Signal Superposition Coded Cooperative Diversity: Analysis and Optimization

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Abstract— **The performance of a coded cooperative diversity system employing the Euclidean superposition of two BPSK-modulated signals is analyzed. For an example using a convolutional code on block fading channels, the results show excellent agreement with computer simulations. The analysis makes it possible to optimize the power allocation between the local and relay signals numerically, circumventing the need for time consuming Monte Carlo simulations.**

I. Introduction

The concept of cooperative diversity is illustrated in Figure 1. During each time slot, Node A and Node B transmit in turn to deliver their packets to a common destination D. To exploit spatial diversity and thereby enhance reliability on fading channels, each source node transmits both its own "local" packet as well as a "relay" packet that originated at its partner [1]–[10]. In the particular configuration under consideration, depicted in Figure 2, each source node encodes and modulates its local packet and the relay packet separately and then transmits the Euclidean superposition of the two [6], [7]. A key design parameter in such a system is the portion of the power dedicated to transmitting the relay signal, referred to as the *superposition factor*. While too little relay power would result in diminished diversity gain, allocating too much of the power to relaying would reduce the cooperation success rate, and in turn cause poor error performance. In [6], the power allocation was determined empirically via simulation, and the resulting signal superposition diversity scheme was shown to outperform designs based on time multiplexing for all error rate regions of interest.

A thorough performance analysis of coded cooperative diversity systems was presented in [8] for time multiplexed based designs, where the local and relay signals are orthogonal to each other. In the signal superposition cooperative diversity scheme proposed in [6], however, the two signals are added in Euclidean space and no longer exhibit such orthogonality. In this paper, we derive a bound on the performance of the signal superposition cooperative diversity scheme; in particular, we focus on a system in which both local and relay information is channel-encoded (using potentially different codes) and then modulated using binary phase-shift keying (BPSK); when the two BPSK-modulated signals are superimposed, the result is a pulse amplitude modulated (PAM) signal with four levels. The analytical results facilitate numerical optimization of the superposition factor without resorting to computationally intensive simulations.

The paper is organized as follows. We present the system model and establish the notation in Section II. The end-to-end packet error rate performance is bounded in Section III. Numerical results and optimization of the superposition factor are given in Section IV, and we present some conclusions in Section V.

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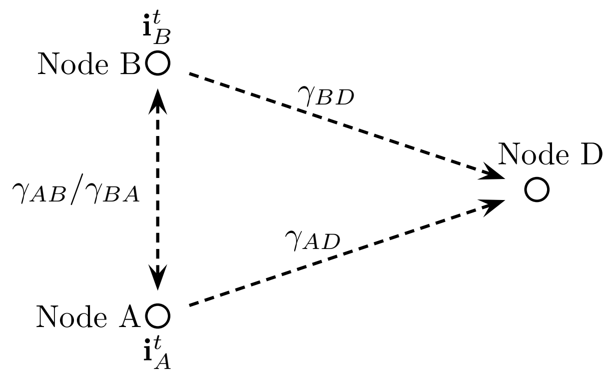

Fig. 1. The system model for cooperative diversity.

II. System Model and Notation

Denote the information packets generated at Node A and B during time slot t as $\mathbf{i}_A^t$ and $\mathbf{i}_B^t$, respectively. In coded cooperation, each packet is encoded (at most) twice – once locally at the originating node and again as a relay packet at the partner node. We denote the two resulting codewords for $\mathbf{i}_A^t$ as $\mathbf{c}_{A,L}^t$ and $\mathbf{c}_{A,R}^t$. Similarly, we define $\mathbf{c}_{B,L}^t$ and $\mathbf{c}_{B,R}^t$ for $\mathbf{i}_B^t$.

The signal superposition cooperative diversity scheme proceeds [6] as follows, focusing on the operations at Node A. (Node B operations are analogous.) Node A first attempts to decode Node B's packet $\mathbf{i}_B^{t-1}$. If successful, Node A modulates the relay codeword $\mathbf{c}_{B,R}^{t-1}$ and the local codeword $\mathbf{c}_{A,L}^t$ separately to generate the relay and the local signals and transmits the Euclidean superposition of the two. The relay signal and the local signal are allocated different powers prior to superposition. Assume Nodes A and B have superposition

