Algebraic Superposition of LDGM Codes for Cooperative Diversity

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***Abstract*— This paper presents a technique for achieving cooperative spatial diversity using serially concatenated low density generator matrix (LDGM) codes. Specifically, we consider a scenario in which a pair of transceivers employ algebraic superposition of error control codes to effect spatial diversity at their common destination. The construction of LDGM codes from a sparse generator matrix makes them a natural fit for such a cooperative diversity scheme. The simple decoder structure for graph based codes reduces the complexity at the destination compared with previously-proposed schemes using algebraic superposition of convolutional codes and turbo-like decoding. The result is a system with low encoding and decoding complexity and improved error performance.**

I. Introduction

In a network with more than one transmitter-receiver pair, a group of source nodes can form a partnership to deliver their collective information to a common destination node. In a fading environment, such a collaboration, often referred to as cooperative diversity, can create spatial diversity [1]–[7] by sharing the independently faded links to the common destination. Perhaps the simplest such system is one in which each of two source nodes serves as the other's partner - transmitting not only the "local" information generated at the node but also the "relayed" information coming from the partner. Since the same information would ideally be transmitted to the destination twice – once in the direct transmission and the other relayed via the partner node – second order spatial diversity can be achieved, assuming the partner nodes can correctly decode each other's message.

Conventional cooperative diversity schemes include a resource allocation arrangement at each partner node – i.e., a scheme that splits the node's transmission time [1]–[3] or power [4], [8] between the locally-generated bits and the relayed bits. In previous work by the authors [7], it was observed that such a separation degrades the quality of the link between the partner nodes, and a design avoiding explicit resource separation was proposed based on the algebraic superposition of convolutional codes. The main idea in [7] was that each partner superimposes – XOR's – the codeword for the local information and a relayed codeword. The decoder at the partner node can cancel the relayed codeword as a scrambling pattern and decode the local information only, while the decoder at the destination observes codewords from both links and iteratively decodes them to extract diversity gain. Similar approaches combining the XOR operation and channel coding were proposed in [9], [10], but for two way relay channels and diversity systems with a dedicated relay node. In [6] a cooperative scheme based on multiplexing irregular repeat-accumulate (IRA) codes was proposed; however, in contrast to the approach in [7] and this paper, the resulting multiplexed IRA code suffers from an inherent rate loss.

While it was demonstrated in [7] that algebraic superposition using convolutional codes provides better performance than designs with explicit resource allocation, such a system pays a price in complexity at the destination node. This is because the number of encoder states grows linearly for the joint detection of two XORed codewords, and hence the complexity of the BCJR algorithm is squared.

In this paper, low density generator matrix (LDGM) codes [11]–[13] are adopted to avoid this increase in decoding complexity. The sparseness of an LDGM code's generator matrix means it has a poor minimum distance and a high error floor [14]. However, it was shown in [12], [13] that the serial concatenation of an inner LDGM code and a high rate outer LDGM code (to clean up residual errors) can yield performance close to that achieved with turbo or LDPC codes. Since the decoding complexity of such a graph-based code scales linearly with the number of ones in the generator matrix, decoding two algebraically superimposed LDGM codes does not bring the exponential growth in complexity experienced with convolutional codes.

The remainder of the paper is organized as follows. The system model is presented in Section II, and Section III includes an outage probability analysis that provides a lower bound on the frame error rate. The details of the transceiver design using LDGM codes is described in Section IV. Simulation results are discussed in Section V, and Section VI concludes the paper.

II. System Model and Notation

Consider the system shown in Figure 1. Denote the local message generated at Node A and Node B at time t as $\mathbf{i}_A^t$ and $\mathbf{i}_B^t$, respectively. Node A and Node B each transmit n BPSK-modulated bits to a common destination Node D using time division multiple access (TDMA). In the first half of time slot t, Node A sends its own local message $\mathbf{i}_A^t$ and it relays Node B's message from the previous time slot, $\mathbf{i}_B^{t-1}$, assuming it was successfully decoded. Similarly, in the second

