ASSESSING SUBJECTIVE PERCEPTION OF AUDIO QUALITY BY MEASURING THE INFORMATION FLOW ON THE BRAIN-RESPONSE CHANNEL

Ketan Mehta

Klipsch School of Electrical
and Computer Engineering
New Mexico State University
Las Cruces, NM 88003, USA

Jörg Kliewer

Department of Electrical
and Computer Engineering
New Jersey Institute of Technology
Newark, NJ 07103, USA

ABSTRACT

In this paper, we use mutual information (MI) as a measure to quantify the subjective perception of audio quality by directly measuring the brainwave responses of human subjects using a high resolution electro-encephalogram (EEG). Specifically, we propose an information theoretic model to interpret the entire "transmission chain" comprising stimulus generation, brain processing by the human subject, and EEG measurements as a nonlinear, time-varying communication channel with memory. In the conducted experiment, subjects were presented with audio whose quality varies between two quality levels. The recorded EEG measurements can be modeled as a multidimensional Gaussian mixture model (GMM). In order to make the computation of the MI feasible, we present a novel approximation technique for the differential entropy of the multidimensional GMM. We find the proposed information theoretic approach to be successful in quantifying audio quality perception, with the results being consistent across different subjects and distortion types.

Index Terms— mutual information, perception, audio quality, electro-encephalography (EEG), Gaussian mixture model (GMM)

1. INTRODUCTION

The current state-of-the-art approach for subjective quality testing of audio is Multi Stimulus with Hidden Anchor (MUSHRA) [1]. One characteristic that MUSHRA and most of the other existing audio and video testing protocols have in common is that each human participant assigns a single quality-rating score to each test sequence. Such testing suffers from a subject-based bias towards cultural factors in the local testing environment and tends to be highly variable. Recently, there has been a growing interest in using EEG to classify human perception of audio [2, 3] and visual [4–6] quality. For example, [3] investigates the use of a time-space-frequency analysis to identify features in EEG brainwave responses corresponding to time-varying audio quality. Further, [2,4] propose to use Linear Discriminant Analysis (LDA) classifiers to extract features based on the P300 evoked response potential (ERP) component [7,8] for classifying noise detection in audio signals and to assess changes in perceptual video quality, respectively. Similarly, in [6] the authors employ a wavelet-based approach for an EEG-classification of commonly occurring artifacts in compressed video, using a single-trial EEG.

In this paper, we provide a novel information theoretic framework to assess the subjective perception of audio quality using EEG data. Our approach here is different compared to the above mentioned studies in that we analyze the overall transmission chain comprising of stimulus generation, processing by the human brain, and the EEG sensors as a time-varying, nonlinear communication channel with memory by determining the corresponding mutual information (MI) [9]. In the neuroscience literature such a channel in general is referred to as the ERP channel [10] and is shown in Figure 1. We are motivated here by the fact that a better understanding of the stochastic characteristics of the end-to-end perceptual processing chain in-turn enables us to create better models/metrics of how the brain responds to changes in observed audio quality. The ERP channel can be considered equivalent to a single-input multiple-output (SIMO) communication channel with unknown characteristics, where the quality of the audio stimulus represents the scalar (single) input, and the observation at the EEG sensors on the scalp is the vector (multiple) output. In particular, we show that the EEG measurements on this channel can be modeled as a Gaussian mixture model (GMM). Further, in order to make the computation of the MI over the ERP channel feasible, we present a novel approximation technique for the differential entropy of the multidimensional GMM based on a Taylor series expansion. Unlike current methods, employing MI does not assume stationary of the EEG signal and does not rely on linear dependencies. Therefore it represents a well suited measure to model nonlinear, time-varying phenomena like brain activity. In the past, previous EEG studies have successfully used MI to analyze corticocortical information transfer [11–14], and for feature extraction and classification purposes (see, e.g., [15–19]). To the best of our knowledge, however, this is the first time that an information theoretic characterization has been used in conjunction with EEG measurements to quantify human perception of audio quality.

