Coding Schemes for an Erasure Relay Channel

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***Abstract*— This paper considers a simple network consisting of a source, a destination, and a relay. In this model, the source-relay and relay-destination links are lossless, while the source-destination link is subject to erasures. Four coding schemes for reliably conveying k symbols from the source to the destination are described. Three of these techniques are adapted directly from well-known point-to-point coding schemes - viz., the use of maximum-distance separable (MDS) codes and Luby Transform (LT) codes. The fourth approach is a new technique using uncoded transmission from the source in conjunction with a relay that transmits a sequence with this property: When the destination subtracts the effects of the unerased symbols from the sequence, what remains is an "LT-like" code for the erased symbols - and this property holds regardless of which symbols were erased on the source-destination link. The four approaches are compared in terms of their complexity and performance.**

I. Introduction

Wireless relay networks modelled as erasure relay channels have been well studied in the literature. For example, in [1] and [2], the capacities of certain relay channels are derived, and practical coding schemes based on maximum-distance separable (MDS) codes are given. In [3] a max-flow min-cut capacity result is obtained for a particular class of interference-free wireless erasure networks assuming the decoder has perfect knowledge about the erasure pattern.

This paper considers a three-node relay network comprised of a source s, a destination d and a relay r. (See Figure 1.) The source-relay and source-destination links together constitute a (physically degraded) *broadcast* channel, while the relay-destination link is a point-to-point channel. We assume time-slotted packetized transmissions (of fixed duration/size) at both the source and relay. Moreover, the source and relay transmissions are assumed to occur in different time slots – i.e., a time-division multiplexing (TDM) strategy is used to control interference between the source and relay transmissions. Finally, the source-relay and relay-destination links are assumed to be *lossless*, while the source-destination channel is modelled as an erasure channel in which each symbol (or packet) is independently erased with probability ε. In the presence of physical-layer error-correction, this is a good higher-layer model for the case when the source-relay and relay-destination distances are much smaller than the source-destination distance.

The goal is to communicate k information symbols from the source to the destination in no more than $k \cdot (1+\Delta)$ time slots, where Δ is close to (but necessarily larger than) ϵ. The feasibility of this task is established in Section II. Then, in Section III, we discuss two possible approaches – one based on MDS codes, and the other based on LT codes [4]. The LT-based scheme requires encoding at the source, whereas the MDS-based scheme does not – i.e., the MDS scheme is *systematic*. However, the complexity of the MDS scheme suffers in comparison to that of the LT-based approach.

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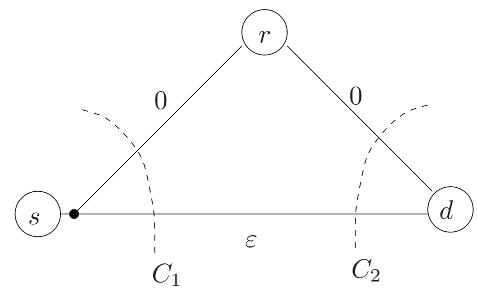

Fig. 1. A three-node erasure relay network

Motivated by this observation, we propose two systematic schemes based on LT codes. The first approach adapts *systematic LT codes* as developed by Shokrollahi [5]. The second technique encodes the symbols at the relay in such a way that the effect of the unerased symbols can be subtracted off at the destination – and what remains resembles an LT sequence for the erased symbols, regardless of which symbols were erased. These new schemes are analyzed in Sections III-C and III-D, respectively.

II. A cut-set bound

The network in Fig. 1 is a particular kind of *degraded erasure relay* channel – one in which the destination receives only the source symbols that are also received by the relay. The capacity of the general degraded relay channel was derived in [6]. This section develops an achievable *cut-set bound* ([7]) for the channel under consideration. Similar bounds for more general erasure relay channels are in [1], [2].

