On the Delay and Energy Performance in Coded Two-Hop Line Networks with Bursty Erasures

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***Abstract*—We consider two-hop line networks where the communication links are bursty packet erasure channels modeled as a simple two-state Gilbert-Elliott channel. The first and second node in the line have local information with Poisson-distributed arrivals available and intend to communicate this information to the receiving node in the line. We consider an online approach and random linear network coding for erasure correction. We provide a queueing-theoretic analysis of two different models, a genie aided full duplex model and a partially genie aided half-duplex model, where the genie only provides the channel state information. Channel-aware policies are shown to reduce delay by up to a factor of 3 in our examples and significantly increase the network's stable throughput region compared to a simple queue-length driven policy.**

I. Introduction

In networks, packets typically are transferred through a series of nodes. Each of these nodes may need to store these packets for subsequent transmission towards the destination, where several input flows may merge at specific nodes in the network. Often, the links between the network nodes become unreliable due to network congestion and buffer overflows. In addition, in wireless networks we need to consider noise, interference, and fading due to node mobility, which typically lead to packet losses on upper layers. Since the duration of these error events typically extends over several uses of the communication channel, packet erasures become busty and time dependent. In these scenarios the end-to-end delay of the associated packet flows is of special interest, and we are interested in coding strategies which lead to a low expected delay under various adversarial scenarios. Similar considerations hold for energy efficient communication in networks with complexity constrained nodes.

One approach to transmit reliably over packet erasure networks is given by random linear network coding [1], [2] over packets buffered at each nodes. For erasure line networks has been demonstrated in [3–5] that network coding can lead to throughput increases and smaller end-to-end delay compared to traditional end-to-end forward error correction. A queueing theoretic analysis of finite buffer lengths has been presented in [6], [7]. Further, busty erasure channels have been addressed in [8] where random linear network coding is employed to increase throughput in broadcast channels.

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In the following we provide a queueing-theoretic analysis of two-hop line networks, where we extend our recent work in [9] to bursty packet erasure channels between the terminals. In particular, we assume that both the first and the second node have local information with Poisson-distributed arrivals available where the corresponding packets of both nodes need to be conveyed to the receiving node. The inter-node communication channels are modeled by a simple two-state Gilbert-Elliott channel model [10]. In this work, we address the particularly interesting online approach [11] where a continuous flow of incoming packets arrives at the nodes in the network. First, we consider the case of a full duplex operation of the second node and inter-session random linear network coding where a genie is assumed to provide information about the channel states to all nodes in the network. We provide an analysis of the average delay and energy consumption as a function of the channel states and the arrival rates at each node, which serves as the ultimate performance limit of our system. We then consider a more practical half-duplex operation at the relays where only a partial genie knowledge is used in the system in the sense that the genie indicates the state of the two channels but not the individual packet loss events. Our results show that a significant increase of the stable network throughput region can be obtained with policies aware of the channel state compared to policies that are aware only to the average performance of the channel.

II. Genie-Aided Inter-Session Coding: Time-dependent Erasures

Let us assume a line network with three nodes, where two adjacent nodes (S_1, S_2) are source nodes and the final node is the destination, R. Each source S_k generates data packets at a rate of λ_k via a Poisson process; we assume that the packet arrivals at each source are independent from each other. This defines the following packet flows, $S_1 \rightarrow S_2 \rightarrow R$ and $S_2 \rightarrow R$, where flow k is the flow originating at S_k. We consider an online approach where input packets arrive continuously. Further, our initial system model has slotted time where parallel transmission channels are assumed and thus node S_2 operates in full-duplex mode. Therefore, at most one packet can be transmitted from S_1 to S_2 and at most one from S_2 to R per slot.

