A New EEG-Based Causal Information Measure for Identifying Brain Connectivity in Response to Perceived Audio Quality

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***Abstract*—In this paper, electroencephalography (EEG) measurements are used to assess cortical functional connectivity in response to perceived audio quality. Specifically, in the conducted experiment the brainwave response patterns of human subjects are directly recorded using a high resolution EEG while they listen to audio whose quality varies with time. A new causal bi-directional information (CBI) measure is proposed which quantifies the information flow between EEG electrodes by appropriately grouping them into specific regions of interest (ROIs) over the cortex. It is shown that CBI can be intuitively interpreted as a causal bi-directional modification of directed information applied to a generalized cortical network setting, and inherently calculates the divergence of the observed data from a multiple access channel with feedback. The proposed measure is used to analyze and compare the information flow between ROI pairs for the case when the subject listens to high quality audio compared to when the subject listens to low quality audio. The results indicate that CBI is a more robust measure for inferring connectivity when compared to using standard directed information measures.**

***Index Terms*—Electroencephalography (EEG), directed information, causal conditioning, functional connectivity, audio quality**

I. Introduction

Functional connectivity [1], [2] refers to the quantification of statistical dependencies between neural data recorded from spatially distinct regions in the brain. Identifying connectivity is important towards understanding how neuronal populations and anatomically segregated regions in the brain interact with each other during a particular task. Information theoretic measures are fundamentally well suited for modeling neural information flow and estimating connectivity [3]. In particular, several recent studies have used measures such as Massey directed information [4] or Schreiber's transfer entropy [5] to capture the causal interactions between neural signals originating from different areas in the brain. For example, transfer entropy has been used in [6] for analyzing connectivity in neuroimaging data and in [7] to infer neural connectivity, respectively. In a similar manner, [8], [9] demonstrate the use of Massey directed information to identify connectivity in neural spike recordings. In [10], the authors propose using Massey directed information for identifying regions in the brain responsible for seizure onsets in epileptic patients. Further, [11]–[13] explore the use of an alternate definition of directed information as proposed by Kamitake [14] to identify the information flow between EEG electrodes.

Our focus here is on determining cortical connectivity in conjuction with EEG for the purposes of evaluating perceived audio quality. We are motivated here by the fact that EEG measurements directly capture brainwave response patterns that depend only on the perceived variation of the signal quality and are potentially well suited for audio-quality assessment [15]–[17]. In particular, in our previous work [17] we were able to quantify the information flow from audio stimulus to EEG sensors using block-based mutual information. In this work, we extend these considerations to assessing information flow *within* the brain using *causal* information measures. By identifying interaction in between cortical regions and inferring changes in connectivity we aim to in turn better understand how the brain perceives and responds to changes in audio quality.

To this end, we propose a novel information measure which can be considered as a causal bi-directional modification of directed information applied to a generalized cortical network setting. In particular, we show that the proposed casual bi-directional information (CBI) measure assesses the direct connectivity between any two given nodes of a multiterminal cortical network by inherently calculating the divergence between the induced conditional distributions and those for a multiple access channel (MAC) with feedback. We employ CBI to calculate the pairwise information flow between EEG sensors by appropriately grouping them into regions of interest (ROIs) over the cortex while using causal conditioning to account for the influence from all other ROIs. We also validate the performance of CBI by comparing how well it is able to distinguish between the perceived audio quality, showing that CBI is a more robust measure for inferring connectivity in contrast to state-of-the-art directional information measures.

II. Background

We conduct subjective listening trials in which the brain response activity of human subjects is recorded using a 128 electrode channel EEG system as they listen to the audio

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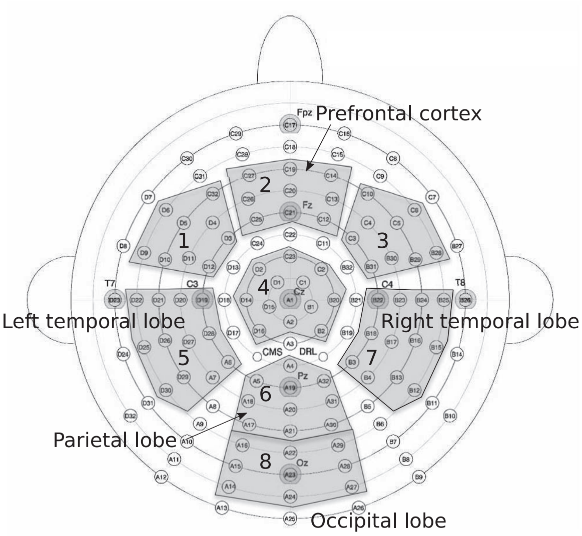