Let θ be the fraction of time for which the source is transmitting. Then, across the cut C_1 (*broadcast* cut) in Figure 1, the maximum rate of information transfer is bounded by θ symbols/time slot. This follows from the fact that every symbol received by the destination is also received by the relay, and the capacity of the ideal source-relay channel, when active, is just one symbol/time slot. Similarly, across the cut C_2 (*multiple-access* cut), the maximum rate of information transfer is bounded by $(1-\theta)+\theta(1-\varepsilon) = 1-\theta\varepsilon$. Consequently, the maximum achievable end-to-end rate R is bounded as

$$R \leq \max_{\theta}\{\min(\theta, 1-\theta\varepsilon)\} \quad (1)$$

$$= \frac{1}{1+\varepsilon} \text{ symbols / time-slot} \quad (2)$$

The maximizing value of θ occurs when the two terms inside $\min(\cdot,\cdot)$ are equal and is hence given by $\theta^* = 1/(1+\varepsilon)$. This bound is achievable, and a coding scheme demonstrating this is discussed in the next section.

Note that, without the relay, the capacity is $1-\varepsilon < \frac{1}{1+\varepsilon}$. Further, a capacity of $\frac{1}{1+\varepsilon}$ implies that at least $k(1+\varepsilon)$ time slots are needed to convey k symbols of information, and so it is imperative that the value of Δ for any scheme satisfies $\Delta \geq \varepsilon$. Finally, $\Delta \geq 1$ allows routing of all the information through the relay, thus trivially solving the problem. Hence, the range of interest for Δ is $\varepsilon \leq \Delta < 1$.

III. Four Coding Schemes

Throughout this section, we assume that if n symbols are transmitted by the source, then exactly $n(1-\varepsilon)$ of those symbols are successfully received at the destination. Of course, this is not accurate; rather, the law of large numbers can be invoked to guarantee that the fraction of symbols successfully received is *arbitrarily close* to $(1-\varepsilon)$. However, assuming that the fraction is exactly $(1-\varepsilon)$ does not substantively change the coding techniques described, and it simplifies notation.

Since the source-relay and relay-destination links are lossless, the optimal TDM strategy is for the source to first complete its entire transmission and thereby convey all of its information to the relay, before the relay begins transmitting to the sink.

Finally, we refer to a scheme in which the source transmits *uncoded* information symbols as a *systematic* scheme.

A. MDS-based scheme

We first consider a simple coding scheme that demonstrates the achievability of the cut-set bound. Assume that the symbols to be transmitted are drawn from a finite field. Then the scheme is as follows:

1) The source first transmits k *uncoded* information symbols. The relay successfully receives all the symbols, whereas the destination receives $k(1-\varepsilon)$ of them.
2) The relay now encodes the k information symbols into $k\varepsilon$ code symbols such that any set of $k\varepsilon$ information symbols erased on the source-destination link can be recovered from these code symbols. In other words, the $k \times k\varepsilon$ *generator matrix* $\mathbf{G}$ used by the relay must have the property that every set of $k\varepsilon$ *rows* are linearly independent. This implies that $\mathbf{G^T}$ must be the generator matrix of a $(k, k\varepsilon)$ *maximum-distance separable* (MDS) code - e.g., a Reed-Solomon code.

The total number of transmissions in this scheme is $k(1+\varepsilon)$, exactly what is indicated by the cut-set bound. Since this scheme does not involve any encoding at the source, it is a *systematic* scheme. Consequently, all this scheme's computational complexity is localized to the relay and the destination.

To estimate complexity, we assume standard Reed-Solomon encoder/decoder implementations. This means that encoding requires $O(k^2\varepsilon)$ symbol operations; this is also the total complexity at the relay. For decoding at the destination, the contribution of the $k(1-\varepsilon)$ known symbols is first subtracted from the $k\varepsilon$ symbols sent by the relay – this requires $O(k^2\varepsilon(1-\varepsilon))$ operations; then, we have $k\varepsilon$ linear equations in $k\varepsilon$ unknowns and these can be solved with quadratic complexity $O(k^2\varepsilon^2)$ (as these equations involve a Vandermonde matrix)[1]. Thus, the total complexity at the destination is $O(k^2\varepsilon)$.

Finally, note that the overall code describing symbols sent from both the source and the relay – the code with generator $[I_{k\times k}|\mathbf{G}]$ – is *not necessarily* MDS, i.e., it *need not* have the property that every $k \times k$ sub-matrix is invertible. So this scheme explicitly exploits the assumption that all erasures will occur on the source-to-destination link.