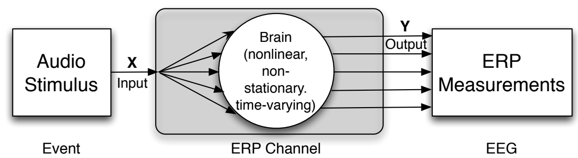

Fig. 1: Communicating over the ERP channel.

2. EXPERIMENT

To collect the data required for the study in our experiments, test subjects were presented with a variety of different audio sequences whose qualities varied with time. A total of 19 test subjects, all with normal hearing capability, participated in the experiment with the majority of them being male. We employed an ActiveTwo Biosemi EEG system, which captures data on 128 spatial channels, sampled at 1024 Hz. An average re-referencing and baseline removal was performed, and the EEG data was passed through a high-pass filter with a cut-off frequency at 1 Hz.

All stimulus test-sequences were created from three funda-

This work was supported in part by NSF grant CCF-1065603.

mentally different base-sequences sampled at a 'high' quality of 44.1 kHz, with a precision of 16 bits per sample. Two different types of distortions were considered — scalar quantization and frequency band truncation. To generate the 'distorted' quality each of these base-sequences were passed through a 2048 MDCT (with 50% overlap) and either the frequency truncation or the scalar quantization was applied to the coefficients prior to reconstruction. The frequency truncation was implemented by setting all the coefficients above 1.1 KHz to zero, while the scalar quantization was performed by only retaining up to the second most significant bit of the MDCT coefficient. The test-sequence for a specific trial was then created by selecting one of the two distortion types and applying it over the duration of the entire sequence using a time-varying pattern of non-overlapping five second blocks, each comprising of a piecewise constant 'high' or 'distorted' quality [20]. Multiple of such trials were conducted for each subject by choosing all possible combinations of sequences, distortion types, and time-varying patterns.

To better manage the large amount of collected data while effectively mapping the activity across different regions of the brain, we suggest grouping the 128 electrodes of the EEG-system into specific regions of interest (ROI). A potential grouping scheme [21] and the one that we use is shown in Figure 1. While a large number of grouping schemes are possible, this scheme is favored for our purposes as it efficiently covers the different cortical regions (lobes) of the brain with a relatively low number of ROIs.

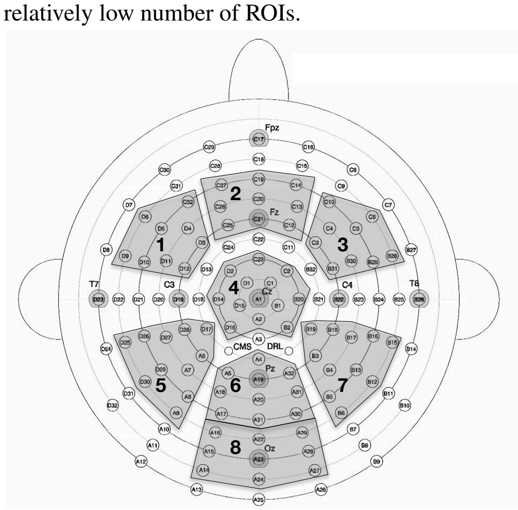

Fig. 2: Schematic representation of the 128-channel EEG system. The electrodes are grouped into eight regions of interest (ROI) to effectively map the activity across different regions of the brain.

3. INFORMATION THEORETIC ANALYSIS

3.1. ERP channel

The input random variable X of the ERP channel is uniformly distributed over a set of class indices $\mathcal{X}$ which describe the quality of the stimulus sequences at any given time interval. The audio quality of the input sequence can then be represented as an equiprobable Bernoulli distribution

$$X = \begin{cases} x_1, & \text{if the input stimulus is of high quality,} \\ x_2, & \text{if the input stimulus is of distorted quality,} \end{cases}$$

with *a priori* probabilities $p(x_1) = p(x_2) = 1/2$.