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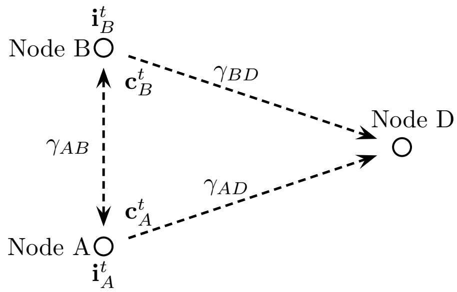

Fig. 1. System model and notation in cooperative diversity.

half of time slot t, Node B transmits its own new message $\mathbf{i}_B^t$ and also relays $\mathbf{i}_A^t$ (assuming it was successfully decoded). Denote the codewords modulated at time t by $\mathbf{c}_A^t$ and $\mathbf{c}_B^t$ for Node A and Node B, respectively. Denote the signal-to-noise ratio (SNR) of the individual links as γ_{AB}, γ_{AD}, and γ_{BD}, as shown in Figure 1. These channels are assumed to be frequency nonselective, block Rayleigh faded with additive white Gaussian noise; the instantaneous SNRs are statistically independent and follow the distribution

$$p_\gamma(\gamma) = \frac{1}{\Gamma}\exp(-\frac{\gamma}{\Gamma}), \tag{1}$$

where $\gamma \geq 0$ and Γ is the average SNR. Finally, we assume Node A and Node B are constant bit rate sources, each generating a packet of k bits per time slot.

III. Outage Probability Analysis

In this section, we derive the outage probability for the system under consideration as well as a simple lower bound on the frame error rate (FER). The destination node observes transmissions over two channels with instantaneous SNRs γ_{AD} and γ_{BD}. With the TDMA constraint and fixed power, the maximum rate the two nodes can jointly deliver to the destination is bounded by sum of the capacities of the individual links. Let $C(\gamma)$ represent the capacity of a particular link with instantaneous SNR γ, and let $r = \frac{k}{n}$ be the rate of each source. To derive a lower bound on the FER for very large n and k ($n, k \to \infty$), we divide the sum capacity into three regions

- $C(\gamma_{AD}) + C(\gamma_{BD}) < r$: In this region, the two channels are not able to support either of the sources, and successful decoding of either source at the destination is not possible. The frame error rate in this region is one.
- $r \leq C(\gamma_{AD}) + C(\gamma_{BD}) < 2r$: The sum capacity of the two channels cannot simultaneously support both sources. However, one source of rate r might be transmitted to the destination if the two nodes cooperate. In this case, at least one source cannot be correctly decoded. Hence the FER is lower bounded by one half.
- $C(\gamma_{AD}) + C(\gamma_{BD}) \geq 2r$: The sum capacity is larger than the sum rate of the two sources. It is possible through cooperation to deliver packets from both sources to the destination. The FER in this case is lower bounded by zero.

Note that the first two cases taken together describe when the system is in outage - i.e., when the the sum capacity cannot support the sum rate. Thus the outage probability is given by

$$P_O = \mathbb{P}\mathrm{r}\big\{C(\gamma_{AD}) + C(\gamma_{BD}) < 2r\big\}. \tag{2}$$

The frame error rate is lower bounded by

$$\begin{aligned} P_F &\geq \mathbb{P}\mathrm{r}\big\{C(\gamma_{AD}) + C(\gamma_{BD}) < r\big\} \\ &+ \frac{1}{2}\mathbb{P}\mathrm{r}\big\{r \leq C(\gamma_{AD}) + C(\gamma_{BD}) < 2r\big\} \\ &= \frac{1}{2}\left[P_O + \mathbb{P}\mathrm{r}\big\{C(\gamma_{AD}) + C(\gamma_{BD}) < r\big\}\right]. \end{aligned} \tag{3}$$

Assuming BPSK modulation and Rayleigh fading, the probability $\mathbb{P}\mathrm{r}\{C(\gamma_{AD}) + C(\gamma_{BD}) < X\}$ can be evaluated through the numerical integration as

$$\begin{aligned} &\mathbb{P}\mathrm{r}\{C(\gamma_{AD}) + C(\gamma_{BD}) < X\} \\ &= \int_0^{\frac{(J^{-1}(X))^2}{8}} \frac{1}{\Gamma_{AD}}\exp\big(-\frac{\gamma_{AD}}{\Gamma_{AD}}\big) \\ &\times \left[1 - \exp\big(-\frac{[J^{-1}(X - J(\sqrt{8\gamma_{AD}}))]^2}{8\Gamma_{BD}}\big)\right]\mathrm{d}\gamma_{AD}, \end{aligned} \tag{4}$$

where the definition of $J(\cdot)$ and its inverse $J^{-1}(\cdot)$, together with closed-form approximations, can be found in [15].

