

Nested Codes with Multiple Interpretations

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Abstract—This paper proposes a new approach to channel code design for wireless network applications. The resulting nested codes can be decoded at different *effective rates* by different receivers – rates that depend on the prior knowledge possessed by each receiver; we say these codes have *multiple interpretations*. We have identified several applications in wireless networks where this property is useful. Specific nested code constructions as well as efficient soft and hard decision decoding algorithms are described. The concept of a nested code with multiple interpretations provides flexibility in the design of error protection schemes for multi-terminal wireless networks.

I. INTRODUCTION

Forward error control coding is effective in improving the reliability of data transmitted over noisy channels. In the past, the art and practice of error control coding was largely confined to point-to-point communication links [1]. In wireless network applications, the traditional layered approach breaks the network into a number of point-to-point links and then applies classical error control coding techniques to protect the data in each link. However, codes designed and optimized for point-to-point links might not be efficient in a network setting.

There has recently been significant research interest in developing error control methodologies that address not just the point-to-point link but the larger network as well. In [2], the authors discussed the use of redundancy in network coding and generalized the Hamming and Gilbert-Varshamov bounds to cover network error correcting codes; the codes described in [2] were defined over a finite field and only hard decoding was considered. In a more recent paper [3], the authors discussed the possibility of constructing codes on-the-fly, adapting to the time varying connectivity of wireless relay networks, and proposed a de-centralized framework termed adaptive network coded cooperation. In such a framework, each relay node combines (XORs) a selected set of the bits it received correctly; the direct transmissions from source nodes can be viewed as systematic bits while the relayed XORed bits can be viewed as parity bits in either a low density generator matrix (LDGM) code or a lower-triangular low density parity check (LDPC) code [3]. Diversity can be obtained at the destination node by decoding the LDGM or LDPC code constructed on-the-fly. The approach in [3] requires a reconfigurable LDGM/LDPC decoder at the destination and is only applicable to the scenario where a large number of relays

is available. In [4], the authors applied network coding over the binary field to exploit the shared nature of wireless networks and presented an opportunistic network coding scheme called COPE; in COPE, data packets are XORed at the network layer, resulting in significant throughput advantages compared to the current 802.11 mesh network. The potential benefit arising from the interaction of channel coding and the proposed XOR operation, however, was not considered in [4].

This paper proposes a new approach to channel code design – an approach in which multiple information packets are separately encoded and the resulting encoded packets are then XORed together at the physical layer prior to transmission. These encoded-and-XORed packets can be decoded in different ways depending on the prior information available to the receiver. If a receiver already knows the information in one or more of the packets, then the effect of the known packet(s) can be stripped away by regarding the (known) codeword(s) as a “scrambling” pattern; the net result is to decode the information that is *not* known to the receiver at a lower *effective rate* – i.e., with more redundancy and thus more error protection. Alternatively, if a receiver has no prior information about any of the packets, then its decoder regards the XORed codewords as being produced by a higher-rate “nested” code. In this manner, the same codeword broadcast in a wireless medium is interpreted and decoded differently at different receivers depending on the receivers’ prior knowledge of the broadcast information.

The paper is organized as follows. The concept of a nested code as well as encoding methods and efficient hard-decision and soft-decision decoding algorithms are introduced in Section II. Three applications and specific choices of codes for these applications are discussed in Section III. Finally, Section IV concludes the paper.

II. NESTED CODES WITH MULTIPLE INTERPRETATIONS

Consider the scenario depicted in Figure 1, in which a transmitter (Tx) seeks to deliver N information vectors $\mathbf{i}_1, \dots, \mathbf{i}_N$ of k bits each to a number of receivers (Rx). Assume that the codeword \mathbf{c} that is broadcasted to the receivers is Nn bits long, where $n \geq k$.