Fig. 1: The 128 electrodes are grouped into eight regions of interest (ROI) to effectively cover the different cortical regions (lobes) of the brain (adopted from [17]).

sequences whose quality varies with time. Specifically, the audio quality of each sequence is alternated in a time-varying pattern between different quality levels, with each quality being presented for a full five second duration before it changes. For the sake of simplicity and analytical tractability we restrict ourselves to only two audio quality levels — 'high' and 'low'. The low quality audio is obtained by applying a selected distortion (either frequency truncation or scalar quantization) to the original uncompressed high quality audio. Each subject is in turn presented with multiple of such trials by picking different combinations of sequences, distortion types, and time-varying patterns [18]. Also, we only select a subset of eight subjects who showed a maximum response to the changing audio quality over the event related potential (ERP) channel connecting the audio stimulus and the quantized EEG sensor outputs (see [17] for details).

To better map and visualize the information flow over the cortex we group the EEG electrodes into eight specific regions of interest with each ROI representing a distinct cortical region (lobe) as shown in Fig. 1. Every electrode in a given ROI is also considered an independent realization of the same random process. Further, in our earlier work [17] we have demonstrated that the EEG output of an ROI converges to a Gaussian distribution with zero mean. Since the EEG electrode at any given time-instant receives a superposition of responses of a large number of neurons, the distribution of a sufficiently high number of these trials taken at different time instances converges to a Gaussian distribution as a result of the Central Limit Theorem.

III. Assessing information flow between ROI

Notation. Let $X^n = [X_1, X_2, \ldots, X_n]$ denote a random vector of n discrete valued random variables with realizations $x^n = [x_1, x_2, \ldots, x_n]$, respectively, and drawn from the joint probability distribution denoted by $p(x_1, x_2, \ldots, x_n)$. Further, the expected value of a random variable is denoed by $\mathbb{E}[\cdot]$.

A. Causal conditioning

Given two random processes, the entropy of X^N conditioned on Y^N is defined as [19]

$$H(Y^N|X^N) = \sum_{n=1}^{N} H(Y_n|Y^{n-1}X^N). \tag{1}$$

In addition, the entropy of X^N *causally* conditioned on Y^N is defined as [20]

$$H(Y^N||X^N) = \sum_{n=1}^{N} H(Y_n|Y^{n-1}X^n). \tag{2}$$

The above definition differs from conditional entropy by replacing the X^N in the conditioning of (1) by X^n. Here, causality is used to refer to the conditioning on only the current and past n samples, and not on the future X_{n+1}^N samples.

Directed information as proposed by Massey [4] quantifies the information flow in the direction from the input X^N to the output Y^N of a communication channel with feedback. The feedback link allows the users to see the previous outputs of the channel and then use these to choose subsequent channel inputs. Directed information is defined as

$$\begin{aligned} DI(X^N \to Y^N) &= H(Y^N) - H(Y^N||X^N) && (3)\\ &= \sum_{n=1}^{N} I(X^n; Y_n|Y^{n-1}), && (4) \end{aligned}$$

where $I(\cdot;\cdot|\cdot)$ denotes conditional mutual information. The expression for directed information can be extended to account for the causal influence of additional random processes as follows:

Definition 1. *Causally conditioned Massey directed information is defined as the information flowing from X^N to Y^N causally conditioned on the sequence Z^{N-1} as*

$$\begin{aligned} &DI(X^N \to Y^N||Z^{N-1})\\ &\quad \triangleq H(Y^N||Z^{N-1}) - H(Y^N||X^N Z^{N-1}) && (5)\\ &\quad = \sum_{n=1}^{N} D_{KL}\Big(p(y_n|x^n y^{n-1} z^{n-1})\,||\,p(y_n|y^{n-1}z^{n-1})\Big) && (6)\\ &\quad = \sum_{n=1}^{N} I(X^n; Y_n|Y^{n-1}Z^{n-1}), && (7) \end{aligned}$$

where $D_{KL}(\cdot|\cdot)$ denotes the Kullback-Leibler (KL) divergence.