IV. Algebraic Superposition of LDGM Codes

A. LDGM Codes

An LDGM code is defined by a sparse generator matrix $\mathbf{G}$. Systematic LDGM codes with generator matrix of the form $\mathbf{G} = [\mathbf{I}\,\mathbf{P}]$ have a low density parity check matrix $\mathbf{H} = [\mathbf{P}^T\,\mathbf{I}]$, and the decoding of systematic LDGM codes can be carried out on the corresponding Tanner graph. A systematic LDGM code is fully described by the parity matrix $\mathbf{P}$; if $\mathbf{P}$ is $M \times N$ then the code has rate $\frac{M}{M+N}$. It worth noting that, even in nonsystematic LDGM codes, there should be a certain number of degree one parity bits (i.e., systematic bits) to ensure the propagation of soft values in the graph [16]. In this paper, systematic LDGM codes are used throughout.

Serial concatenation of LDGM codes consists of an inner LDGM code and a high rate LDGM outer code [12]. Since the inner LDGM code determines low SNR performance, while the outer coder helps to reduce the error floor, no iterations between the two LDGM decoders are required [12]. Hence, for simplicity, the rest of this section focuses on the algebraic superposition and decoding of the inner code at the partner and destination nodes. We assume that the packets $\mathbf{i}_A^t$ and $\mathbf{i}_B^t$ have already been encoded with a high rate LDGM code, and that the decoding of the outer code is performed only when a hard decision on the bits is needed.

B. Partner Node: Encoding

Invoking symmetry, we consider only the encoding operation at Node B in slot t. Node B must transmit its own information $\mathbf{i}_B^t$ and it also must relay the information $\mathbf{i}_A^t$ that originated at Node A – assuming it was able to successfully decode $\mathbf{i}_A^t$ in the first half of time slot t.

If Node B failed to decode $\mathbf{i}_A^t$, the locally generated information $\mathbf{i}_B^t$ is the only packet to encode, and the modulated codeword is

$$\mathbf{c}_B^t = \mathbf{i}_B^t\mathbf{G}_L = [\mathbf{i}_B^t \;\; \mathbf{i}_B^t\mathbf{P}_L], \tag{5}$$

where $\mathbf{G}_L = [\mathbf{I} \;\; \mathbf{P}_L]$ is a systematic LDGM for the locally generated information.

If Node B successfully decoded $\mathbf{i}_A^t$ in the first half slot, the transmitted codeword is the XOR of the two codewords generated by $\mathbf{i}_B^t$ and $\mathbf{i}_A^t$. However, the algebraic superposition of two systematic codes is not systematic; in fact, the LDGM decoder will have a problem decoding such a "fully superimposed" code due to a lack of degree one check nodes in the graph. (The absence of degree one check nodes inhibits the decoding process from getting started [16].) This would not not be a problem for the decoder at the partner node, because it can always cancel the relayed portion of the received signal, as seen in the next subsection. However, the decoder at the destination node must decode information from both sources, and it does not possess any information to cancel prior to decoding. For this reason, the codeword consists of the locally generated packet $\mathbf{i}_B^t$ and a set of parity bits generated by both $\mathbf{i}_B^t$ and the relayed packet $\mathbf{i}_A^t$. That is,

$$\begin{aligned}\mathbf{c}_B^t &= \begin{bmatrix}\mathbf{i}_A^t & \mathbf{i}_B^t\end{bmatrix}\begin{bmatrix}\mathbf{0} & \mathbf{P}_R \\ \mathbf{I} & \mathbf{P}_L\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{i}_B^t & \mathbf{i}_A^t\mathbf{P}_R \oplus \mathbf{i}_B^t\mathbf{P}_L\end{bmatrix}. \end{aligned} \tag{6}$$

The relayed information does not have to be sent in systematic form because it has already been transmitted in systematic form by the other partner – i.e., it was sent in systematic form as local information.