Also assume that the j -th receiver knows *a priori* the value taken by some of the information vectors; let \mathcal{K}_j denote the indices of the information vectors known *a priori* to receiver j – i.e., receiver j knows the value of \mathbf{i}_ℓ prior to transmission if and only if $\ell \in \mathcal{K}_j$. This scenario is realistic in many wireless networks; for example, a source node has knowledge of the

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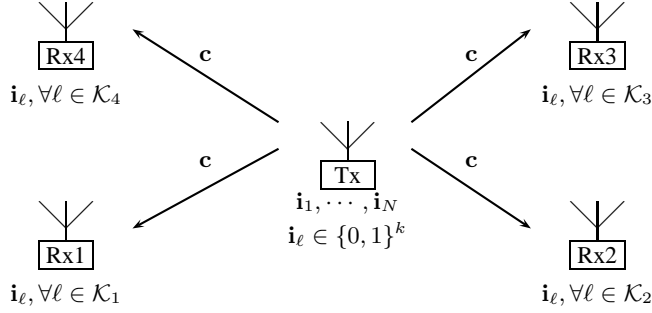


Fig. 1. One transmitter broadcasts a codeword \mathbf{c} , representing N information vectors $\mathbf{i}_1, \dots, \mathbf{i}_N$ of k bits each. Each of the multiple receivers knows the value of some subset of the information vectors.

information that it originated, and that same information might be relayed by another user in a multihop network. The wireless nodes could also perform opportunistic listening [4] to reveal information that is intended to be delivered to other users and utilize this additional knowledge to assist in decoding packets addressed to itself.

A. Encoding

A straightforward approach is to encode each information vector separately with a rate $R = k/n$ code and then concatenate the resultant codewords. This approach is not efficient because it does not exploit the fact that some receivers already know some of the information vectors; the resources that the transmitter uses to convey to receiver j the $|\mathcal{K}_j|k$ bits already known to receiver j are, in effect, wasted.

Alternatively, consider encoding each information vector with a low rate $R/N = k/(nN)$ code and then XORing the resulting codewords, i.e.,

$$\mathbf{c} = \mathbf{i}_1 \mathbf{G}_1 \oplus \mathbf{i}_2 \mathbf{G}_2 \oplus \dots \oplus \mathbf{i}_N \mathbf{G}_N \quad (1a)$$

$$= [\mathbf{i}_1, \mathbf{i}_2, \dots, \mathbf{i}_N] \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \\ \dots \\ \mathbf{G}_N \end{bmatrix}, \quad (1b)$$

where $\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_N$ are generator matrices of rate $k/(nN)$ codes. The representations of the XORed codewords in (1a) and (1b) suggest different interpretations of how the (possibly corrupted) codewords should be processed at the receiver. Representation (1a) suggests that, if the receiver knows some of the information vectors, then the effects of those information vectors can be “stripped off” by treating the associated codewords as a scrambling pattern. Alternatively, if none of the information vectors are known at the receiver, then the N information vectors can be estimated based on the nested code associated with the “stacked” generator matrix in (1b).

It is worth pointing out that if we choose the generator matrices $\mathbf{G}_1 = \mathbf{G}_2 = \dots = \mathbf{G}_N = \mathbf{G}$ and then XOR after channel coding (i.e., we XOR the codewords), this is equivalent to XORing the information vectors before channel

coding, as in [4], i.e.,

$$\mathbf{i}_1 \mathbf{G} \oplus \mathbf{i}_2 \mathbf{G} \oplus \dots \oplus \mathbf{i}_N \mathbf{G} = (\mathbf{i}_1 \oplus \mathbf{i}_2 \oplus \dots \oplus \mathbf{i}_N) \mathbf{G}. \quad (2)$$

In the case where more than one information vector among $\mathbf{i}_1, \mathbf{i}_2, \dots, \mathbf{i}_N$ is unknown (or only known with soft a priori values) to the receivers, however, it becomes necessary to use different generator matrices for $\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_N$, since the “stacking” of identical generator matrices in (1b) results in rank deficiency, and consequently the encoding operation is non-invertible even in the absence of noise. We can also interpret the difference between XORing the information vectors (prior to channel coding) and XORing the coded vectors (after channel coding) from a code rate prospective. The XOR operation is, in essence, a form of compression that increases the rate, while channel encoding lowers the rate by adding redundant bits. When the XOR operation is effected before channel encoding, the result, in the stage between the XOR and channel encoding, is a vector that conveys information at a rate higher than one. Such a high rate can only be resolved by *perfect* a priori knowledge at the receiver of all but one of the information vectors XORed at the transmitter. Alternatively, If we first introduce redundancy through channel encoding and then compress the low rate codewords, we can still recover the information vectors even when more than one of the information vectors are not known perfectly at the receiver, assuming that the code is properly designed to avoid rank deficiency in the “stacked” generator matrix and the overall rate – the rate considering both the channel encoding and the XOR compression – of the unknown vectors is less than one. Hence, by choosing different (linearly independent) generator matrices, we avoid a catastrophic condition and make nested codes with multiple interpretations more flexible and suitable for a variety of applications.