B. LT-based scheme

We now consider a scheme that is *non-systematic*, i.e., in which the source transmits encoded symbols.

The codes we consider here are Luby Transform (LT) codes [4]. LT codes have the property that slightly more than k code symbols are needed to recover k information symbols – unlike MDS codes which require *exactly* k code symbols. The symbols are binary strings and symbol operations consist of bit-wise XORs. The i^{th} code symbol is generated by XORing d_i randomly chosen information symbols. The integer d_i, called the *degree* of the code symbol, is itself chosen randomly according to a *degree distribution* – the choice of which is crucial to the performance of the code. The degree distribution $\mu(i), 1 \leq i \leq k$, used in [4] is the *robust soliton distribution* (RSD), defined as follows:

$$\mu(i) = \frac{\rho(i)+\tau(i)}{\beta}, \quad \text{for} \quad 1 \leq i \leq k, \quad (3)$$

with:

$$\beta = \sum_{i=1}^{k}\Big(\rho(i)+\tau(i)\Big), \quad (4)$$

$$\rho(i) = \begin{cases} 1/k, & \text{for} \quad i = 1, \\ 1/(i(i-1)), & \text{for} \quad 2 \leq i \leq k, \end{cases} \quad (5)$$

$$\tau(i) = \begin{cases} S/ik, & \text{for} \quad 1 \leq i \leq \frac{k}{S}-1, \\ S\log(S/\delta)/k, & \text{for} \quad i = \frac{k}{S}, \\ 0, & \text{otherwise}, \end{cases} \quad (6)$$

$$S = c\sqrt{k}\log\left(\frac{k}{\delta}\right), \quad (7)$$

for constants $c > 0$ and $\delta \in [0,1]$. It is shown in [4] that $k + O(\sqrt{k}\cdot\log^2(k/\delta))$ LT code symbols suffice to recover k information symbols with probability at least $1-\delta$. Furthermore, the average encoding complexity per code symbol is $O(\log(k/\delta))$, and the average complexity for decoding *all* the information is $O(k\cdot\log(k/\delta))$.

[1] There are asymptotically faster algorithms for this class of MDS codes; however, in [4] and [5] it is observed that these are often in practice more complicated to implement than the quadratic complexity schemes.

We now describe a coding scheme for our relay network based on LT codes:

1) The source transmits $n_s = k + c\sqrt{k}\log^2(k/\delta)$ LT code symbols constructed from the k information symbols; of these the destination receives $n_s(1-\varepsilon)$, while the relay receives all n_s symbols.
2) From the received symbols, the relay decodes the information and re-encodes it into $n_r = n_s\varepsilon$ LT code symbols, which are transmitted.
3) The destination decodes the information from the n_s LT code symbols received from both the source and relay.

The relay fails to decode with probability at most δ; conditioned on successful decoding at the relay, the destination fails with probability at most δ. Thus, the overall failure probability is at most $2\delta - \delta^2$. Further, the average complexity at the source is $O(n_s \log(k/\delta)) = O(k\log(k/\delta))$. The average complexity at the relay (decoding + encoding) is given by $O((1+\varepsilon)\cdot k\log(k/\delta))$. Similarly, the average complexity at the destination is $O(k\log(k/\delta))$.

Another important aspect of this coding scheme is the *overhead* – defined as the number of transmissions required in *excess* of the theoretical minimum ($k(1+\varepsilon)$) indicated by the cut-set bound. It can be verified that the overhead of this scheme is given by $c(1+\varepsilon)\sqrt{k}\log^2(k/\delta)$. However, since the overhead is of the form $o(k)$, for large k, the *normalized* overhead (= overhead$/k$) can be made smaller than its maximum permitted value $\Delta - \varepsilon$.

Despite their overhead and non-zero failure probability, LT codes may be preferred over MDS codes owing to their ease of implementation, especially since they operate over the binary field while the field size needs to scale as the blocklength for Reed-Solomon codes. Also, for a given blocklength, the complexity of encoding and decoding LT codes is significantly lower than the standard quadratic complexity of MDS codes.