In the encoding operation described above, a flag bit must be transmitted along with each codeword to indicate whether the codeword includes relayed information or whether it includes only locally-generated information. In practice, this flag bit should be protected by a very strong code. In our simulations, we assumed that the receivers at both the partner node and the destination node can recover the flag bit associated with each codeword without error.

Encoding at Node A is similar, with the locally generated packet being $\mathbf{i}_A^t$ and the relayed packet being $\mathbf{i}_B^{t-1}$. The particular LDGM codes used at the two partner nodes do not, in general, have to be identical. However, we assume here, for simplicity, that the two source nodes are indeed using the same code, as indicated by the notation $\mathbf{G}_L$, $\mathbf{P}_L$, and $\mathbf{P}_R$.

C. Partner Node: Decoding

Consider the decoding of $\mathbf{i}_B^t$ at Node A. The received codeword $\mathbf{c}_B^t$ during time slot t contains $\mathbf{i}_B^t$ and (probably) $\mathbf{i}_A^t$. Note that $\mathbf{i}_A^t$ is known to Node A, since this is where it originated. If the flag bit indicates only local information was included in the encoding procedure, i.e., the codeword was created using (5), then decoding is straightforward. Alternatively, if the flag bit indicates that the encoder worked in the cooperative mode – i.e., the codeword was formed by (6) – then the decoder observes the log-likelihood ratios (LLR's) of $\mathbf{i}_B^t$ and those of an algebraic superposition of parity bits given by $\mathbf{i}_A^t\mathbf{P}_R \oplus \mathbf{i}_B^t\mathbf{P}_L$. Let $[\mathbf{c}](i)$ and $L[\mathbf{c}](i)$ denote the i-th bit in the codeword $\mathbf{c}$ and its LLR value, respectively. Then the LLR value of $\mathbf{i}_B^t\mathbf{P}_L$ can be obtained by canceling the effect of $\mathbf{i}_A^t\mathbf{P}_R$ – i.e.,

$$\begin{aligned}&L[\mathbf{i}_B^t\mathbf{P}_L](i) \\ &= \begin{cases} L[\mathbf{i}_A^t\mathbf{P}_R \oplus \mathbf{i}_B^t\mathbf{P}_L](i) & \text{if } [\mathbf{i}_A^t\mathbf{P}_R](i) = 0 \\ -L[\mathbf{i}_A^t\mathbf{P}_R \oplus \mathbf{i}_B^t\mathbf{P}_L](i) & \text{if } [\mathbf{i}_A^t\mathbf{P}_R](i) = 1. \end{cases}\end{aligned} \tag{7}$$

The receiver collects the LLR of $\mathbf{i}_B^t$ from the demodulator and that of $\mathbf{i}_B^t\mathbf{P}_L$ from (7) and proceeds to process the message using the LDGM decoder for $\mathbf{G}_L = [\mathbf{I} \ \mathbf{P}_L]$. In this manner, all the channel resources available to Node B's transmission are used in decoding $\mathbf{i}_B^t$ at Node A - in contrast to cooperative schemes based on time or power sharing.

Analogous operations are used to decode $\mathbf{i}_A^{t-1}$ at Node B. This approach guarantees a high probability that the partner nodes can decode each other's packets and therefore provide spatial diversity via relaying.

D. Destination Node

The destination node observes signals from two source nodes through fading channels and makes a decision on the packets. Ideally, when the partner nodes are successful in decoding, the soft values available to the destination can be decomposed into two categories:

- The soft values of the packets $\mathbf{i}_A$ and $\mathbf{i}_B$ from each node's direct transmission. No algebraic superposition is applied to these packets. The destination node stores these LLR's and applies them to the message-passing algorithm whenever they appear in the set of parity bits under consideration.
- The parity bits constructed from algebraic superposition. Since each packet is sent twice, once from its source and again as relayed information, the parity bits of any particular packet can be divided into two sets. The first set of parity bits is generated using $\mathbf{P}_L$ and XORed with the previous packet of the partner node encoded with $\mathbf{P}_R$, while the second set consists of the relayed parity bits XORed with the next packet from the partner node. If we collect all the parity bits, they can be regarded as generated by the encoding function