B. Hard-Decision Decoding

In hard-decision decoding, the output of the demodulator is $\hat{\mathbf{r}} = \mathbf{c} \oplus \mathbf{e}$, where \mathbf{e} is the binary error pattern. At the j -th receiver, $\hat{\mathbf{r}}$ can be interpreted as

$$\hat{\mathbf{r}} = \bigoplus_{l \notin \mathcal{K}_j} \mathbf{i}_l \mathbf{G}_l \oplus \bigoplus_{l' \in \mathcal{K}_j} \mathbf{i}_{l'} \mathbf{G}_{l'} \oplus \mathbf{e}. \quad (3)$$

Since part of $\hat{\mathbf{r}}$ – namely $\bigoplus_{l' \in \mathcal{K}_j} \mathbf{i}_{l'} \mathbf{G}_{l'}$ – is known to the j -th receiver, it can be cancelled by XORing to obtain

$$\hat{\mathbf{r}}_j = \hat{\mathbf{r}} \oplus \bigoplus_{l' \in \mathcal{K}_j} \mathbf{i}_{l'} \mathbf{G}_{l'} = \bigoplus_{l \notin \mathcal{K}_j} \mathbf{i}_l \mathbf{G}_l \oplus \mathbf{e}. \quad (4)$$

The right hand side of (4) represents a corrupted version of a codeword from a rate $(N - |\mathcal{K}_j|)k/n$ code – the nested code obtained by “stacking” the generator matrices corresponding to all the information vectors *not* known to receiver j . The information vectors that are known to the receiver a priori are cancelled as a scrambling pattern using (4) and are transparent to the channel decoder. Thus, when a receiver knows more of the information vectors *a priori*, it can decode at a lower rate – i.e., with more robustness – the information vectors it does *not* know *a priori*. Hence we have a single nested code with multiple interpretations.

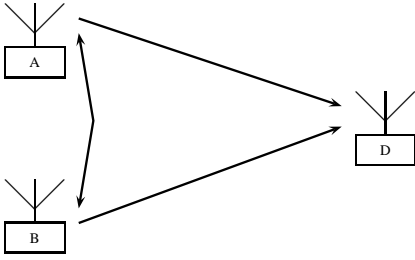


Fig. 2. In cooperative diversity, Node A and Node B work in collaboration to deliver their packets to a common destination Node D.

C. Soft-Decision Decoding

In soft-decision decoding, the output of the demodulator is $L_{c(i)}$, the log-likelihood ratio (LLR) of the i -th bit in \mathbf{c} . Similar to hard-decision decoding, the codeword \mathbf{c} can be decomposed into two parts as

$$\mathbf{c} = \underbrace{\bigoplus_{l \notin \mathcal{K}_j} \mathbf{i}_l \mathbf{G}_l}_{\mathbf{c}_u} \oplus \underbrace{\bigoplus_{l' \in \mathcal{K}_j} \mathbf{i}_{l'} \mathbf{G}_{l'}}_{\mathbf{c}_c}. \quad (5)$$

The first term on the right hand side of (5), \mathbf{c}_u , is a collection of unknown codewords, while the second term, \mathbf{c}_c , is the collection of codewords known to the j -th receiver – which can therefore be cancelled. The LLR of the i -th bit in \mathbf{c}_u can be computed according to

$$L_{\mathbf{c}_u(i)} \triangleq \log \frac{\Pr[\mathbf{c}_u(i) = 0]}{\Pr[\mathbf{c}_u(i) = 1]} = \begin{cases} L_{\mathbf{c}(i)} = \log \frac{\Pr[\mathbf{c}_u \oplus \mathbf{c}_c(i) = 0]}{\Pr[\mathbf{c}_u \oplus \mathbf{c}_c(i) = 1]} & \text{if } \mathbf{c}_c(i) = 0 \\ -L_{\mathbf{c}(i)} = \log \frac{\Pr[\mathbf{c}_u \oplus \mathbf{c}_c(i) = 1]}{\Pr[\mathbf{c}_u \oplus \mathbf{c}_c(i) = 0]} & \text{if } \mathbf{c}_c(i) = 1. \end{cases} \quad (6)$$

The operation in (6) changes only the sign of the LLR and not its magnitude; hence no information from the observation of the channel output is lost. We refer to this as the “flipping” operation. This operation can be regarded as a degenerate case of the box-plus operation introduced in [5] when one of the input LLR’s takes a value of either infinity or minus infinity. With the LLR $L_{\mathbf{c}_u(i)}$ available from the flipping operation, we can employ a soft decoder for the rate $(N - |\mathcal{K}_j|)k/Nn$ codeword \mathbf{c}_u to estimate the $N - |\mathcal{K}_j|$ information vectors that are unknown *a priori* to receiver j .