The output of the channel is given by the index set $\mathcal{Y}$ containing all possible values of the EEG potential at any given time interval. For a total of n ROIs we therefore get a (multivariate) output

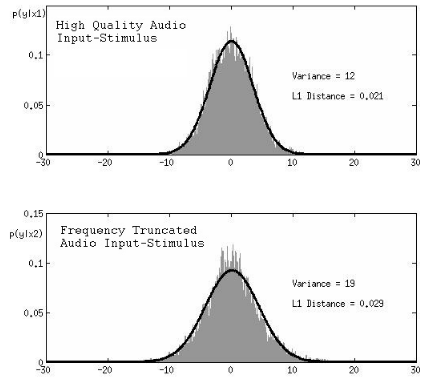

Fig. 3: Normalized probability distributions $p(y|x_1)$ and $p(y|x_2)$ of a subject over a single ROI, for high quality and frequency truncated audio input-stimulus, respectively. The Gaussian fit is obtained by using an estimator that minimizes the L_1 distance between the fitted Gaussian distribution and the histogram data.

vector of random processes $\underline{Y}(t) = (Y_1(t), \ldots, Y_n(t))$. To reduce the variance of the ERP channel measurements we consider every electrode in the i-th ROI as an independent realization of the same random process $Y_i(t)$, $i = 1, \ldots, n$. Further, we assume the random process to be ergodic within the five second non-overlapping blocks of a trial with constant audio quality, and therefore without any loss of generality we can set $\underline{Y}(t) = \underline{Y}$. Note that this assumption does not rule out any non-stationary behavior between sections of different audio quality within the same trial.

Since the input can take on two distinct values, there are two conditional distributions $p(y_i|x_1)$ and $p(y_i|x_2)$ corresponding to any given ROI. Figure 3 shows the normalized conditional distributions obtained via histogram measurements of a single ROI output over time. A detailed inspection using different subjects, input sequences, and ROIs allows us to assert two important facts about this distribution. *First*, the conditional distribution converges to a Gaussian with zero mean. The potential recorded at the EEG electrode at any given time-instant can be considered as the superposition of responses of a large number of neurons. Thus, the distribution of a sufficiently high number of these trials taken at different time instances converges to a Gaussian distribution as a result of the Central Limit Theorem (CLT). It then follows directly from the CLT that the probability distribution for n ROIs will also converge to a n-dimensional multivariate Gaussian distribution. *Second*, we observe from Figure 3 that there is a difference between the variances of the distributions $p(y_i|x_1)$ and $p(y_i|x_2)$. This indicates that Y_i is dependent on X, i.e., the EEG data is related to and contains "information" about the input stimulus. In information theory this is measured with the MI $I(X; Y_i)$, typically measured in *bits* [9]. In particular, the MI quantifies the information transfer over the communication channel $X \to Y_i$. Further, if we consider the conditional probability distributions corresponding to n ROIs chosen simultaneously, then each of the distributions $p(\underline{Y}|X = x_1)$ and $p(\underline{Y}|X = x_2)$ are n-dimensional Gaussian distributions, with K_1 and K_2 being the $n \times n$ covariance matrices of each of the distributions, respectively. The conditional differential entropy $h(\underline{Y}|X)$ in bits is then defined as [9]

$$\begin{aligned} h(\underline{Y}|X) &\triangleq -\sum_{x \in X} p(x) \int_{-\infty}^{\infty} p(\underline{y}|x) \log p(\underline{y}|x)\, d\underline{y} \\ &= \frac{1}{4} \left\{ \log(2\pi e)^n |K_1| + \log(2\pi e)^n |K_2| \right\}, \quad (1) \end{aligned}$$

where here and in the following all logarithms are taken with base 2. Using the the law of total probability we can rewrite $p(y)$ in terms of the Gaussian conditional probabilities as

$$p(\underline{y}) = \sum_{x \in X} p(\underline{Y}{=}\underline{y}|x)p(x) = \frac{1}{2}\left\{p(\underline{y}|x_1) + p(\underline{y}|x_2)\right\}. \quad (2)$$