$$\begin{aligned}&\big[\cdots \mathbf{i}_A^{t-1}\mathbf{P}_R \oplus \mathbf{i}_B^{t-1}\mathbf{P}_L, \ \mathbf{i}_B^{t-1}\mathbf{P}_R \oplus \mathbf{i}_A^{t}\mathbf{P}_L, \\ &\quad \mathbf{i}_A^{t}\mathbf{P}_R \oplus \mathbf{i}_B^{t}\mathbf{P}_L, \ \mathbf{i}_B^{t}\mathbf{P}_R \oplus \mathbf{i}_A^{t+1}\mathbf{P}_L, \\ &\quad \mathbf{i}_A^{t+1}\mathbf{P}_R \oplus \mathbf{i}_B^{t+1}\mathbf{P}_L, \ \mathbf{i}_A^{t-1}\mathbf{P}_R \oplus \mathbf{i}_B^{t-1}\mathbf{P}_L \ \cdots\big] \\ &= \big[\cdots \mathbf{i}_A^{t-1} \ \mathbf{i}_B^{t-1} \ \mathbf{i}_A^{t} \ \mathbf{i}_B^{t} \ \mathbf{i}_A^{t+1} \ \mathbf{i}_B^{t+1} \cdots\big] \\ &\cdot \begin{bmatrix} \ddots & \mathbf{P}_R & & & & & \\ & \mathbf{P}_L & \mathbf{P}_R & & & & \\ & & \mathbf{P}_L & \mathbf{P}_R & & & \\ & & & \mathbf{P}_L & \mathbf{P}_R & & \\ & & & & \mathbf{P}_L & \mathbf{P}_R & \\ & & & & & \mathbf{P}_L & \ddots \end{bmatrix}\end{aligned} \tag{8}$$

  All parity bits form a chain structure, with adjacent packets "glued together" by the XOR operation.

The destination node buffers soft values and makes decisions with a fixed maximum delay by truncating the XORed parities into a graph of finite size. Define the buffer size B as the maximum number of received codewords that can be stored and jointly processed at the destination node and the maximum delay D as the maximum number of transmissions between the first time a packet is sent from its own host and the time the destination makes a decision on the packet. Since the relaying of the packet comes after its transmission from its source,

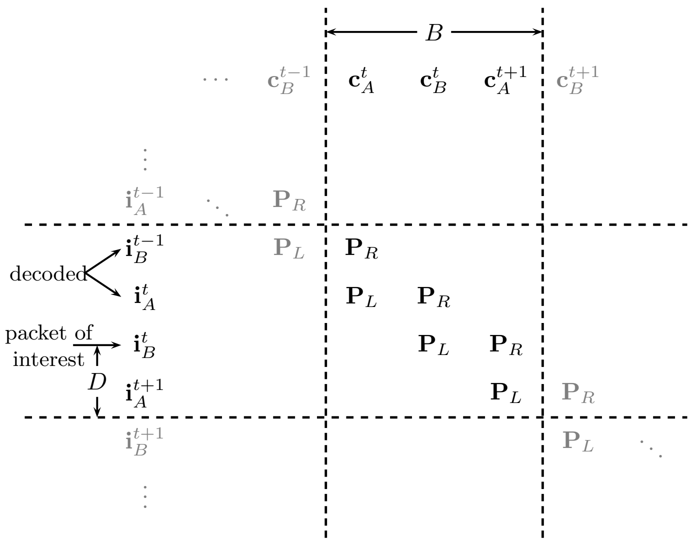

Fig. 2. Illustration of sliding window decoding with $D = 1$ and $B = 3$. The column on the left denotes the related packets while the row on top represents the codeword containing the parity bits. The matrices in the middle represents the structure of the parity matrix. The dashed lines denotes the boundary of the decoding window. Grey vectors and matrices are outside the decoding window and are not involved in the decoding of $\mathbf{i}_B^t$. The window shifts one block to the right and one block down after the current decoding to process $\mathbf{i}_A^{t+1}$. The incoming codeword $\mathbf{c}_B^{t+1}$ then enters the window, and the obsolete codeword $\mathbf{c}_A^t$ is removed from the buffer, with the check degree output on edges corresponding to the top-left $\mathbf{P}_L$ in the window summed over $\mathbf{i}_A^t$ and added to the systematic LLR's of $\mathbf{i}_A^t$.