We assume an independent Gilbert-Elliott model for the channel of each link, where $p_{(g,i)}$ and $p_{(b,i)}$ are the corre-

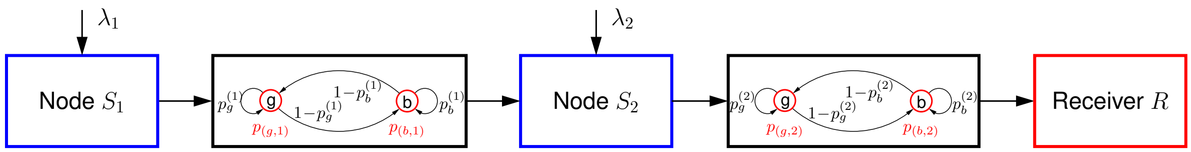

Fig. 1: System setup.

sponding erasure probabilities on the i-th link for the good and bad channel, respectively. The probability of link i to remain in state $c \in \{b, g\}$ is given by $p_c^{(i)}$. Each source node performs inter-session random linear network coding, where at S_2 all incoming flows are linearly combined. We also assume that each node in the network has full system knowledge provided by a genie. This system setup is shown in Fig. 1

As in previous works (see, e.g., [4], [6], [7]) we model the system as a Markov process. A state $\mathcal{S} = (i_1, i_2, C)$ is defined by i_1 (or i_2) which denotes the number of degrees of freedom (dof) at S_1 (or S_2) that have not been seen at S_2 (or R). The state variable i_k represents the number of (coded) packets in the queue S_k because all remaining packets can, without loss of generality, be discarded from the queue [12]. The state variable $C = (c_1, c_2)$ represents the status of the channel at each link, with $c_i \in \{g, b\}$. Let us consider $\bar{Y} = (y_1, y_2)$ as a variable that indicates if a change in state occurred in each Gilbert-Elliott channel, where $y_i = 1$ represents a switch in state and $y_i = 0$ otherwise. We define $\mathcal{C}\langle(\bar{Y}|C)$ as the probability of transitioning from C. Given channel independence, we have $\mathcal{C}\langle(\bar{Y}|C) = \mathcal{C}\langle_1(y_1|c_1)\mathcal{C}\langle_2(y_2|c_2)$ with $\mathcal{C}\langle_k(y|c) = p_c^{(k)}\mathbf{1}_{\{y=0\}} + (1 - p_c^{(k)})\mathbf{1}_{\{y=1\}}$ where $\mathbf{1}_{\{s\in S\}}$ denotes the indicator function which is one when $s \in S$ and zero otherwise. Thus, we can obtain the mean probability of channel i easily as

$$\bar{p}_i = p_{(g,i)}\frac{1 - p_b^{(i)}}{2 - p_g^{(i)} - p_b^{(i)}} + p_{(b,i)}\frac{1 - p_g^{(i)}}{2 - p_g^{(i)} - p_b^{(i)}}.$$

We define $a_{(x_1,x_2)}\{b\}$ as the probability of x_i packets being generated at S_k in b time slots, $k = 1, 2$. Given independence we obtain $a_{(x_1,x_2)}\{b\} = a_{(x_1)}^{(1)}\{b\}a_{(x_2)}^{(2)}\{b\}$, where $a_{(x_i)}^{(i)}\{b\} = \frac{e^{-\lambda_i b}(\lambda_i b)^{x_i}}{x_i!}$. Further, let $d_{(y_1,y_2)|\mathcal{S}}\{b_1, b_2\}$ be the probability of y_k packets being transmitted successfully from S_k conditioned on the current state $\mathcal{S}$ when b_k coded packets, generated from the i_k packets in the queue, are transmitted. Since we have parallel transmission channels, $d_{(y_1,y_2)|\mathcal{S}}\{b_1, b_2\} = d_{(y_1)|i_1,c_1}\{b_1\}d_{(y_2)|i_2,c_2}\{b_2\}$, where

$$d_{(y_k)|i_k,c_k}\{b\} = \mathbf{1}_{\{i_k=0,y_k=0\}} + \mathbf{1}_{\{i_k>0,y_k=0,..,\min(i_k,b)-1\}}\cdot \left(\sum_{l=0}^{b}\sum_{r=0}^{\min(y_k,l)} P_{GE}(b,l,c_k)\mathrm{Bi}_{(y_k-r;b-l,p_{(g,k)})}\mathrm{Bi}_{(r;l,p_{(b,k)})}\right) \left(1 - \sum_{m=0}^{\min(i_k,b)-1} d_{(m)|i_k,c_k}\{b\}\right)\mathbf{1}_{\{i_k>0,y_k=\min(i_k,b)\}}$$