To illustrate the performance of the LT-based scheme, Figure 2 shows the total number of source/relay transmissions required for successful delivery of k information symbols. Specifically, these results indicate that, to convey $k = 2000$ information symbols with a source-to-destination erasure probability of $\epsilon = 0.25$, an average (mean) of about 2750 transmissions are required – which results in an average overhead of about 250 symbols.

A naive systematic scheme based on LT codes could be implemented by transmitting uncoded symbols from the source and then constructing the relay's sequence as before - i.e., by taking randomly chosen linear combinations of the data, with the degree of each relay-transmitted symbol selected according to the RSD. However, this approach performs quite poorly. For the parameters considered above ($k = 2000$ and $\epsilon = 0.25$), about 3280 transmitted symbols are required (on average) to recover all the source symbols, much worse than the 2750 required with non-systematic LT encoding. This motivates the use of more sophisticated systematic schemes based on LT codes, provided in the next two sections.

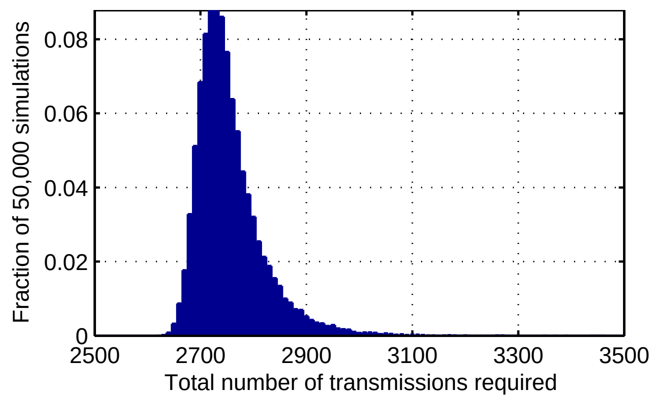

Fig. 2. Histogram of the number of source and relay LT symbols required to decode. Parameters: $k = 2000$, $\varepsilon = 0.25$.

C. Systematic scheme based on LT codes and pre-processing at the relay

We now describe a scheme based on a less naive approach to systematic LT coding – one proposed by Shokrollahi in [5]. The scheme is as follows:

1) The source transmits k uncoded symbols. Denote these symbols by the $1\times k$ vector $\mathbf{u}$.
2) The relay performs a decode-and-reencode operation on the received vector $\mathbf{u}$ as follows:
   - An invertible $k\times k$ binary matrix $\mathbf{G}$ is precomputed at the relay: First, $k + o(k)$ vectors (each of length k) are generated by sampling the RSD $k + o(k)$ times and then randomly selecting from the k-tuples of appropriate weight. With high probability, these vectors contain a linearly independent set of size k, which may be identified using the belief-propagation algorithm. The identified vectors are the columns of $\mathbf{G}$.
   - Given the k symbols from the source, the relay generates a $1\times k$ vector $\mathbf{v}$ such that $\mathbf{u} = \mathbf{v}\cdot\mathbf{G}$. This can be done without inverting $\mathbf{G}$ by merely using the BP algorithm to solve for $\mathbf{v}$ from $\mathbf{u}$, as the columns of $\mathbf{G}$ can be permuted to obtain an upper-triangular matrix [5].
   - The result is that the vector $\mathbf{u}$ "looks like" an LT code formed from $\mathbf{v}$. The relay now generates and transmits an additional $k\varepsilon + o(k)$ LT code symbols derived from $\mathbf{v}$.
3) The sink receives a total of $k + o(k)$ code symbols from which it attempts to decode $\mathbf{v}$. From $\mathbf{v}$, the sink can recover the $k\varepsilon$ erased symbols of $\mathbf{u}$, using the relation $\mathbf{u} = \mathbf{v}\cdot\mathbf{G}$.

In practice, the performance of this code can be seen in Figure 3, a histogram of the number of relay symbols required for successful decoding when $k = 2000$ and $\epsilon = 0.25$. Figure 3 indicates than an average of 650 relay symbols are required under these circumstances. This translates to an average overhead of about 150 symbols.