Another widely used causal measure is transfer entropy [5]. It calculates the directional information flow between two interacting random processes using conditional probabilities based on Markovian dependencies. Transfer entropy too can be extended to incorporate causal conditioning from an additional random process.

Definition 2. *Causally conditioned transfer entropy from X^n to Y^{n-1} is defined as follows:*

$$TE(X^{n-1} \to Y^n || Z^{n-1})$$
$$\triangleq D_{KL}\Big(p(y_n|y^{n-1}x^{n-1}z^{n-1}) \,||\, p(y_n|y^{n-1}z^{n-1})\Big) \quad (8)$$
$$= H(Y_n|Y^{n-1}Z^{n-1}) - H(Y_n|Y^{n-1}X^{n-1}Z^{n-1}). \quad (9)$$

Directed information and transfer entropy are closely linked, and the analytical relationship between them is examined in [21], [22]. Additionally, [23] also establishes the relation between causally conditioned transfer entropy and the two different directed information measures as proposed by Massey in (7) and by Kamitake [14], respectively.

As pointed out in the discussion so far, directional measures have been used extensively to infer statistical causal influences in complex interconnected neuronal networks. Therefore, it becomes important here to make the distinction between causal influences versus the direction of information flow. While it might be tempting to declare the direction of causal influence as the direction in which there is a higher information rate, this might not necessarily be true [5], [24]. Especially in the case of EEG each electrode measures the sum electric potential resulting from the large scale synchronous activity of several hundred million neurons averaged over tissue masses, making it difficult to make a one-to-one correspondence between directionality and causality. This argument is also supported by our experimental observation that the information transfer between most of the regions in Fig. 1 does not have a preferred direction. Instead, in the following we propose a new causal bi-directional measure to analyze functional connectivity in EEG.

B. Causal bi-directional information measure

In order to derive the CBI measure let us first consider as a preliminary result how causally conditioned directed information can be used to express the information rate over a two user MAC with feedback. Fig. 2 shows such a MAC with inputs $\mathcal{X}$ and $\mathcal{Y}$, and output $\mathcal{Z}$. Also, let the corresponding random processes associated with these nodes be X^N, Y^N, and Z^N, respectively. The capacity region for a MAC with feedback can be lower bounded using directed information, wherein the information rate[1] from $\mathcal{X}$ to $\mathcal{Z}$ is shown to be [20]

$$R_1 \leq \frac{1}{N} DI(X^N \to Z^N || Y^N), \quad (10)$$

for all

$$p(x_n y_n | x^{n-1} y^{n-1} z^{n-1})$$
$$= p(x_n|x^{n-1}z^{n-1}) \cdot p(y_n|y^{n-1}z^{n-1}), \quad n \leq N. \quad (11)$$

Now consider the scenario of a general multi-terminal network as shown in Fig. 3(a), where each node sends information and receives feedback from every other node in the network.

[1]The other rates R_2 from $\mathcal{Y}$ to $\mathcal{Z}$ and the sum rate $R_1 + R_2$ are not of interest in the following.

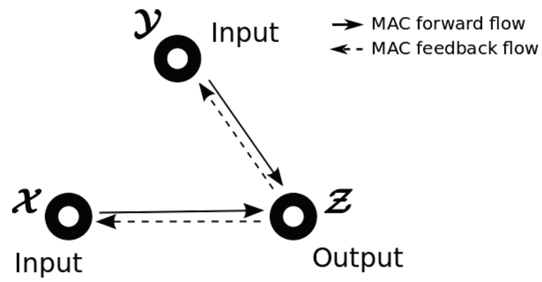

Fig. 2: A multiple access channel with feedback.