The differential entropy of $\underline{Y}$ in bits is therefore given as

$$h(\underline{Y}) = -\frac{1}{2}\int_{\Re^n}[p(\underline{y}|x_1)+p(\underline{y}|x_2)]\cdot\log\frac{1}{2}[p(\underline{y}|x_1)+p(\underline{y}|x_2)]d\underline{y}. \quad (3)$$

where $\Re^n$ denotes the support region of the distribution $p(\underline{y}|x_i) + p(\underline{y}|x_2)$. The MI between the output EEG data and the input audio stimulus can then be calculated by using $I(X;\underline{Y}) = h(\underline{Y}) - h(\underline{Y}|X)$.

3.2. Entropy approximation

It turns out that, to the the best of our knowledge, there is no closed form solution for the entropy of a mixture of Gaussian random variables (3) which is a recurring open problem in the literature. This is due to the fact that (3) consists of a multiple integral over a logarithm of a sum of exponential functions which makes it difficult to formulate a general closed-form analytical solution. In the absence of an analytical solution it is usually common to use numerical methods to calculate a sufficiently accurate estimate of the solution. However, as the dimensionality of the Gaussian multivariate random variable increases, it becomes computationally infeasible to evaluate the n-th order multiple integral using numerical integration.

Therefore, we propose an approximation for the entropy by performing a component-wise Taylor series expansion of the logarithmic function [22]. This approximation makes no prior assumptions and is suitable in general for estimating the entropy of any given multidimensional GMM. If we assume $p_i(\underline{z})$ to be the probability distribution for the i-component of the GMM associated with the n-dimensional Gaussian distribution $\mathcal{N}(\underline{z};\underline{\mu}_i,\mathbf{C}_i)$ with mean $\underline{\mu}_i \in \mathbb{R}^n$ and covariance matrix $\mathbf{C}_i \in \mathbb{R}^{n\times n}$ then the probability distribution of the GMM is given by

$$p(\underline{z}) = \sum_{i=1}^{L} w_i p_i(\underline{z}), \quad (4)$$

where L is the number of mixture components, and w_i denotes the weight of the i-th component of the GMM, respectively. Let

$$f(\underline{z}) = \log p(\underline{z}) = \log\left(\sum_{j=1}^{L} w_j p_j(\underline{z})\right). \quad (5)$$

If we then use the Taylor series to expand the function $f(\underline{z})$ around $\underline{\mu}_i$, we obtain

$$f(\underline{z}) = f(\underline{\mu}_i) + \frac{f^{'}(\underline{\mu}_i)}{1!}(\underline{z}-\underline{\mu}_i) + \frac{f^{''}(\underline{\mu}_i)}{2!}(\underline{z}-\underline{\mu}_i)^2 + \dots \quad (6)$$

The odd central moments of a Gaussian distribution are zero, therefore all the odd order terms in the Taylor series expansion are also zero. The differential entropy of the Gaussian mixture therefore reduces to

$$h(\underline{z}) = -\int_{\Re^n} p(\underline{z})\log p(\underline{z})\, d\underline{z} = -\int_{\Re^n} p(\underline{z}) f(\underline{z})\, d\underline{z}$$

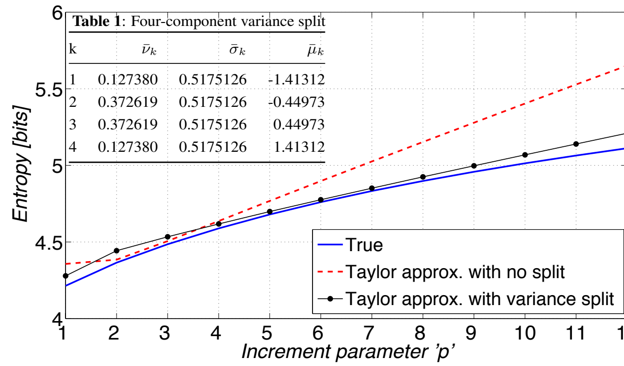

Fig. 4: Entropy approximation using a second-order Taylor expansion for a sample synthetic GMM with zero mean vectors, equal weights and $\mathbf{C}_1 = [4,2;2,4]$,$\mathbf{C}_2 = [5+p,2;2,5+p]$, where the variance of the second distribution is incremented using the parameter p. The true entropy was calculated using numerical integration.