with $\mathrm{Bi}_{(k;n,p)} = \binom{n}{k}(1-p)^k p^{n-k}$. Here $P_{GE}(b, l, c_k)$ is the probability of having a sequence with $b - l$ good and l bad states, resp., given that the first state was c_k. Its associated probability generating function (PGF) is defined as $M_{GE|c_k}\{b\}(z)$. This probability and the corresponding PGF are directly linked to the dynamics of the Gilbert-Elliott channel model and are omitted in this discussion for brevity. Finally, we define $M_{d_k|i_k,c_k}\{b\}(z)$ as the PGF of $d_{(y_k)|i_k,c_k}\{b\}$.

Let us further define $P(T|\mathcal{S}) = P_{\mathcal{S}\to\mathcal{S}'}$ as the transition probability between states $\mathcal{S} = (i_1, i_2, C)$ and $\mathcal{S}' = (i_1', i_2', C')$. This effect is captured by the probability of the random vector $T = (\Delta_1, \Delta_2, C')$, where $\Delta_k = i_k' - i_k$. Thus, the transition probability between states $\mathcal{S} = (i_1, i_2, C)$ and $\mathcal{S}' = (i_1', i_2', C')$ can be written as

$$P(\Delta_1, \Delta_2, C'|\mathcal{S}) = \sum_{y_1\in\{0,1\},y_2\in\{0,1\}} a_{(\Delta_1,\Delta_2-f(y_1,y_2))}\{1\}\, d_{(y_1,y_2)|\mathcal{S}}\{1,1\}\mathcal{C}\langle(\bar{Y}|C)$$

with $f(y_1, y_2) = \mathbf{1}_{\{y_2=0,y_1=1\}} - \mathbf{1}_{\{y_2=1,y_1=0\}}$. After some intermediate steps we can express $P(\Delta_1, \Delta_2, C'|\mathcal{S})$ as follows,

$$\begin{aligned} P(\Delta_1, \Delta_2, C'|\mathcal{S}) = \mathcal{C}\langle(\bar{Y}|C)\cdot\Big(& d_{(0)|i_1,c_1}\{1\}\, a_{(\Delta_1)}^{(1)}\{1\}\,\mathbf{1}_{\{\Delta_1\geq 0\}}\cdot \\ & \Big[d_{(0)|i_2,c_2}\{1\}\, a_{(\Delta_2)}^{(2)}\{1\}\,\mathbf{1}_{\{\Delta_2\geq 0\}} + \\ & d_{(1)|i_2,c_2}\{1\}\, a_{(\Delta_2+1)}^{(2)}\{1\}\,\mathbf{1}_{\{\Delta_2\geq -1\}}\Big] + \\ & d_{(1)|i_1,c_1}\{1\}\, a_{(\Delta_1+1)}^{(1)}\{1\}\,\mathbf{1}_{\{\Delta_1\geq -1\}}\cdot \\ & \Big[d_{(0)|i_2,c_2}\{1\}\, a_{(\Delta_2-1)}^{(2)}\{1\}\,\mathbf{1}_{\{\Delta_2\geq 1\}} + \\ & d_{(1)|i_2,c_2}\{1\}\, a_{(\Delta_2)}^{(2)}\{1\}\,\mathbf{1}_{\{\Delta_2\geq 0\}}\Big]\Big), \end{aligned} \quad (1)$$

where $y_i = \mathbf{1}_{\{c_i'\neq c_i\}}$. This expression is relevant to model the queue dynamics and to determine the stationary distribution of this Markov chain. Further, positive recurrence of the Markov chain can be shown by using the criteria in [13], which guarantees existence of a unique stationary probability.