III. APPLICATIONS

In this section, we present three scenarios in wireless networks that describe a situation like the one depicted in Figure 1. In particular, we demonstrate how nested codes with multiple interpretations can be used to improve the performance of wireless systems compared with more conventional schemes.

A. Cooperative Diversity

Consider first a cooperative diversity system in which two partners – call them Node A and Node B – cooperate in

transmitting information to a common destination node, Node D, as illustrated in Figure 2. (See [6] and the references therein.) Each partner node transmits both its locally generated information and relayed information that originated at the other partner. Spatial diversity can be exploited from the reception of the same information twice – from both the direct transmission and the relay.

Conventional cooperative schemes [6] adopt a time division approach, wherein the transmission time of each partner node is split into two parts – one part for broadcasting its locally-generated information and the other part for relaying the information that originated at its partner node. Much like the straightforward concatenated approach discussed in Section II-A, the fact that Node A knows Node B’s relayed information prior to its transmission – and Node B knows Node A’s relayed information prior to *its* transmission – is not exploited in conventional time division based designs.

As an alternative that *does* exploit this knowledge, consider the use of nested codes with multiple interpretations. Node A encodes its own local information using a channel code with generator \mathbf{G}_L while it interleaves and encodes the relayed information from Node B using a channel code with generator \mathbf{G}_R ; what is transmitted is the XOR of these two codewords. Node B, with its prior knowledge of node A’s relayed information, can cancel the part of the codeword encoded by \mathbf{G}_R , as described in Section II, and decode only the information vector encoded by \mathbf{G}_L . From the perspective of Node B, all the resources used in Node A’s transmission are used to convey Node A’s locally generated information; no resources are “wasted” in conveying the relayed information, which Node B already knows. Since Node B will be able to cooperate (i.e., relay information) only when it decodes Node A’s locally generated information correctly, the adoption of nested codes with multiple interpretations makes the cooperative diversity system work in a cooperative mode (i.e., a diversity enabling mode) more often than in conventional time division designs.

The destination – Node D – has prior knowledge of *neither* of the information vectors, so it views the received codeword as one encoded with the “stacked” generator $\begin{bmatrix} \mathbf{G}_L \\ \mathbf{G}_R \end{bmatrix}$. Diversity is achieved at node D by iteratively decoding the direct transmission and the relayed transmission encoded with an interleaved version of the same information. Of course, the bits generated at the two partner nodes are formed by XORing together two codewords, and this must be accounted for at Node D. Take the decoding of Node A’s information as an example. Node D processes a codeword from Node A containing Node A’s local information and relayed information that originated at Node B; at the same time it also processes a codeword from Node B containing a new information vector generated at Node B and relayed information that originated at Node A. For the codeword from Node A, the relayed part (Node B’s information) is decoded iteratively together with the direct transmission from Node B received previously, and we can use the extrinsic information generated by the previous decoding as *a priori* information for the relayed bits. For the codeword from Node B, relaying Node A’s relevant information, however, the decoding of Node B’s local

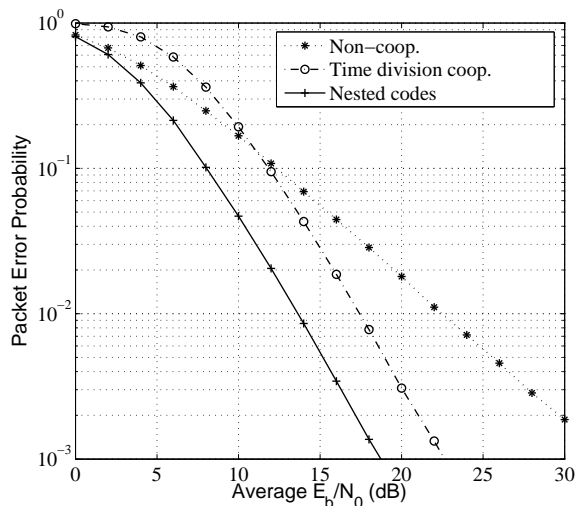


Fig. 3. An SNR gain of about 4 dB is obtained using nested codes with multiple interpretations, compared with the conventional approach to cooperative diversity based on time division multiplexing.

information cannot be completed since Node A might relay Node B's information in its next transmission. We use zero *a priori* values for Node B's local bits in the soft-decoding of this codeword.