In terms of complexity, this scheme does not involve any computations at the source. At the relay, the complexity lies

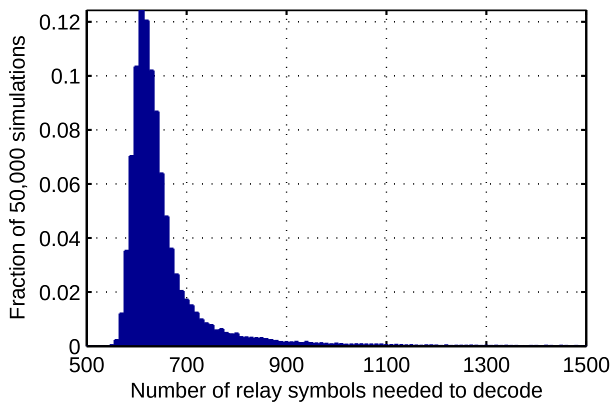

Fig. 3. Histogram of the number of relay symbols required for the scheme based on systematic LT encoding [5]; parameters: $k = 2000$, $\varepsilon = 0.25$.

in generating $\mathbf{v}$ (given $\mathbf{G}$) and $k\varepsilon$ LT symbols – and is $O(k(1+\varepsilon)\log(k))$. Likewise, the complexity at the destination is $O(k(1+\varepsilon)\log(k))$. (The dependence on δ is omitted.) The computation of $\mathbf{G}$ is a one-time operation and is hence not considered here.

D. A new systematic scheme

This section describes a novel approach to the problem – a systematic scheme in which the relay transmits an "LT-like" sequence that the destination uses to recover the erased symbols. This approach uses a different (non-RSD) distribution to generate the code symbol degrees at the relay, but it does so in such a way that the "target" distribution – the distribution of the code symbol degrees over the *erased* symbols – has properties similar to the RSD.

Assume that the source transmits k uncoded symbols and the relay transmits $m = k\Delta$ code symbols. Ideally, the m code symbols would constitute an LT sequence for *exactly* the $k\varepsilon$ symbols that were erased on the source-destination link, so the destination could recover the lost symbols with high probability for m only slightly larger than $k\varepsilon$. However, this is impossible, as the relay does not know what symbols were erased on the source-destination channel.

An alternative is to generate m coded symbols at the relay in such a way that *after subtracting* the contribution of the known symbols at the destination, the m code symbols constitute an LT code over the $k\varepsilon$ unknown symbols. Assume the relay follows the LT encoding procedure but uses a different degree distribution $p(i), 1 \leq i \leq k$. Then, ideally, we would want the corresponding degree distribution viewed over any set of $k\varepsilon$ information symbols to be the RSD.

Without loss of generality, assume that the first $\ell = k\varepsilon$ symbols transmitted on the source-destination channel are erased. Now consider a code symbol of degree i generated by the relay, with i chosen according to $p(i)$. Let the *reduced degree* j be the number of symbols in the first ℓ positions that contribute to i. Letting I and J denote the corresponding random variables, we have:

$$Pr(J = j | I = i) = \frac{\binom{\ell}{j}\binom{k-\ell}{i-j}}{\binom{k}{i}}, \tag{8}$$

for $0 \leq j \leq \ell,\ j \leq i \leq (k - \ell + j)$. Then, the *reduced degree distribution* is given by:

$$\begin{aligned} p^r(j) &= Pr(J = j) && (9)\\ &= \sum_{i=1}^{k} Pr(J = j | I = i) \cdot p(i). && (10) \end{aligned}$$

So $p^r(\cdot)$ is the distribution on the number of information symbols (among the first ℓ such symbols) that contribute to a particular code symbol. Ideally, we would like to choose $p(i)$ such that $p^r(j)$ is the RSD $\mu(j)$ for $1 \leq j \leq l$. However, this is clearly impossible since $p^r(0) \neq 0$. Instead we consider the class of distributions $p(i)$ such that, when $p^r(\cdot)$ is computed using (10), it satisfies

$$p^r(j) \geq \frac{1}{\alpha} \cdot \mu(j), \quad 1 \leq j \leq \ell, \tag{11}$$

for some $\alpha > 1$. If (11) is satisfied, then, by following Chernoff bound-type arguments, with high probability the number of symbols required to decode the information is at most α times the number required if the RSD had been used to select the degrees. The main challenge is to obtain a value of α that is as close to one as possible to minimize overhead.