A. *Probability generating function*

In the following, we consider the probability generating function (PGF) for the state transition probabilities $P(T|\mathcal{S})$. The PGF is useful for computing the steady-state distribution of the underlying Markov process as discussed below. The PGF for a random vector $X = \{x_1, x_2, \ldots, x_n\}$ is defined as

$$M_X(Z) = \sum_K P(X = K)\prod_i z_i^{k_i} \quad (2)$$

where $Z = \{z_1, z_2, \ldots, z_n\}$ and $K = \{k_1, k_2, \ldots, k_n\}$. Clearly,

$$\frac{\partial^{k_1}}{\partial z_1^{k_1}} \cdots \frac{\partial^{k_n}}{\partial z_n^{k_n}} M_X(Z)\Big|_{z_1=0,\ldots,z_n=0} = P(x_1 = k_1, \ldots, x_n = k_n), \tag{3}$$

which simplifies the computation of the indivitual transition probabilities in our approach. For our system we define $M_{T|\mathcal{S}}(Z)$ as the PGF for the state transition probability when starting in state $\mathcal{S}$. We have the following lemma.

Lemma 1. *Let $P(T|\mathcal{S}) = P_{\mathcal{S}\to\mathcal{S}'}$ be the transition probability between $\mathcal{S} = (i_1, i_2, c_1, c_2)$ and $\mathcal{S}' = (i'_1, i'_2, c'_1, c'_2)$, where $T = (i'_1 - i_1, i'_2 - i_2, y_1, y_2)$ and $y_i = \mathbf{1}_{\{c'_i \neq c_i\}}$. The PGF for the genie-aided case with inter-session coding is given as*

$$M_{T|\mathcal{S}}(Z) = e^{\lambda_1(z_1-1)} e^{\lambda_2(z_2-1)} \cdot \left(p^{(1)}_{c_1} + (1 - p^{(1)}_{c_1}) z_3\right) \cdot \left(p^{(2)}_{c_2} + (1 - p^{(2)}_{c_2}) z_4\right) \cdot \left(d_{(0)|i_2,c_2}\{1\} + d_{(1)|i_2,c_2}\{1\} z_2^{-1}\right) \cdot \left(d_{(0)|i_1}, c_1\{1\} + d_{(1)|i_1,c_1}\{1\} z_2 z_1^{-1}\right).$$

Proof (sketch):. The proof follows by inserting (1) into the definition of the PGF and by using the fact that

$$\sum_{k_h \in \mathbb{Z}} a^{(h)}_{(k_h+\alpha)}\{b\} \mathbf{1}_{\{k_h+\alpha \geq 0\}} z_h^{k_h} = e^{\lambda_h \tau b (z_h - 1)} z_h^{-\alpha} \tag{4}$$

for $\alpha \in \mathbb{Z}$ and $h = \{1, 2\}$, and that $\sum_{y_h \in \{0,1\}} \mathcal{C}\langle_h (y_h|c_h) z^{y_h} = p^h_{c_h} + (1 - p^h_{c_h}) z$ for $h = \{1, 2\}$. ■

Let us define $\pi_{\mathcal{S}} = \pi_{(m,l,C)}$ as the stationary probability of state $\mathcal{S} = (m, l, C)$. Using the fact that

$$\pi_{\mathcal{S}} = \sum_{m' \geq 0, l' \geq 0, C' \in \{g,b\}^2} \pi_{(m',l',C')} P_{(m',l',C') \to (m,l,C)} \tag{5}$$

the PGF for $\pi_{(m,l,C)}$ can be written as $\Pi(Z) = \sum_{m \geq 0, l \geq 0, C \in \{g,b\}^2} \pi_{(m,l,C)} M_{T|(m,l,C)}(Z)$. Given that each node can process at most one packet per time slot, the PGF has a limited number of cases of interest, namely 16 cases, corresponding to one or both queues being empty and to the scenario in which both queues have at least one packet. The latter translates into $M_{T|(1,1,C)}(Z) = M_{T|(m,l,C)}(Z)$ for $m \geq 1, l \geq 1$. This is clear as the transition probabilities for states with $m \geq 1, l \geq 1$ is the same, i.e., the system can at most transmit one packet from each source while the Poisson distribution driving the packet arrivals remains the same. The same holds true for $M_{T|(1,0,C)}(Z) = M_{T|(m,0,C)}(Z)$ with $m \geq 1$ and for $M_{T|(0,1,C)}(Z) = M_{T|(0,l,C)}(Z)$ with $l \geq 1$. We can leverage techniques similar to those in our previous work [9] to determine the $\Pi(Z)$.