The performance improvement in terms of packet error probability from using nested codes with multiple interpretations is shown in Figure 3. This simulation assumes BPSK modulation on an AWGN channel with i. i. d. block Rayleigh fading and ten iterations at the destination node. Packets of 500 information bits each and rate 1/3-convolutional codes were used. CRC-12 was adopted to identify decoding failures. $\mathbf{G}_R = \begin{bmatrix} 02 & 07 & 1 \\ 15 & 15 & 1 \end{bmatrix}_8$ was chosen to maximize the free distance of the nested code, given $\mathbf{G}_L = \begin{bmatrix} 1 & 13 & 17 \\ 15 & 15 & 1 \end{bmatrix}_8$. Figure 3 also includes the performance of two reference systems – namely a non-cooperative point-to-point system and the conventional time-division based coded cooperative diversity system [6] with a cooperation level of 50 percent.

Unlike conventional iterative decoding, we only exchange extrinsic information for some of the encoded information bits between the two constituent decoders. (Recall that in our case two information vectors are encoded into an XORed codeword, but only one information vector is processed in iterative decoding.) Extrinsic information transfer (EXIT) charts [7] provide a useful tool for studying the convergence behavior of iterative decoding. Figures 4 and 5 depict the extrinsic information transfer relation for two constituent codes, i.e., the code from the direct transmission and the code from the relay. For the direct transmission constituent code, the relayed portion has been previously processed, and the amount of extrinsic information received from the previous decoding parameterizes the curves in Figure 4. For the relayed constituent code in Figure 5, no previously processed codeword is available, and the local information bits use zero a priori values. A typical decoding trajectory is shown in Figure 6. In this example, decoder one processes the codeword from the direct transmission, which is transmitted over an AWGN channel with $E_b/N_0 = 0$ dB. It is assumed that the previous

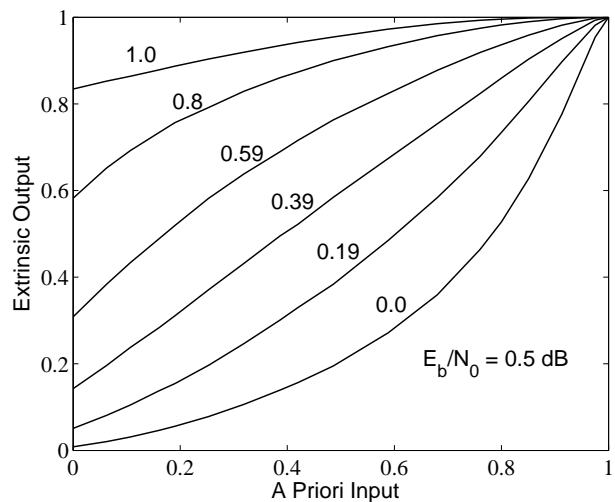


Fig. 4. The extrinsic information input-output relation for the constituent code from the direct transmission.

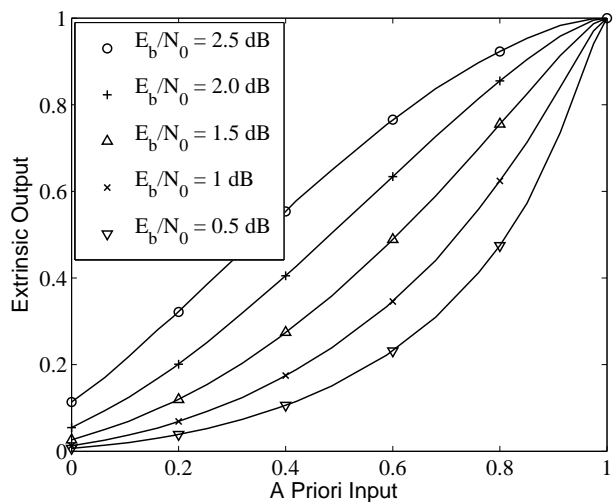


Fig. 5. The extrinsic information input-output relation for the constituent code from the relay.

decoding provides an a priori mutual information of 0.76 for the relayed bits to decoder one. Decoder two processes the codeword from the relay, which is transmitted over an AWGN channel with $E_b/N_0 = 1.5$ dB. From both the EXIT curves (Figures 4 and 5) and the decoding trajectory (Figure 6), it is observed that, for either information vector, the extrinsic information at the decoder output can reach a value of one even when the a priori information for the other information vector is small or even zero.