$$= -\sum_{i=1}^{L}\int_{\Re^n} w_i p_i(\underline{z})\cdot\left\{f(\underline{\mu}_i) + \frac{f^{''}(\underline{\mu}_i)}{2!}(\underline{z}-\underline{\mu}_i)^2 + \dots\right\} d\underline{z}. \quad (7)$$

The Gaussian mixture is a smooth function and the derivatives of the function $f(\underline{z})$ will therefore always exist. The Taylor series approximation is exact only if an infinite order of terms are considered, and the accuracy of the approximation therefore depends on the number of expansion terms used. We provide closed form expressions for the zeroth-order and second-order terms of the Taylor series expansion. We obtain the zeroth-order Taylor series expansion term of (7) as

$$h_0 = -\sum_{i=1}^{L}\int_{\Re^n} w_i p_i(\underline{z}) f(\underline{\mu}_i)\, d\underline{z} = -\sum_{i=1}^{L} w_i \log p(\underline{\mu_i}). \quad (8)$$

The second-order expansion term requires us to calculate the gradient $\mathbf{g}(\underline{z})$ and Hessian $\mathbf{H}(\underline{z})$ of the distribution

$$\mathbf{g}(\underline{z}) \triangleq \nabla p(\underline{z}) = p(\underline{z})\mathbf{C}^{-1}(\underline{\mu}_i - \underline{z}), \quad (9)$$

$$\mathbf{H}(\underline{z}) \triangleq (\nabla\nabla^T)p(\underline{z}) = \frac{\mathbf{g}(\underline{z})\mathbf{g}(\underline{z})^T}{p(\underline{z})} - p(\underline{z})\mathbf{C}^{-1}, \quad (10)$$

where ∇ is the gradient with respect to $\underline{z}$. Accordingly, the Hessian for the log-density $f(\underline{z})$ is

$$\mathbf{H}_l(\underline{z}) \triangleq (\nabla\nabla^T)f(\underline{z}) = -\frac{1}{p(\underline{z})^2}\mathbf{g}(\underline{z})\mathbf{g}(\underline{z})^T + \frac{1}{p(\underline{z})}\mathbf{H}(\underline{z}). \quad (11)$$

Finally, the second-order Taylor expansion term of (7) can be obtained as

$$h_2 = -\sum_{i=1}^{L}\int_{\Re^n} w_i p_i(\underline{z})\cdot\frac{1}{2}\mathbf{H}_l(\underline{\mu}_i)\cdot(\underline{z}-\underline{\mu_i})(\underline{z}-\underline{\mu_i})^T d\underline{z}$$

$$= -\frac{1}{2}\sum_{i=1}^{L} w_i\cdot\mathbf{H}_l(\underline{\mu}_i)\circ\mathbf{C}_i, \quad (12)$$

where $\circ$ is the Frobenius product, defined as $\mathbf{A}\circ\mathbf{B} = \sum_i\sum_j a_{ij}\cdot b_{ij}$.

The second-order Taylor expansion of the differential entropy can then simply be calculated from (7), (8), and (12) as $h(\bar{z}) = h_0 + h_2$.