B. Delay

Let us define D_k as the time that a packet in S_k experiences between being received and being seen at the next hop, and thus discarded from the queue of S_k. By Little's Law we obtain the average delay $E[D_1]$ and $E[D_2]$. Note that a packet from flow 1 will experience an average delay of $E[D_1] + E[D_2]$ before been seen at the end receiver R, while a packet from flow 2 will experience an average delay of $E[D_2]$ before being seen at R.

C. Energy

We study the average total energy invested per successfully transmitted packet for each of the two transmitting nodes, S_1 and S_2. We consider E_k to be the overall energy to convey a packet over a time slot S_k (including transmission and reception energy). Each source is considered to operate in cycles, where each cycle has two phases. First, we have an "idle" phase where the queue for S_k is empty, which requires T_k^0 time slots. Second, there is a "busy" phase where the queue is not-empty, which requires T_k time slots. This constitutes the time the system needs to obtain $i_k = 0$ for the first time, given that the system starts at $i_k > 0$ after the reception of packets at the end of the previous idle phase.

Theorem 2 ([9]). *The average overall energy per transmitted packet at node S_k, $\mathcal{E}_k$, for the genie-aided case with inter-session coding is given by*

$$\mathcal{E}_1 = (1 - P_{em_1}) E_1/\lambda_1, \quad \mathcal{E}_2 = (1 - P_{em_2}) E_2/(\lambda_1 + \lambda_2),$$

where $P_{em_1} = \frac{E[T_1^0]}{E[T_1] + E[T_1^0]}$, $P_{em_2} = \frac{E[T_2^0]}{E[T_2] + E[T_2^0]}$ *and*

$$E[T_1^0] = \frac{1}{1 - e^{-\lambda_1}}, \; E[T_2^0] = \frac{1}{1 - e^{-\lambda_2}(P_{em_1} + p_1(1 - P_{em_1}))},$$

Further, the extension to the intrasession coding case, where both flows are separated by performing random linear coding only within a single flow, is equivalent to [9].

III. Genie-Aided Half-Duplex Inter-Session Case

We now introduce a half-duplex constraint on the problem in the sense that node S_2 can only transmit or receive packets, but not both, in a single time slot. We assume that a genie indicates the state C of the two channels, i.e., the probabilities of packet loss in each channel, at each time slot. However, the event of a packet loss is not known *a priori* to the genie. The genie has also perfect knowledge of the queue lengths of every queue. The key problem in this scenario is to schedule transmissions from S_1 and S_2 to (i) maintain stability of the queues, and (ii) exploit the nature of the channel in the scheduling process. The genie uses specific policies to schedule a transmission from S_1, S_2 or schedule no transmission in each and every time slot.

Let us consider the state $\mathcal{S} = (i_1, i_2, c_1, c_2, S_t)$, where i_1 and i_2 represent the dof missing at node S_2 and R, resp., c_1 and c_2 represent the state of the Gilbert-Elliott channels, and S_t indicates the node that will be actively transmitting in the upcoming round. We define a policy $\mathcal{P} \in \{S_1, S_2\}$ as a function that schedules the following transmitter based on the queue lengths and channel states. For a transmission from S_1 the transition probability is given by

$$P(\Delta_1, \Delta_2, C', S'_t | i_1, i_2, c_1, c_2, S_1) = \mathcal{C}\langle(\bar{Y}|C) \cdot \mathbf{1}_{\{\mathcal{P} = S'_t\}} \cdot \Bigg(d_{(0)|i_1,c_1}\{1\}\, a^{(1)}_{(\Delta_1)}\{1\}\, a^{(2)}_{(\Delta_2)}\{1\}\, \mathbf{1}_{\{\Delta_1 \geq 0, \Delta_2 \geq 0\}} + d_{(1)|i_1,c_1}\{1\}\, a^{(1)}_{(\Delta_1+1)}\{1\}\, a^{(2)}_{(\Delta_2)}\{1\}\, \mathbf{1}_{\{\Delta_1 \geq -1, \Delta_2 \geq 0\}} \Bigg).$$