$D \geq 1$ is required to gain diversity. Moreover, $B \geq D + 1$, because it is possible to buffer the soft values of packets for which decisions have already been made.

Initially, the destination node has an empty buffer and collects soft values as long as they are generated in the cooperative mode. The decoder performs message passing on the graph each time there are more than D codewords in the buffer and outputs one packet to satisfy the maximum delay constraint. Fig. 2 depicts the operation of such a decoder, taking the decoding of $\mathbf{i}_B^t$ as an example. The decoding window is then shifted by one codeword. If the decoder already has B codewords buffered, the least recent codeword (i.e., the parity bits in $\mathbf{c}_A^t$ in Figure 2) is removed. To minimize the loss caused by graph truncation, the check node outputs to the edges corresponding to the $\mathbf{P}_L$ matrix of the removed parity bits – i.e., the edges corresponding to $\mathbf{P}_L$ in the $\mathbf{i}_A^t$ row and $\mathbf{c}_A^t$ column in Figure 2 – are collected in the last iteration and added to the LLR values of the systematic bits.

There are no XORed common parity bits when decoding is unsuccessful on the partner-to-partner link. The destination can process all the buffered soft values and output all the pending packets in the buffer upon the reception of such a noncooperative codeword. The buffer is then cleared and only the soft values of the new codeword are stored, since a relayed version may arrive in the next time slot.

V. Simulation Results

A. LDGM Codes

Consider the construction of $\mathbf{P}_L$ and $\mathbf{P}_R$. Because the partner-to-partner link is protected by the LDGM code $\mathbf{G}_L = [\mathbf{I}\mathbf{P}_L]$ and no diversity is available to the receiver at the partner node, it is desirable to make $\mathbf{G}_L$ a strong code. To guarantee diversity at the destination node, the relayed parity matrix $\mathbf{P}_R$ must have full row rank, because $(\mathbf{i}-\hat{\mathbf{i}})\mathbf{P}_R$ should not be zero for any $\hat{\mathbf{i}} \neq \mathbf{i}$. Moreover, the decoder would rely on the relayed codeword if the direct transmission falls into a deep fade, so it is desirable to have a high row degree in $\mathbf{P}_R$ to achieve a large minimum distance. Conversely, a high column degree usually implies poor low SNR performance due to the nature of the box-plus operation, suggesting that a low column degree $\mathbf{P}_R$ should be chosen. It was found empirically via simulations that a column degree of one for $\mathbf{P}_R$ provides a good trade-off between these two apparently conflicting requirements. For an inner code rate lower that 1/2, the columns and the position of the ones are arranged to ensure full row rank $\mathbf{P}_R$ and avoid cycles of length four in $\begin{bmatrix}\mathbf{P}_R \\ \mathbf{P}_L\end{bmatrix}$.

The sliding window decoder from Section IV can be used with arbitrary buffer size B and maximum delay D. Simulation results indicate that choosing $D = 1$ and $B = 3$ gives a good trade-off between performance and decoding complexity. Taking into consideration that $\mathbf{P}_R$ is a column degree one matrix, the XOR operation introduces almost no increase in complexity. Hence, decoding at the destination node is about three times more complex than at the partner node, whereas in [7] two 64-state convolutional codes with four branches coming into each state must be iteratively decoded [7] when the partner nodes use two 8-state convolutional codes to encode local and relayed information. Note that the decoding complexity of a convolutional code is proportional to the product of the number of states and the number of branches coming into each state. Thus, the XOR operation increases decoding complexity by a factor of 16 in the convolutional code design.

B. Simulation Setting and Results

To accurately represent a slow fading environment and to prevent the destination's decoder from gaining an advantage from more than two channel realizations in one decoding window, the channel gains over the channels A→D and B→D are fixed for 50 time slots and then change independently. The channel gain of the partner-to-partner channel A↔B is fixed for each time slot, again changing independently. The purpose of these assumptions is to isolate and identify the benefit of (spatial) cooperative diversity.