Also, as the variance of the distribution increases in each dimension more terms are required to reduce the approximation error which becomes increasingly complex and computationally demanding. In order to obtain a high accuracy approximate while keeping the number of expansion terms used constant, we propose using a *variance splitting* approach [22]. In this approach we split and replace the high-variance Gaussian component with a mixture of Gaussians, each with a substantially lower variance than the original. The i-th component for splitting is identified and aligned such that the multidimensional Gaussian has its longest ellipsoid along the principal axis. This is done by diagonalizing the covariance matrix

$$\mathbf{D}_i = \mathbf{\Sigma}^T \mathbf{C}_i \mathbf{\Sigma}, \tag{13}$$

where $\mathbf{D}_i$ is diagonal matrix containing the eigenvalues of $\mathbf{C}_i$, and Σ is the diagonalization matrix whose columns are the the eigenvectors of $\mathbf{C}_i$. The i-th component is then split as

$$w_i \cdot p_i(\underline{z}) = \sum_{k=1}^{M} \hat{w}_k \cdot \hat{p}_k(\underline{z}), \tag{14}$$

where

$$\begin{aligned} &\hat{p}_k(\underline{z}) \sim \mathcal{N}(\underline{z}; \hat{\underline{\mu}}_k, \hat{\mathbf{C}}_k) \quad \text{with} \quad \hat{\mathbf{C}}_k = \mathbf{\Sigma}\mathbf{D}_k\mathbf{\Sigma}^T, \\ &\hat{w}_k = \bar{\nu}_k w_i, \quad \hat{\underline{\mu}}_k = \underline{\mu}_i + \sqrt{\lambda_d} \cdot \bar{\mu}_k \cdot [0, \ldots, 1, \ldots, 0]^T, \\ &\mathbf{D}_k = \text{diag}\left(\lambda_1, \ldots, \lambda_{d-1}, \bar{\sigma}_k^2, \lambda_{d+1}, \ldots, \lambda_n\right), \end{aligned} \tag{15}$$

and d is the index of the largest eigenvalue of $\mathbf{C}_i$. The parameters $\bar{w}_k, \bar{\nu}_k$, and $\bar{\sigma}_k$ in (15) are calculated using the splitting library [23] shown in Figure 4. The figure also shows the simulation results for the entropy approximation for a sample GMM for $n = 2$ as its variance is increased, calculated using a second-order Taylor series expansion, with and without the variance split. While maintaining a high degree of accuracy, this split is not perfect and introduces a marginal amount of error due to the limited number of Gaussians in the splitting library. However, we have observed that the splitting approach is especially helpful at higher variances and, if required, can be performed repeatedly to further refine the approximation.

4. RESULTS

The MI can be trivially upper bounded as

$$I(X; \underline{Y}) = H(X) - H(X|\underline{Y}) \leq H(X) = 1 \text{ bit}, \tag{16}$$

where we have used the fact that X is drawn from an equiprobable Bernoulli distribution. Therefore, the maximum information that can be transferred over the ERP channel for the given input is 1 bit. This upper bound is based on the entropy of the input random variable and depends only on the quality of the audio stimulus. It is also independent of subjective perception, mental state of the individual, any pre-processing to the EEG data, or even the number of ROIs considered.

The output of the ERP channel maps the activity spread over the entire cortex and the total MI is therefore a contribution of all the eight ROIs. To calculate the entropy over these multiple regions we use the second-order Taylor series approximation presented in the previous section. A four-component variance split is then performed twice to further refine the approximation result. Figure 5 shows the final estimates for the MI, calculated for each trial. We observe that the MI over the ERP channel for a given trial is in general moderate-to-high. This shows that the recorded EEG data reveals a significant amount of information about the quality of the audio sequence in the trial. Also, the subset of trials with low MI suggest that the brain activity of the subject in those trials is less aligned in response to the audio quality, e.g., if the subject is either distracted or not paying attention to the presented audio sequences. The individual MI estimates for each subject are summarized in Figure 6. These results indicate the MI to be fairly consistent across the entire pool of test subjects. Further, subjects that show a low level of quality perception appear to do so over both the distortion types, indicating once again that the particular subject did not comply very well with the test procedures. Overall, the MI results are uniformly high across different subjects, trials, and distortion-types, thus demonstrating the viability of both the proposed end-to-end channel model and the MI as a suitable measure for subjective audio quality perception.