This expression is obtained by considering that (i) only S_1 shall transmit in the current time-slot, (ii) S'_t is chosen by the policy as the next transmitter, otherwise the indicator function drives the probability of a transition to a state where S'_t transmits to zero, (iii) the channel state transitions are independent, and (iv) the transition can be related to two events, namely, that transmission of a packet was successful or not, and the associated packet arrivals. A similar expression can be found for a starting state $\mathcal{S} = (i_1, i_2, c_1, c_2, S_2)$.

Efficient scheduling policies for this problem are inherently dependent on the channel characteristics and the length of the queue. Since packet loss events are unknown at the time of scheduling a transmission, the policy should take into account this randomness as part of the scheduling process. Scheduling policies can be probabilistic, as in Section III-A, or deterministic, as the ones presented in Section III-B.

A. Optimal policy

In the following we define an optimal policy for the minimization of the mean channel utilization. Let us first define the *effective* mean erasure probability of S_i, $\bar{p}_{\text{eff}}(i)$ as the mean erasure probability seen by the transmitter as a result of the scheduling policy. We define $\alpha_{(i,C)}$ and $\text{Pr}_{(i,C)}$ as the fraction of the rate associated with S_i and the probability of transmission of S_i during the channel state C, respectively. Thus,

$$\bar{p}_{\text{eff}}(i) = \sum_{C=(c_1,c_2)\in\{g,b\}^2} p_{(c_1,i)}\alpha_{(i,C)}\pi_C, \tag{6}$$

where π_C constitutes the stationary probability of the channel state C, which can be easily determined through standard finite Markov chain techniques. The channel utilization in our half-duplex system is given by

$$U_{\lambda_1,\lambda_2}([p_{\text{eff}}(i)]) = \frac{\lambda_1}{1-p_{\text{eff}}(1)} + \frac{\lambda_1+\lambda_2}{1-p_{\text{eff}}(2)} \leq 1, \tag{7}$$

which is a convex function in the region of interest, i.e., $p_{\text{eff}}(i) \in [p_{(g,i)}, p_{(b,i)}]$.

The optimization problem is stated as

$$\min_{[\alpha_{(i,C)}]} U_{\lambda_1,\lambda_2}\left(\left[\sum_{C=(c_1,c_2)\in\{g,b\}^2} p_{(c_1,i)}\alpha_{(i,C)}\pi_C\right]\right),$$

subject to

$$\text{Pr}_{(1,C)} + \text{Pr}_{(2,C)} \in [0,1], \quad \forall C \in \{g,b\}^2,$$

$$\sum_{C\in\{g,b\}^2} \alpha_{(i,C)} = 1, \quad i = \{1,2\},$$

$$(1-p_{(c_1,1)})\text{Pr}_{(1,C)}\pi_C = \lambda_1\alpha_{(1,C)}, \quad \forall C \in \{g,b\}^2,$$

$$(1-p_{(c_2,2)})\text{Pr}_{(2,C)}\pi_C = (\lambda_1+\lambda_2)\alpha_{(2,C)}, \quad \forall C \in \{g,b\}^2.$$

In the first line, we have used the right hand side of (6) as the argument of $U_{\lambda_1,\lambda_2}(\cdot)$. The last two conditions capture the fact that the probability of S_k transmitting in a given channel state is linked to the mean usage of the channel of S_k during that state, e.g., $\lambda_1\alpha_{(1,C)}/(1-p_{(c_1,1)})$ for S_1. The throughput region can be derived by searching for λ_i's that guarantee $U_{\lambda_1,\lambda_2}([p_{\text{eff}}(i)]) = 1$.