In our case, the information transfer over the cortex can be considered equivalent to such a cortical multi-terminal network with each ROI taking over the role as a communicating node, and X^N, Y^N and Z^N corresponding to the sampled EEG signals from different ROI. Also without any loss of generality, Z^N represents the output of multiple (and potentially all other) ROIs. We define a new casual bi-directional information measure to assess the causal dependency between two nodes while accounting for the feedback from all other nodes as follows:

Definition 3. *Causal bi-directional information (CBI) is defined as the sum of the causally conditioned Massey directed information between X^N to Y^N, and the sum transfer entropy in the reverse direction:*

$$I(X^N \leftrightharpoons Y^N || Z^{N-1})$$
$$\triangleq DI(X^N \to Y^N || Z^{N-1}) + \sum_{n=1}^{N} \Big\{ TE(Y^{n-1} \to X^n || Z^{n-1}) \Big\}. \quad (12)$$

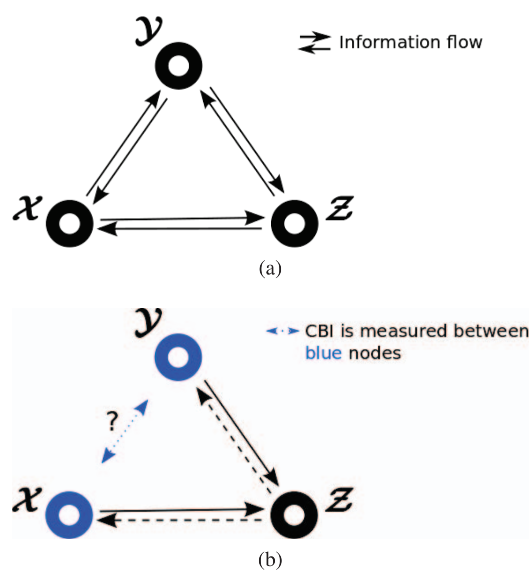

Fig. 3: (a) A general multi-terminal network whose connectivity we are interested in. All three communicating nodes send and receive information from each other. (b) CBI infers the connectivity between $\mathcal{X}$ and $\mathcal{Y}$ by calculating the divergence of the observed joint distribution on the network of Fig. 3(a) from the one of a MAC with feedback in (11).

CBI considers the three node network of Fig. 3(a) and measures the information flow in between nodes $\mathcal{X}$ and $\mathcal{Y}$ by using the MAC as a reference, as shown in Fig. 3(b). In particular, we point our attention towards the relation in (11) specifying the relation between the joint distribution of the two inputs of the MAC. The conditional independence of the inputs in the factorization of (11) arises due to the causal nature of the feedback structure, where the output at the receiver Z^{n-1} is available causally at both $\mathcal{X}$ and $\mathcal{Y}$ for each time n. For the case of a MAC with feedback there is no direct connectivity (path) between the inputs, and all information flows only via $\mathcal{Z}$, i.e., the connectivity can be expressed as $\mathcal{X} \leftrightharpoons \mathcal{Z} \leftrightharpoons \mathcal{Y}$. Any violation of (11) creates dependencies between $\mathcal{X}$ and $\mathcal{Y}$, and these dependencies can be measured by the KL divergence between the joint distribution on the l.h.s. of (11) and the factorization on the r.h.s. of (11). This result is summarized in the following proposition:

Proposition 1. *CBI calculates the KL divergence between the induced distributions of the observed channel and a multiple access channel with feedback:*

$$I(X^N \leftrightharpoons Y^N || Z^{N-1}) = D_{KL}\Big(p(x_n y_n | x^{n-1} y^{n-1} z^{n-1}) \,||\, p(x_n | x^{n-1} z^{n-1}) p(y_n | y^{n-1} z^{n-1})\Big) \tag{13}$$

$$= H(X^N || Z^{N-1}) + H(Y^N || Z^{N-1}) - H(X^N Y^N || Z^{N-1}). \tag{14}$$

Proof. We have

$$\begin{aligned} & I(X^N \leftrightharpoons Y^N || Z^{N-1}) \\ &= DI(X^N \rightarrow Y^N || Z^{N-1}) + \sum_{n=1}^{N} \Big\{ TE(Y^{n-1} \rightarrow X^n || Z^{n-1}) \Big\} \\ &= \sum_{n=1}^{N} \Big\{ I(Y_n; X^n | Y^{n-1} Z^{n-1}) + TE(Y^{n-1} \rightarrow X^n || Z^{n-1}) \Big\}, \end{aligned} \tag{15}$$

where the equality in (15) follows from (7) in Definition 1. Denoting the term inside the summation of (15) by $\Phi(\cdot)$ and rewriting yields