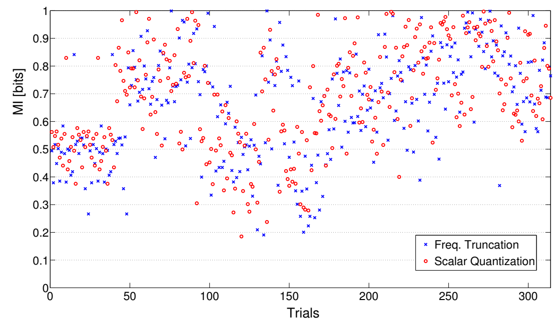

Fig. 5: Mutual information estimates for a total of 314 trials for *each* distortion-type, conducted across all 19 test subjects using different combinations of base sequences and time-varying distortion patterns. The total number of trials presented to each subject varied between 32-36.

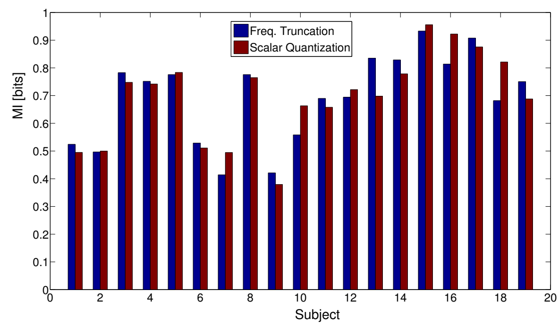

Fig. 6: The median mutual information estimates for each of the 19 test subjects when presented with the same set of trial-sequences, across the two different distortion-types. By taking the median we are able to remove any outlying values corresponding to the trials when the subject was either distracted or not paying attention to the audio sequence.

5. CONCLUSION

We have presented a novel information theoretic framework for subjective quality assessment based on observed EEG data for subjects listening to time-varying distorted audio. By modeling the end-to-end perceptual processing chain as a discrete-input time-varying nonlinear SIMO channel with memory, we aim to better understand its stochastic characteristics and in-turn create better models/metrics of how the brain processes audio stimuli and responds to changes in observed audio quality. The MI estimate computed over the ERP channel quantifies the information transmitted between the input audio stimulus and the EEG measurements, thus serving as a direct performance measure for audio quality perception. The approach presented here can be extended for assessing the subjective perception of video quality and can also be generalized to other assessment techniques like MEG, albeit with a higher complexity.

6. REFERENCES

[1] *Method for subjective assesment of intermediate quality levels of coding systems*, Question ITU-R 220/10 Recommendation ITU-R BS.1534-1, Geneva, Switzerland, 1998-2001.

[2] A. K. Porbadnigk, J. Antons, B. Blankertz, M. S. Treder, R. Schleicher, S. Moller, and G. Curio, "Using ERPs for assessing the (sub) conscious perception of noise," in *Engineering in Medicine and Biology Society (EMBC), 2010 Annual International Conference of the IEEE*, Buenos Aires, Argentina, 2010, pp. 2690–2693.

[3] C. D. Creusere, J. Kroger, S. R. Siddenki, P. Davis, and J. Hardin, "Assessment of subjective audio quality from EEG brain responses using time-space-frequency analysis," in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*, Bucharest, Hungary, 2012, pp. 2704–2708.

[4] S. Scholler, S. Bosse, M. S. Treder, B. Blankertz, G. Curio, K.-R. Muller, and T. Wiegand, "Toward a direct measure of video quality perception using EEG," *IEEE Transactions on Image Processing*, vol. 21, no. 5, pp. 2619–2629, 2012.

[5] L. Lindemann, S. Wenger, and M. Magnor, "Evaluation of video artifact perception using event-related potentials," in *Proceedings of the ACM SIGGRAPH Symposium on Applied Perception in Graphics and Visualization*, Toulouse, France, 2011, pp. 53–58.