The optimal policy for a given channel state C and source rates λ_1, λ_2 is given by the vector $[\text{Pr}_{(i,C)}]$ that results of this optimization. The optimal policy $\mathcal{P}_{\text{opt}}(C)$ can be stated as

Policy 1.

$$\mathcal{P}_{opt}(C) = \begin{cases} S_1 & w.p.\ \text{Pr}_{(1,C)} \\ S_2 & w.p.\ \text{Pr}_{(2,C)} \\ \textit{No transmission} & \textit{otherwise}\ . \end{cases}$$

We emphasize that this policy is probabilistic in nature and relies on the fact that the queues are infinity. The latter allows the system to store the (coded) packets while awaiting a good channel. In practice, deterministic algorithms that link the queue lengths and channel-awareness may be more relevant since they may guarantee better delay performance, albeit with a possibly degraded throughput region.

B. Heuristic policies

We now propose a class of policies for scheduling transmissions between S_i's, which constitute a combination of back pressure and proportional fairness mechanisms.

Policy Class 1.

$$\mathcal{P}_{\alpha,\beta,}(i_1,i_2,C) = \arg\max_{S\in S_1,S_2} W_S \tag{8}$$

where $\alpha,\beta \in [0,\infty)$, $W_{S_2} = i_2\frac{(1-p_{(c_2,2)})^\alpha}{(1-\bar{p}_2)^\beta}$, *and*

$$W_{S_1} = \max\left\{0, i_1\frac{(1-p_{(c_1,1)})^\alpha}{(1-\bar{p}_1)^\beta} - i_2\frac{(1-p_{(c_2,2)})^\alpha}{(1-\bar{p}_2)^\beta}\right\}. \tag{9}$$

The class of policies is given by different values for α and β. The objective of this class of policies is to schedule the transmissions based on the queue lengths, the current erasure probabilities ($p_{(c_1,1)}$, $p_{(c_2,2)}$) and the average erasure probabilities ($\bar{p}_1$, $\bar{p}_2$) of the channels. In the following, we discuss some special cases.

- $\alpha = 1, \beta = 1$ provides a mechanism that effectively incorporates the mean number of transmissions for the queue in S_k, $i_k/(1-\bar{p}_1)$, and the current impact of that queue transmitting in a given time slot, namely $(1-p_{(c_k,k)})$. This is an opportunistic scheme that aims to exploit transmissions when a channel is in a good state.
- $\alpha = 0, \beta = 0$ provides a mechanism that only considers the queue lengths to determine the schedule. This makes the system unaware of the current state of the channels. Thus, the effective erasure probability seen by each transmitter is equal to the average erasure probability of each channel, i.e., $\bar{p}_{\text{eff}}(i) = \bar{p}_i$.

As a final comment, we observe that our deterministic policies have a $\bar{p}_{\text{eff}}(i)$ larger than the optimal value determined in Section III-A in general. The main reason is that the optimal policy may choose between three outcomes, including "no transmission", while our deterministic policies always choose S_1 or S_2 to transmit, unless the queues are empty. Thus, transmissions from each source S_k will experience bad channel states more often.

IV. Numerical Results

Fig. 2 compares genie-aided inter-session coding under full duplex and half-duplex constraints by illustrating the delay

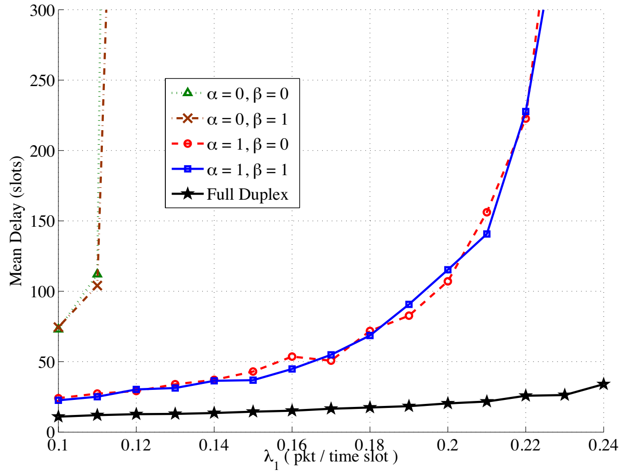

Fig. 2: Delay vs. arrival rate λ_1 for genie-aided half-duplex inter-session coding. Parameters: $\lambda_2 = 0.25$, $p_{(g,1)} = p_{(g,2)} = 0.1$, $p_{(b,1)} = p_{(b,2)} = 0.8$, $p_b^{(1)} = p_b^{(2)} = 0.9$, $p_g^{(1)} = p_g^{(2)} = 0.9$.