$$\begin{aligned} & \Phi(x^n, y^n, z^{n-1}) \\ &= I(Y_n; X^n | Y^{n-1} Z^{n-1}) + TE(Y^{n-1} \rightarrow X^n || Z^{n-1}) && (16) \\ &= \mathbb{E}\left[\log \frac{p(Y_n | X^n Y^{n-1} Z^{n-1})}{p(Y_n | Y^{n-1} Z^{n-1})}\right] \\ &\quad + \mathbb{E}\left[\log \frac{p(X_n | Y^{n-1} X^{n-1} Z^{n-1})}{p(X_n | X^{n-1} Z^{n-1})}\right] && (17) \\ &= \mathbb{E}\left[\log \frac{p(Y_n X_n | Y^{n-1} X^{n-1} Z^{n-1})}{p(Y_n | Y^{n-1} Z^{n-1}) p(X_n | X^{n-1} Z^{n-1})}\right], && (18) \end{aligned}$$

where in (17) we have made use of (6) and (8) respectively, and (18) follows from the chain rule of joint probability

$$\begin{aligned} & p(y_n x_n | x^{n-1} y^{n-1} z^{n-1}) \\ &= p(y_n | x_n x^{n-1} y^{n-1} z^{n-1}) \cdot p(x_n | x^{n-1} y^{n-1} z^{n-1}). \end{aligned} \tag{19}$$

Replacing (18) back into the summation in (15) provides the claim. □

It follows directly from Proposition 1 (13) that CBI ascertains the direct connectivity between two nodes in a general multi-node network and is zero only if: (i) X^n and Y^n are independent for all $n \leq N$, i.e., there is no information flow between the two nodes, (ii) there is no direct link between $\mathcal{X}$ and $\mathcal{Y}$ and all information flows only via an additional node $\mathcal{Z}$.

We now show that CBI is a symmetric bidirectional measure.

Corollary 2. *CBI is a symmetric measure, i.e.,*

$$\begin{aligned} & I(X^N \leftrightharpoons Y^N || Z^{N-1}) \\ &= DI(X^N \rightarrow Y^N | Z^{N-1}) + \sum_{n=1}^{N} \Big\{ TE(Y^{n-1} \rightarrow X^n | Z^{n-1}) \Big\} && (20) \\ &= DI(Y^N \rightarrow X^N | Z^{N-1}) + \sum_{n=1}^{N} \Big\{ TE(X^{n-1} \rightarrow Y^n | Z^{n-1}) \Big\}. && (21) \end{aligned}$$

Proof. Without any loss of generality, the joint probability distribution in (19) can alternatively be expanded as

$$\begin{aligned} & p(y_n x_n | y^{n-1} x^{n-1} z^{n-1}) \\ &= p(x_n | y_n y^{n-1} x^{n-1} z^{n-1}) \cdot p(y_n | y^{n-1} x^{n-1} z^{n-1}). \end{aligned} \tag{22}$$

Using the expansion (22) in (18) yields

$$\begin{aligned} & \Phi(x^n, y^n, z^{n-1}) \\ &= \mathbb{E}\left[\log \frac{p(Y_n X_n | Y^{n-1} X^{n-1} Z^{n-1})}{p(Y_n | Y^{n-1} Z^{n-1}) p(X_n | X^{n-1} Z^{n-1})}\right] && (23) \\ &= \mathbb{E}\left[\log \frac{p(X_n | Y^n X^{n-1} Z^{n-1})}{p(X_n | X^{n-1} Z^{n-1})}\right] \\ &\quad + \mathbb{E}\left[\log \frac{p(Y_n | Y^{n-1} X^{n-1} Z^{n-1})}{p(Y_n | Y^{n-1} Z^{n-1})}\right] && (24) \\ &= I(X_n; Y^n | X^{n-1} Z^{n-1}) + TE(X^{n-1} \rightarrow Y^n || Z^{n-1}). && (25) \end{aligned}$$

Finally, taking summations on both sides

$$\begin{aligned} & I(X^N \leftrightharpoons Y^N || Z^{N-1}) \\ &= \sum_{n=1}^{N} \Big\{ I(X_n; Y^n | X^{n-1} Z^{n-1}) + TE(X^{n-1} \rightarrow Y^n || Z^{n-1}) \Big\} \\ &= DI(Y^N \rightarrow X^N | Z^{N-1}) + \sum_{n=1}^{N} \Big\{ TE(X^{n-1} \rightarrow Y^n | Z^{n-1}) \Big\} \end{aligned} \tag{26}$$

yields the required result. □

Note that even though CBI is symmetric, it is still a causal measure on account of the fact that the conditioning is restricted to only the past sample values of the involved random processes.