B. Network Coded Duplex Relay

In network coded duplex relay systems [4], [8], two nodes, Node A and Node B, wish to exchange packets through a common relay, Node R. Instead of relaying A's packet \mathbf{i}_A and B's packet \mathbf{i}_B in sequence, the relay node XORs the decoded and re-encoded packets from A and B and subsequently broadcasts the XORed information to both A and B [4], [8]. When a channel encoder \mathbf{G} is used to protect the bits against

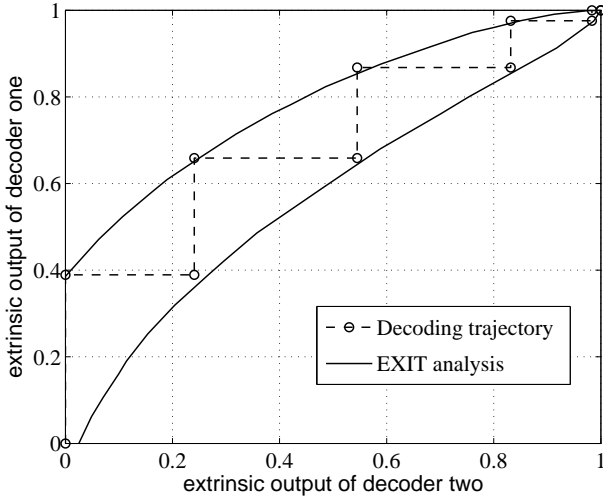


Fig. 6. A typical decoding trajectory for a block size of 50000 bits.

channel noise, the broadcast codeword can be written as

$$(\hat{\mathbf{i}}_A \oplus \hat{\mathbf{i}}_B)\mathbf{G} = \hat{\mathbf{i}}_A\mathbf{G} \oplus \hat{\mathbf{i}}_B\mathbf{G}, \quad (7)$$

where $\hat{\mathbf{i}}_A$ and $\hat{\mathbf{i}}_B$ are the estimates of \mathbf{i}_A and \mathbf{i}_B at Node R. This scheme is illustrated in Figure 7 with $\mathbf{G}_A = \mathbf{G}_B = \mathbf{G}$. Upon receiving the corrupted version of $(\hat{\mathbf{i}}_A \oplus \hat{\mathbf{i}}_B)\mathbf{G}$, nodes A and B can decode the XOR of the two information packets as $\hat{\mathbf{i}}_A \oplus \hat{\mathbf{i}}_B \oplus \mathbf{e}$, where \mathbf{e} is the error pattern introduced by decoding failures. Node A, for instance, can form an estimate of \mathbf{i}_B using

$$\begin{aligned} \tilde{\mathbf{i}}_B &= \hat{\mathbf{i}}_A \oplus \hat{\mathbf{i}}_B \oplus \mathbf{e} \oplus \mathbf{i}_A \\ &= \hat{\mathbf{i}}_B \oplus \mathbf{e} \oplus (\hat{\mathbf{i}}_A \oplus \mathbf{i}_A). \end{aligned} \quad (8)$$

Note that if the relay only sends $\hat{\mathbf{i}}_B$ to Node A, Node A would decode $\hat{\mathbf{i}}_B \oplus \mathbf{e}$ under the same channel conditions. The last term $(\hat{\mathbf{i}}_A \oplus \mathbf{i}_A)$ in (8) is an error propagation term resulting from the superposition of $\hat{\mathbf{i}}_A$, which will be different from \mathbf{i}_A when the decoding of \mathbf{i}_B fails at Node R. Note that the error propagation term is transparent to the channel decoders, since we cancel \mathbf{i}_A after channel decoding. From the perspective of nested codes with multiple interpretations, this transparency is caused by the rank deficiency of the “stacked” generator matrix $\begin{bmatrix} \mathbf{G} \\ \mathbf{G} \end{bmatrix}$, which is noninvertable even in the absence of noise.