[6] M. Mustafa, S. Guthe, and M. Magnor, "Single-trial EEG classification of artifacts in videos," *ACM Transactions on Applied Perception (TAP)*, vol. 9, no. 3, pp. 12:1–12:15, 2012.

[7] T. W. Picton, "The P300 wave of the human event-related potential," *Journal of Clinical Neurophysiology*, vol. 9, no. 4, pp. 456–479, 1992.

[8] S. Sutton, P. Tueting, and J. Zubin, "Information delivery and the sensory evoked potential," *Science*, vol. 155, pp. 1436–1439, 1967.

[9] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. John Wiley & Sons, 2012.

[10] J. J. Vidal, "Real-time detection of brain events in EEG," *Proceedings of the IEEE*, vol. 65, no. 5, pp. 633–641, 1977.

[11] J. Xu, Z. Liu, R. Liu, and Q. Yang, "Information transmission in human cerebral cortex," *Physica D: Nonlinear Phenomena*, vol. 106, pp. 363–374, 1997.

[12] J. Jeong, J. C. Gore, and B. S. Peterson, "Mutual information analysis of the EEG in patients with Alzheimer's disease," *Clinical Neurophysiology*, vol. 112, pp. 827–835, 2001.

[13] S. H. Na, S. Jina, S. Y. Kima, and B. Hamb, "EEG in schizophrenic patients: mutual information analysis," *Clinical Neurophysiology*, vol. 113, pp. 9154–1960, 2002.

[14] Z. J. Wang, P. W. Lee, and M. J. McKeown, "A novel segmentation, mutual information network framework for eeg analysis of motor tasks," *Biomedical engineering online*, vol. 8, no. 9, pp. 1–19, 2009.

[15] S. Aviyente, L. A. Brakel, R. K. Kushwaha, M. Snodgrass, H. Shevrin, and W. J. Williams, "Characterization of event related potentials using information theoretic distance measures," *IEEE Transactions on Biomedical Engineering*, vol. 51, no. 5, pp. 737–743, 2004.

[16] R. K. Kushwaha and W. J. Williams, "An information flow technique for category related evoked potentials," *IEEE Transactions on Biomedical Engineering*, vol. 39, no. 2, pp. 165–175, 1992.

[17] L. Wu, P. Neskovic, E. Reyes, E. Festa, and H. William, "Classifying n-back EEG data using entropy and mutual information features," in *European Symposium on Artificial Neural Networks*, Bruges, Belgium, 2007, pp. 61–66.

[18] A. Temko, G. Boylan, W. Marnane, and G. Lightbody, "Speech recognition features for EEG signal description in detection of neonatal seizures," in *Engineering in Medicine and Biology Society (EMBC), 2010 Annual International Conference of the IEEE*, Buenos Aires, Argentina, 2010, pp. 3281–3284.

[19] D. H. Johnson, "Information theory and neural information processing," *IEEE Transactions on Information Theory*, vol. 56, no. 2, pp. 653–666, 2010.

[20] C. D. Creusere and J. C. Hardin, "Assessing the quality of audio containing temporally varying distortions," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 19, no. 4, pp. 711–720, 2011.

[21] T. Gruber, N. J. Trujillo-Barreto, C.-M. Giabbiconi, P. A. Valdés-Sosa, and M. M. Müller, "Brain electrical tomography (BET) analysis of induced gamma band responses during a simple object recognition task," *NeuroImage*, vol. 29, no. 3, pp. 888–900, 2006.

[22] M. F. Huber, T. Bailey, H. Durrant-Whyte, and U. D. Hanebeck, "On entropy approximation for Gaussian mixture random vectors," in *IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems*, Seoul, S.Korea, 2008, pp. 181–188.

[23] U. D. Hanebeck, K. Briechle, and A. Rauh, "Progressive bayes: A new framework for nonlinear state estimation," in *Proc. SPIE 5099, Multisensor, Multisource Information Fusion: Architectures, Algorithms, and Applications*, Apr. 2003, pp. 256–267.