For this purpose, we focus on a choice of $p(i)$ with support over only those degrees of the form $i = j/\varepsilon$ for $1 \leq j \leq \ell$. Using Stirling's approximation $n! \approx n^n e^{-n}\sqrt{2\pi n}$ in (8), then

$$Pr\Big(J = j | I = \frac{j}{\varepsilon}\Big) \geq \frac{1}{\sqrt{2\pi j}}. \tag{12}$$

Hence:

$$\begin{aligned} p^r(j) &> Pr\Big(J = j | I = \frac{j}{\varepsilon}\Big) \cdot p\Big(\frac{j}{\varepsilon}\Big) && (13)\\ &> \frac{1}{\sqrt{2\pi j}} \cdot p\Big(\frac{j}{\varepsilon}\Big). && (14) \end{aligned}$$

Note from (3)-(6) that the RSD $\mu(j)$ is of the form:

$$\mu(j) = \begin{cases} \frac{1}{\beta_\ell}\Big(\frac{1}{j(j-1)} + \frac{1}{s_\ell j}\Big), & 2 \leq j < s_\ell, \\ \frac{1}{\beta_\ell} \cdot \frac{1}{j(j-1)}, & s_\ell < j \leq \ell, \end{cases} \tag{15}$$

where β_ℓ and s_ℓ are constants that depend on ℓ. Specifically the degree s_ℓ is given by

$$s_\ell = \frac{\sqrt{\ell}}{c\log(\ell/\delta)}, \tag{16}$$

and $\beta_\ell > 1$.

The degrees 1 and s_ℓ have been omitted in (15) and also in the following development; they are dealt with separately. Consequently, if we choose $p(i)$ to be of the form

$$p(i) = \begin{cases} \frac{1}{\alpha_\ell\beta_\ell}\Big(\frac{1}{\sqrt{j}(j-1)} + \frac{1}{s_\ell\sqrt{j}}\Big), & i = \frac{j}{\varepsilon},\ 2 \leq j < s_\ell, \\ \frac{1}{\alpha_\ell\beta_\ell} \cdot \frac{1}{\sqrt{j}(j-1)}, & i = \frac{j}{\varepsilon},\ s_\ell < j \leq \ell, \\ 0, & \text{otherwise}. \end{cases} \tag{17}$$

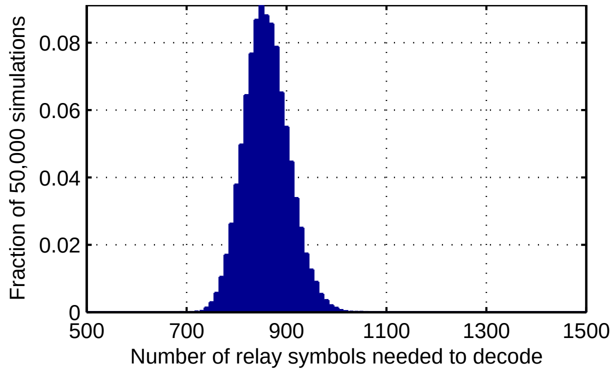

Fig. 4. Histogram of the number of relay symbols required for the new systematic scheme; parameters: $k = 2000$, $\varepsilon = 0.25$.

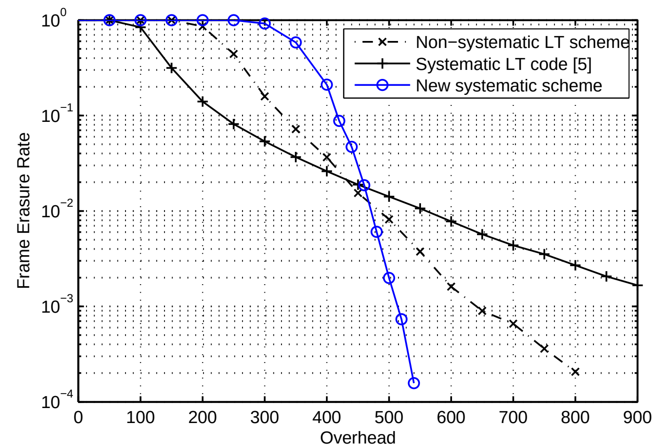

Fig. 5. Frame erasure rate as a function of overhead for $k = 2000$, $\varepsilon = 0.25$.