Finally, we compare CBI to conditional mutual information. Conditional mutual information measures the divergence between the distributions induced by the actual observations from the ones induced by the Markovian chain $X^N \leftrightarrow Z^{N-1} \leftrightarrow Y^N$ and is defined as [19]

$$I(X^N; Y^N | Z^{N-1})$$
$$= \mathbb{E}\left[\log \frac{p(X^N Y^N | Z^{N-1})}{p(X^N | Z^{N-1})\, p(Y^N | Z^{N-1})}\right] \tag{27}$$
$$= \sum_{n=1}^{N} \mathbb{E}\left[\log \frac{p(X_n Y_n | X^{n-1} Y^{n-1} Z^{N-1})}{p(X_n | X^{n-1} Z^{N-1})\, p(Y_n | Y^{n-1} Z^{N-1})}\right], \tag{28}$$

where (28) follows from the chain rule of probability. Comparing conditional mutual information (28) with the expression for CBI in (13), we notice that CBI uses causal conditioning and replaces Z^{N-1} with Z^{n-1}.

IV. Results

We apply the proposed CBI measure to the experimental EEG data collected during the audio trials. We select a source and destination ROI $\mathcal{X}$ and $\mathcal{Y}$, respectively, while representing all other ROIs as side information $\mathcal{Z}$. Since our focus here is on audio perception, we select the regions which are actively involved with audio processing in the brain. In particular, based on Fig. 1 we select regions on the direction and order of the auditory sensing pathway in the brain. The primary auditory cortex (PAC) located in the left (ROI 5) and right (ROI 7) temporal lobes is the first region of the cerebral cortex to receive auditory input. The higher executive functions and subjective responses are a result of the information exchange between the PAC and the other cortical regions, predominantly including the prefrontal cortex (ROI 2).

Knowing that the interacting random processes under consideration are Gaussian allows us to formulate an analytical closed form expression for calculating CBI. The joint entropy of a n-dimensional multivariate Gaussian distribution with probability density $p(z_1 \dots z_n)$ is given by [19]

$$H(Z_1 \dots Z_n) = \frac{1}{2} \log (2\pi e)^n |\mathsf{C}(Z_1 \dots Z_n)|, \tag{29}$$

where $\mathsf{C}(\cdot)$ is the covariance matrix and $|\cdot|$ is the determinant of a matrix. Using (29) in (13) CBI can be reduced to

$$I(X^N \leftrightharpoons Y^N || Z^{N-1}) = \frac{1}{2} \sum_{n=1}^{N} \log \left\{ \frac{|\mathsf{C}(X^{n-1} Y^{n-1} Z^{n-1})|}{|\mathsf{C}(X^n Y^n Z^{n-1})|} \cdot \frac{|\mathsf{C}(X^n Z^{n-1})|}{|\mathsf{C}(X^{n-1} Z^{n-1})|} \cdot \frac{|\mathsf{C}(Y^n Z^{n-1})|}{|\mathsf{C}(Y^{n-1} Z^{n-1})|} \right\}. \tag{30}$$

The corresponding covariance matrices are calculated by extracting samples using 125 milliseconds long overlapping sliding windows ($N = 32$ samples at a sampling rate of 256 Hz). We include the data from all EEG trials for all (eight) subjects across different test sequences and distortion types.

In our analysis, we also separately extract the EEG response data for each of the two audio quality levels and calculate the information measures individually for each of them. This allows us to compare the differences in the information flow between the ROIs for the case where the subjects listen to good quality audio as opposed to the case where the subjects listen to bad quality audio. Fig. 4 illustrates the instantaneous CBI, shown in red and blue color, calculated for four different ROI pairs for a second of the EEG trial data. The vertical axis is the estimated instantaneous information (bits), each calculated in accordance with (30) over a 125 ms long sliding time window. The 0 ms marker on the horizontal axis is the stimulus onset time, i.e., the time instant when the audio quality changed. The results indicate a significant difference between the amount of information flow for high and degraded audio. In particular, there appears to be a higher amount of information flow between the regions when the subject was listening to degraded quality audio. This strongly indicates an increase in brain activity, possibly as a result of paying increased attention to identify the drop in audio quality. For comparison, Fig. 4 also includes causally conditioned directed information rates, shown in black and green color, calculated in accordance to (7) for the same set of trial data. We observe from Fig. 4 that CBI is able to better distinguish between the audio qualities compared to directed information.

