On the Utilization of Residual Source Redundancy for Iterative Joint Source-Channel Decoding of Variable-Length Codes

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Abstract — **We present a novel symbol-based a-posteriori probability (APP) decoder for packetized variable-length encoded source indices transmitted over wireless channels, which exploits residual index correlations after source encoding for error correction. By additionally protecting the variable-length encoded bitstream with channel codes, an iterative decoding scheme can be obtained where the proposed APP source decoder serves as constituent decoder.**

We consider a transmission system where indices I_k from a finite alphabet $\mathcal{I} = \{0, 1, \ldots, 2^M - 1\}$ are obtained from packetized source symbols U_k by a (vector-) quantization with M bit. Due to delay and complexity constraints for the quantization stage, it can generally be assumed that there is some redundancy between adjacent indices I_k. This correlation is modeled by a first-order stationary Markov process with index transition probabilities $P(I_k = \lambda \,|\, I_{k-1} = \mu)$ for $\lambda, \mu = 0, 1, \ldots, 2^M - 1$. The variable-length encoder maps each fixed-length index I_k to a variable-length bit vector $\mathbf{c}(\lambda) = \mathcal{C}(I_k = \lambda)$ using the prefix code $\mathcal{C}$, and the resulting bitstream is then transmitted over the channel.

The proposed decoding technique is based on the non-stationary variable-length code (VLC) trellis representation derived in [1]. The states $S_k = n$, where S_k refers to the state at time instant k and n denotes its value, represent all possible bit positions n for a given k in the variable-length encoded bit sequence. In the following we denote this set of all possible states $S_k = n$ as $\mathcal{N}_k$. The transition from state $S_{k-1} = n_1$ to $S_k = n_2$ is caused by the source symbol $I_k = \lambda$, $\lambda \in \mathcal{I}$, and the corresponding variable-length codeword $\mathbf{c}(\lambda)$ with length $l(\mathbf{c}(\lambda)) = n_2 - n_1$, respectively.

Utilizing this VLC trellis, the APPs $P(I_k = \lambda \,|\, \hat{\mathbf{v}})$ may be calculated via a modified BCJR algorithm [2], where

$$P(I_k = \lambda | \hat{\mathbf{v}}) = \frac{1}{C} \sum_{n_2 \in \mathcal{N}_k} \sum_{n_1 \in \mathcal{N}_{k-1}} \beta_k(n_2)\, \gamma_k(\lambda, n_2, n_1)\, \alpha_{k-1}(n_1). \quad (1)$$

The constant C ensures that the $P(I_k = \lambda \,|\, \hat{\mathbf{v}})$ are true probabilities. It can now be shown that by including the first-order Markov model for the quantized indices we obtain the following expression for the term $\gamma_k(\lambda, n_2, n_1)$:

$$\gamma_k(\lambda, n_2, n_1) = p(\hat{\mathbf{v}}_{n_1+1}^{n_2} \,|\, I_k = \lambda)\, \frac{\alpha_{k-2}(n_0)}{C_1(n_1)} \cdot \sum_{\mu=0}^{2^M - 1} \gamma_{k-1}(\mu, n_1, n_0) \underbrace{P(I_k = \lambda, S_k = n_2 | I_{k-1} = \mu, S_{k-1} = n_1)}_{=: P_c} \quad (2)$$

with $n_0 = n_1 - l(\mathbf{c}(\mu))$, $C_1(n_1)$ denoting a constant depending on n_1, and $\hat{\mathbf{v}}_{n_1+1}^{n_2}$ representing the received soft-bit sequence from bit position $n_1 + 1$ to n_2. The conditional probability abbreviated as P_c in (2) represents the transition probability of the Markov source adapted to the VLC trellis according to

$$P_c = \frac{1}{C_2(n_1, \mu)} \begin{cases} P(I_k = \lambda \,|\, I_{k-1} = \mu) & n_2 - n_1 = l(\mathbf{c}(\lambda)), \\ 0 & \text{otherwise}. \end{cases} \quad (3)$$

The normalization factor $C_2(n_1, \mu)$ considers that due to the non-stationary VLC trellis only certain transitions from $S_{k-1} = n_1$ to $S_k = n_1 + l(\mathbf{c}(\lambda))$ are possible. Finally, the α- and β-terms in (1) are obtained with the recursions stated in [1], where for the α-recursion the modified γ-term in (2) is used.

In the following we assume that the interleaved output of the VLC encoder is protected by a systematic (convolutional) channel code prior to transmission. Then, an iterative decoding scheme can be applied where extrinsic information is exchanged between two constituent decoders. The outer one is given by the APP VLC source decoder from above, and as inner decoder an APP channel decoder [2] is employed. This technique is denoted with JSCD(1).

The simulation results for a fully interleaved flat Rayleigh channel are shown in Fig. 1 for a mean-squares (MS) estimation, where the reconstructed source symbol is obtained via $\hat{U}_k = \sum_{\lambda=0}^{2^M-1} U_q(\lambda)\, P(I_k = \lambda | \hat{\mathbf{v}})$ with $U_q(\lambda)$ denoting the λ-th entry of the quantization table. The packet length (100 samples) and the number of bits in a packet are assumed to be transmitted without errors to the decoder. The JSCD(0) approach only exploits the probability distribution of the indices I_k [1], whereas the JSCD(1) technique also utilizes the residual source index correlation. For $E_b/N_0 > 4\,\text{dB}$ the JSCD(1) technique almost achieves clear-channel quality, which verifies the good performance of the proposed transmission system.

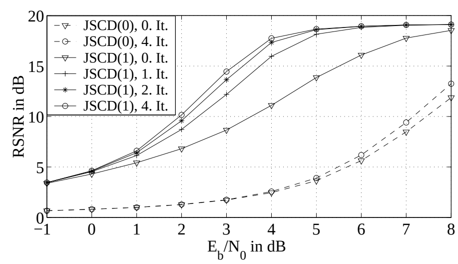

Fig. 1: Reconstruction SNR (RSNR) for a MS estimation over E_b/N_0 (Huffman code, AR(1) source with $a = 0.9$, $M = 4$ bits, terminated rate-3/4 RSC code).

References

[1] R. Bauer and J. Hagenauer, "Symbol-by-symbol MAP decoding of variable length codes", *Proc. 3. ITG Conf. on Source and Channel Coding*, Munich, Germany, Jan. 2000, pp. 111-116.

[2] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," in *IEEE Trans. on Information Theory*, pp. 287-294, Mar. 1974.