frequency domain introduced by (existing or intentionally) unallocated subcarriers as a real-valued BCH channel code, where the clipping error correction at the receiver consists of a simple linear reconstruction based on the pseudo-inverse of a full-rank DFT submatrix. Furthermore, the obtained solution is optimal in the least-squares error sense as long as the number of unused channels is larger than or equal to the number of clipped values for a single received data vector. Simulation results have shown that by applying the proposed technique to DMT transmission over noisy channels we may obtain a strong decrease of the overall BER while clipping is still present at the transmitter.

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