In order to compare the performance of CBI with typically used directed information measures we calculate the mean square error (MSE) between the high and low information transfer rates for different ROI pairs as listed in Table 1. MSE measures the average deviation between two data sets and is defined as $MSE = (1/n) \sum_{i=1}^{n} \{P_i - Q_i\}^2$, where P_i and Q_i correspond to the ith element of length n discrete

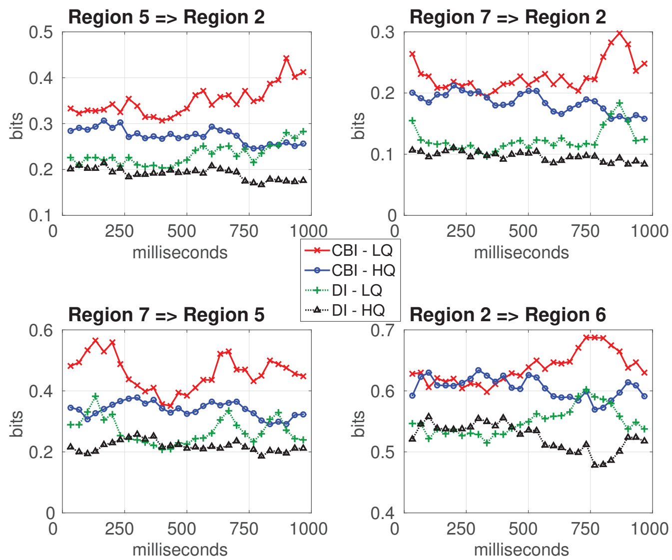

Fig. 4: The instataneous information transfer rates for the proposed CBI measure calculated between four different ROI pairs, for a second of the trial data after stimulus onset. Also included for comparison purposes are the causally conditioned Massey directed information rates calculated using (7) for the same set of trial data.

TABLE I: Mean square error (MSE) of the information rate between high quality and low quality audio. The information transfer rates used for CBI and causally conditioned Massey directed information are the same as in Fig. 4, while causally conditioned Kamitake [14] directed information rates are calculated using the results from [23].

Source ROI	Destination ROI	Massey	Kamitake	CBI
5	2	0.0025	0.0014	0.0077
7	2	0.0012	0.0009	0.0032
5	7	0.0048	0.0064	0.0187
7	5	0.0053	0.0070	0.0187
2	6	0.0025	0.0008	0.0026
6	2	0.0020	0.0001	0.0026
6	8	0.0029	0.0003	0.0024
8	6	0.0030	0.0001	0.0024

valued vectors. In our case, a higher MSE indicates a more pronounced difference between the information rates of the two audio qualities.

Table 1 shows that the MSE for the temporal (ROI 5 and 7) and frontal (ROI 2) regions are in general much higher when using the CBI measure, compared to employing both Massey and Kamitake directed information. As CBI is a bi-directional symmetric measure the information rates and thereby the MSE in between a given set of ROI pairs is equal. Also, since the parietal (ROI 6) and occipital (ROI 8) regions are not directly active during auditory processing, we expect a lower MSE between the obtained transfer rates for high and low audio qualities, respectively. This is verified by the results in Table 1.

V. Conclusion

A new causal information measure was presented to assess the connectivity in EEG measurements as subjects listen to time varying audio quality. The proposed measure is shown to be a causal bi-directional modification of directed information applied to a generalized cortical network setting, which inherently calculates the divergence of the induced distributions from a MAC with feedback. The inferred connectivity results obtained using CBI demonstrate that typically a change in the information flow between different brain regions occurs as the subject listens to different audio qualities, with the information rate being markedly higher for low quality audio. Compared to other directed information measures CBI performs significantly better in being able to distinguish between the audio qualities. In future work, the differences in the inferred connectivity could be used to build an inference model for blind audio quality classification using EEG.

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