As an alternative, we can use nested codes with linearly independent generators \mathbf{G}_A and \mathbf{G}_B for Node A’s and Node B’s information vectors and cancel the known information at nodes A and B prior to either soft-decision or hard-decision decoding, as shown in Section II. In this case, still taking decoding at Node A as an example, the remaining error $\hat{\mathbf{i}}_A\mathbf{G}_A \oplus \mathbf{i}_A\mathbf{G}_A$ from an erroneously decoded packet at Node R will be a valid codeword for \mathbf{G}_A , but not a valid codeword for \mathbf{G}_B thanks to the linear independence between \mathbf{G}_A and \mathbf{G}_B . Hence the channel decoder estimating \mathbf{i}_B will attempt to remove both the effect of noise and the effect of the error propagation term at Node B. The performance improvement from using nested codes with multiple interpretations is shown

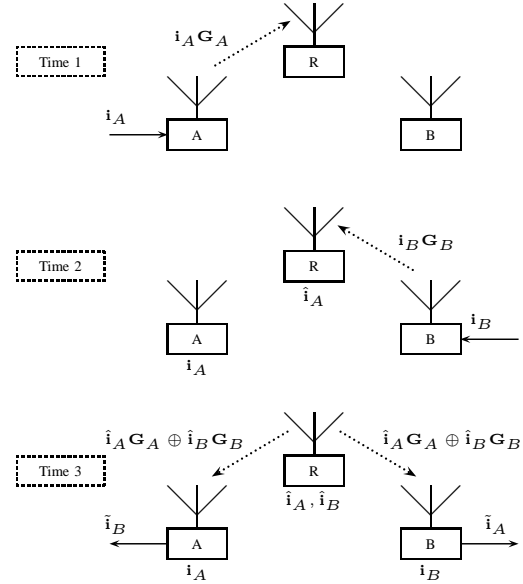


Fig. 7. In network coded duplex relay systems, Node A and Node B exchange their packets using a common relay Node R.

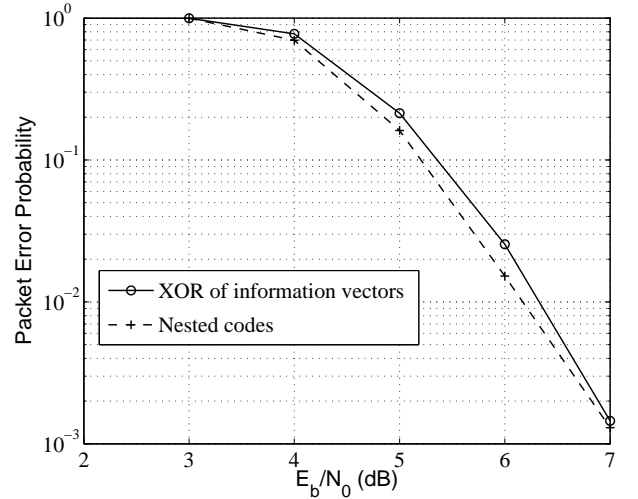


Fig. 8. The effect of adopting nested codes with multiple interpretations in a network coded duplex relay system.

in Figure 8, where an AWGN channel is assumed, the same 8-state convolutional code $[15, 13, 17]_8$ is used for A’s and B’s bits, but a random interleaving of B’s coded bits is adopted prior to the XOR operation to avoid the rank deficiency that results from “stacking” identical generator matrices. Since few errors occur for high channel SNRs, the improvement is most visible in the medium channel SNR range.

Note that the channel decoder is only optimum in detecting codewords on an AWGN channel, but that the error propagation term is clearly not Gaussian distributed. A joint codeword and error pattern detector can be used to further reduce the effect of the error propagation term. The key to this joint processing is to treat the information, after the cancellation or

“flipping” described in Section II, as being from the codeword

$$[\mathbf{i}_A \oplus \hat{\mathbf{i}}_A, \hat{\mathbf{i}}_B] \begin{bmatrix} \mathbf{G}_A \\ \mathbf{G}_B \end{bmatrix} \quad (9)$$

and to decode both the desired information $\hat{\mathbf{i}}_B$ and the error propagation term $\mathbf{i}_A \oplus \hat{\mathbf{i}}_A$ using the bit error probability in decoding \mathbf{i}_A at Node R as a priori information for $\mathbf{i}_A \oplus \hat{\mathbf{i}}_A$. The performance improvement from this nested code approach with joint detection, though, is rather small compared to the nested code approach that detects only the desired information vectors.