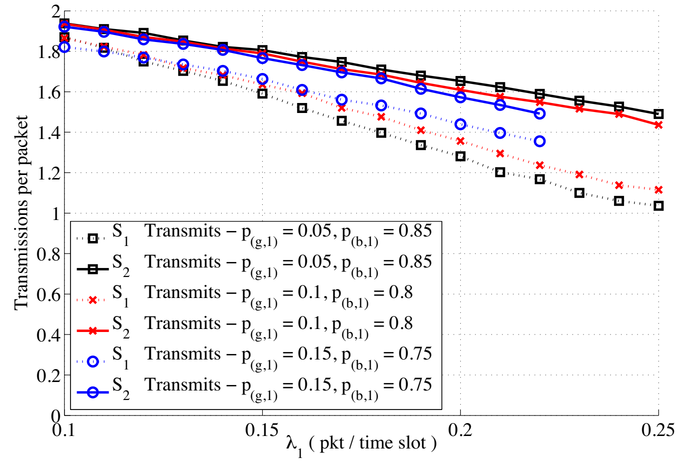

Fig. 3: Energy (transmission/packet) vs. arrival rate λ_1 for genie-aided half-duplex inter-session coding. Parameters: $\lambda_2 = 0.25$, $p_{(g,2)} = 0.1$, $p_{(b,2)} = 0.8$, $p_b^{(1)} = p_b^{(2)} = 0.9$, $p_g^{(1)} = p_g^{(2)} = 0.9$, $\alpha = \beta = 1$.

performance for flow 1 i.e., $E[D_1]+E[D_2]$. For the half-duplex case we use our deterministic policy, under different values of α and β, to determine the transmitter at any given time slot. For the case of genie aided half-duplex with $\alpha = 0$, we observe that the delay is significantly higher than the cases with $\alpha = 1$. In fact, the queues using the policy with $\alpha = 0$ are only stable for $\lambda_1 \leq 0.11$ according to our numerical results. The case of $\alpha = 0$ represents policies that do not incorporate the current state of the channels in the scheduling process. As can be seen from Fig. 2 the choice of β seems to have little effect on the overall delay performance.

Fig. 3 shows the energy performance of the deterministic policy for $\alpha = \beta = 1$ for the genie-aided half-duplex inter-session coding case. We observe that the lower bound to the number of transmissions per packet on the first link (S_1 transmitting) is given by $1/(1 - p_{(g,1)})$, where $p_{(g,1)}$ is the erasure probability of the 'good' channel. In the case of $p_{(g,1)} = 0.1$, this lower bound amounts to 1.1 which corresponds to the average number of transmissions per packet for S_1 transmissions. The figure also illustrates that our deterministic policy becomes more efficient in terms of energy as the system load increases because the policy can take more advantage of the time-dependency of the channel.

V. Conclusions

We have considered a two-hop line network with bursty erasures, where each of the first two nodes intends to send local information packets with Poisson-distributed arrivals to the last node in the line. We discuss an online approach where the correction of the erasures is carried out by random linear network coding. Queuing-theoretic results are provided for a simple two-state Gilbert-Elliott channel model, where two different models were considered: (i) a genie aided scheme with a full duplex operation at the middle node, where the genie provides all information about the system state; and, (ii) a partially genie aided half duplex scheme where the genie only provides information about the channel state. Optimal random and practical deterministic policies are derived which show that the stable throughput region of the scheme can be significantly extended compared to a simple queue-length driven (backpressure) policy. We observe from the simulation results that a higher time-dependency of the channel, i.e., a larger probability that both channels stay in the same state, leads to larger gains in both throughput increase and delay reduction for both flows.

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