where α_ℓ is a normalization constant, then from (14), it is easily seen that (11) is satisfied for all j (except $1, s_\ell$) with

$$\alpha \leq \alpha_\ell \sqrt{2\pi}. \tag{18}$$

We now obtain an upper bound for α_ℓ. Note that

$$\alpha_\ell < \frac{1}{\beta_\ell}\sum_{j=2}^{\infty}\frac{1}{\sqrt{j}(j-1)} + \frac{1}{\beta_\ell s_\ell}\sum_{j=2}^{s_\ell}\frac{1}{\sqrt{j}} \tag{19}$$

$$< \sum_{j=2}^{\infty}\frac{1}{\sqrt{j}(j-1)} + \frac{1}{s_\ell}\sum_{j=2}^{s_\ell}\frac{1}{\sqrt{j}} \tag{20}$$

since $1/\beta_\ell < 1$. Further, clearly $1/(\sqrt{j}(j-1)) \sim j^{-3/2}$ whose infinite sum converges. Also, note that

$$\frac{1}{s_\ell}\sum_{j=2}^{s_\ell}\frac{1}{\sqrt{j}} < \frac{1}{\sqrt{2}}. \tag{21}$$

Summarizing, $\alpha_\ell < 3$, which gives $\alpha < 3\sqrt{2\pi}$.

Recall that degrees 1 and s_ℓ were not included in (15) – correspondingly, the degrees $1/\varepsilon$ and s_ℓ/ε were not included in (17). However, $\mu(1)$ and $\mu(s_\ell)$ tend to zero for large ℓ, so incorporating symbols of these degrees in our code requires negligible overhead. More precisely: It can be shown that enough symbols of reduced degree one can be generated using $p(\cdot)$ as given in (17). Similarly, it can be shown that symbols of reduced degree s_ℓ can be accounted for by incorporating a small additional contribution to $p(i)$ at $i = s_\ell/\varepsilon$.

The bound $\alpha < 3\sqrt{2\pi}$ suggests that as many as 7ℓ relay symbols may required so that the destination can recover from ℓ erasures. This is clearly impractical. However, the bounding techniques are quite loose, and, in practice, far fewer symbols are actually needed. An example is given in Figure 4.

Figure 4 suggests that, on average, the new scheme requires about 860 relay symbols to recover from $\ell = \epsilon k = 500$ erasures when $k = 2000$ and $\epsilon = 0.25$; this is a larger average overhead – 360 symbols – compared to the 250 symbols required by the LT-based scheme (Section III-B) and the 150 symbols required by the scheme based on systematic LT codes (Section III-C). On the positive side, there is a smaller "tail" of large redundancy for the new scheme; this is illustrated in Figure 5, which shows the frame erasure rate (FER) as a function of the overhead for all three schemes, when $k = 2000$ and $\epsilon = 0.25$. This figure demonstrates that for frame erasure rates less than (approximately) 10^{-2}, the new scheme requires less overhead.

Regarding asymptotic complexity: Using $p(i)$ in (17) as the degree distribution, the average degree per code symbol is $O(\sqrt{\ell}/\varepsilon)$ (compared to an average degree of $O(\log(\ell))$ for the RSD). Consequently, letting $\ell = k\varepsilon$, the average encoding and decoding complexity scale as $O(k\sqrt{k\varepsilon})$.

IV. Regarding an Extension to Raptor Codes

This paper focused on LT codes because they are a key component in the design of modern erasure correcting codes. Raptor codes ([5]) consist of an outer "pre-code" and an inner LT code. The pre-code ensures that, even with a constant average degree per LT code symbol, good performance is obtained. Specifically, the degree distribution may be capped at a maximum degree independent of k. Thus in the context of the relay network, by using a pre-code at the source, the degree distribution in (17) could also be capped, ensuring that the encoding complexity at the relay remains constant.

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