C. Broadcast Infrastructure Aided Multicasting

In this scenario, the base station relays multicasting content from one of the users while broadcasting its own information to all the users at the same time. This could happen, for example, in a trunked radio system, where one user multicasts its packets to a “talk-group” and the base station’s information is from the central office.

In conventional designs, the base station would adopt a multiplexing scheme to separate the broadcasting of its own bits and the relaying of its talking user’s multicast information. This neglects the fact that the talking user knows its own message, and moreover, the users close to the talking user might successfully decode the same message when the packet is delivered to the base station for relay.

Nested codes with multiple interpretations can be used in this scenario to make more efficient use of available resources at the base station. The base station encodes its own broadcasting packet \mathbf{i}_B using encoder \mathbf{G}_B , while it encodes the relayed multicasting packet from the talking user, \mathbf{i}_M , using a different encoder \mathbf{G}_M . The transmitted codeword is then $\mathbf{i}_B \mathbf{G}_B \oplus \mathbf{i}_M \mathbf{G}_M$. The user where the multicast packet originated, together with the users that have obtained the multicasted packet through the transmission to the base station, can cancel the known information $\mathbf{i}_M \mathbf{G}_M$ and decode only the broadcasted packet \mathbf{i}_B at a low rate. Other users, without knowing either of the packets, can decode both packets at the same time using the decoder for the nested code $\begin{bmatrix} \mathbf{G}_B \\ \mathbf{G}_M \end{bmatrix}$. The choice of code in this application is very different compared with the choice of code in the cooperative diversity scenario, where the link between the two source nodes is vital to ensure diversity and thus requires good protection. In this scenario, the users capable of cancelling $\mathbf{i}_M \mathbf{G}_M$ have already enjoyed a rate benefit, and hence it is more desirable to ensure that the users decoding $\begin{bmatrix} \mathbf{G}_B \\ \mathbf{G}_M \end{bmatrix}$ are using a strong code.

In Figure 9, we illustrate the benefit of using nested codes with multiple interpretations on an AWGN channel. The best 64-state rate 2/3 convolutional code $\begin{bmatrix} 06 & 13 & 13 \\ 13 & 06 & 17 \end{bmatrix}_8$ [1] is used as the “stacked” generator matrix $\begin{bmatrix} \mathbf{G}_B \\ \mathbf{G}_M \end{bmatrix}$. We choose $\mathbf{G}_B = [13 \ 06 \ 17]_8$ and $\mathbf{G}_M = [06 \ 13 \ 13]_8$, since the former has better distance properties than the latter. With nested codes, the users with knowledge of the multicast packets gain around 2 dB compared to the other users. Note that in a

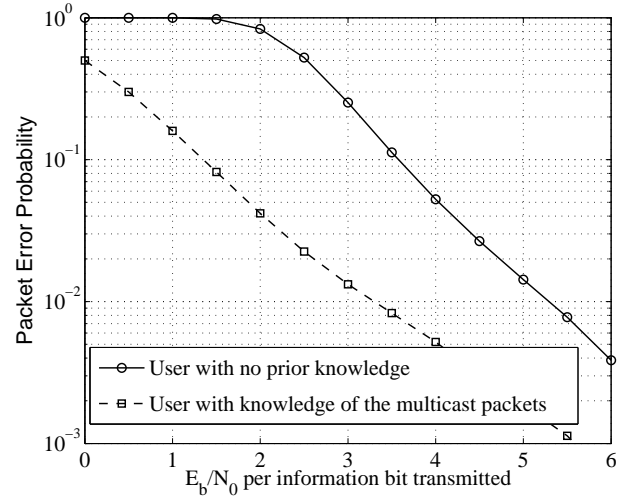


Fig. 9. Users with knowledge of the multicast packets gain from using nested codes with multiple interpretations.

conventional reference scheme, the information is transmitted at rate 2/3, and the packet error performance is similar to that of the users without knowledge of the multicast packets. Hence we improve the performance of the “knowledgeable” users without affecting that of the “ignorant” users.

IV. CONCLUSIONS

A framework for nested codes with multiple interpretations has been proposed, and simple and efficient decoding methods that reduce the *effective code rate* based on a receiver’s *a priori* knowledge have been described. Three application examples have been provided to show how the nested code can improve system performance compared to conventional designs. These codes, together with the efficient hard-decision and soft-decision decoding described in this paper, are flexible and useful tools in the design of error control schemes in a wireless network setting.

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