To determine decoding success at the partner nodes, a CRC-12 code is used to encode each packet prior to the outer LDGM code; the code rate is calculated taking into consideration all three codes. The lower bounds on FER in (3) and outage probability in (2) are derived as functions of the average SNR per transmitted (coded) bit. For comparison purpose, the average SNR is normalized by the number of information bits in the plots.

Simulation results for different rates, together with the outage probability and the lower bound on the FER, are shown in Figures 3 and 4. The inner LDGM codes are of rate 1/2 and 1/3, respectively, and both have a column weight six parity matrix $\mathbf{P}_L$ (with row weight six and nine,

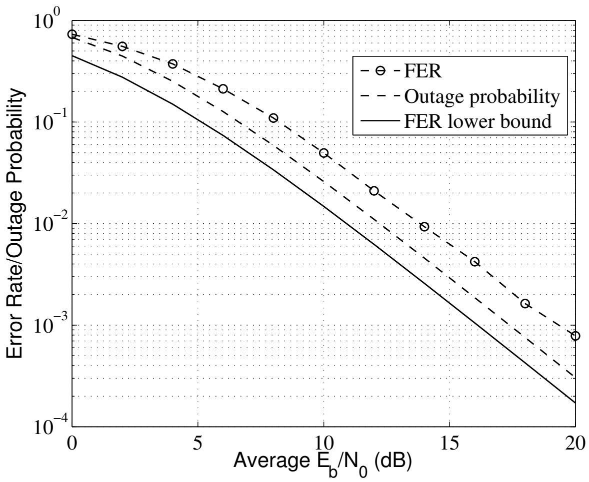

Fig. 3. Frame error rate, outage probability, and a lower bound on FER for a cooperative diversity system operating at an overall rate of 0.464.

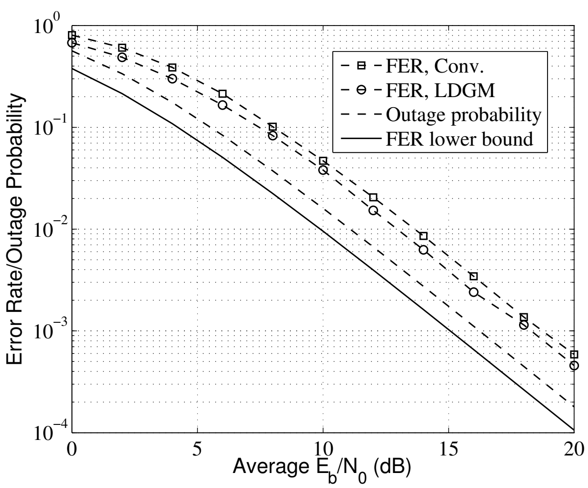

Fig. 4. Frame error rate, outage probability, and a lower bound on FER for a cooperative diversity system operating at an overall rate of 0.304444. A previous convolutional code design with an overall rate of 0.3255 is also given for comparison.

respectively), since it was shown in [12] that row weight six gives good low SNR performance. Note that it would be hard to design a system close to rate 1/2 using an algebraic superposition of convolutional codes because the destination would be required to decode an overall code with rate close to one. Other simulation parameters are summarized in Table I. It is observed that the new LDGM code design is about 2 dB away from the outage probability bound, and a little more than 3 dB away from the simple lower bound on FER. The larger gap to the lower bound is explained by the fact that we assume the partners are intelligent enough to determine whether to transmit one packet or both packets depending on the channel conditions in deriving the lower bound, whereas our cooperative partners always relay information if the the partner-to-partner transmission was decoded successfully. Comparing the new LDGM-based system with a similar-rate system based on convolutional codes, the new design enjoys an SNR gain of about 0.5 dB.

TABLE I
SIMULATION SUMMARY

Inner code rate	1/2	1/3
Codeword size	2000	1800
Outer code rate	0.940	0.933
Outer parity row weight	4	5

VI. CONCLUSIONS

A novel application of LDGM codes to provide cooperative diversity using algebraic superposition has been presented. Compared to a previous design based on convolutional codes and iterative decoding, the graph based decoding proposed here circumvents the need for a large memory BCJR decoder at the destination node, while at the same time providing improved error performance. Future studies will include the use of density evolution techniques to design irregular LDGM codes for such applications.

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