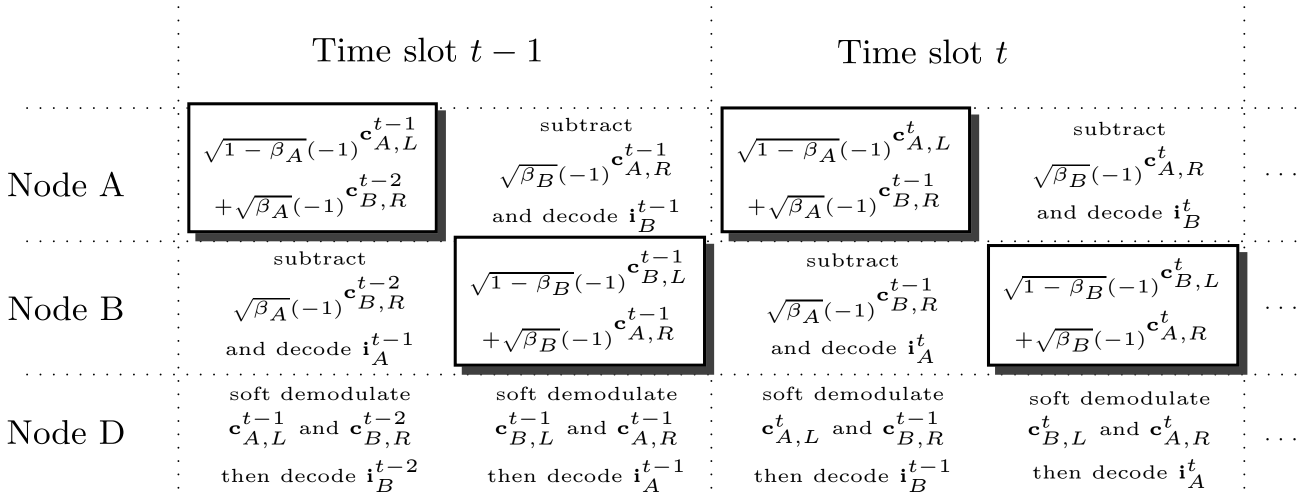

Fig. 2. The operations in the signal superposition cooperative diversity scheme. The boxes depict the transmitted signals.

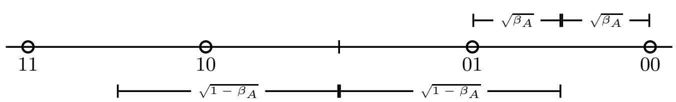

Fig. 3. The superposition modulation employed at Node A. Node B adopts a similar modulation with a possibly different superposition factor β_B. The left bit in the label is the local bit, while the right bit is the relay bit.

factors β_A and β_B, respectively, *i.e.*, Node A uses a fraction β_A of its power for relaying while Node B uses a fraction β_B for relaying. Assuming both the local and relay signals are BPSK modulated, Node A thus transmits the signal [6]

$$y_A^t = \sqrt{1-\beta_A}(-1)^{\mathbf{c}_{A,L}^t} + \sqrt{\beta_A}(-1)^{\mathbf{c}_{B,R}^{t-1}}, \qquad (1)$$

where the signal constellation is normalized and the transmit power is included in the channel gain (see Figure 2, time slot t, Node A). The weighted sum of two BPSK signals in (1) forms an asymmetric 4-PAM constellation due to the different signal powers for local and relayed symbols, as illustrated in Figure 3.

We assume the signal transmitted by Node A is received at both Node B and Node D through two independent block fading channels. Denote the instantaneous signal-to-noise ratio (SNR) and average SNR from Node X to Node Y as γ_{XY} and Γ_{XY}, respectively, where X and Y can be A, B, or D. The pdf of γ_{XY} is denoted as $p_{XY}(\gamma_{XY})$. In time slot t, Node B subtracts the relay signal $\sqrt{\beta_A}(-1)^{\mathbf{c}_{B,R}^{t-1}}$, which it knows, from the received signal and decodes $\mathbf{i}_A^t$ from the noisy, faded version of $\sqrt{1-\beta_A}(-1)^{\mathbf{c}_{A,L}^t}$. Node D observes an asymmetric 4-PAM signal and employs a soft demodulator to calculate the log-likelihood ratios (LLRs) of $\mathbf{c}_{A,L}^t$ and $\mathbf{c}_{B,R}^{t-1}$. Its decision on $\mathbf{i}_B^{t-1}$ can be made using the LLRs for $\mathbf{c}_{B,R}^{t-1}$ and the previously received $\mathbf{c}_{B,L}^{t-1}$ using a decoder for the low rate codeword $[\mathbf{c}_{B,L}^{t-1}\ \mathbf{c}_{B,R}^{t-1}]$, whereas the LLRs for $\mathbf{c}_{A,L}^t$ are buffered. Figure 2 describes the operations at Nodes A, B, and D in time slots $t-1$ and t for the signal superposition cooperative system. For simplicity, we assume the receivers know the channel and coherent demodulation is available. If a source node fails to decode its partner's packet or initiates the cooperation, it BPSK-modulates only its own packet and transmits the result using full power. The paper considers convolutionally encoded packets; moreover, the code from [4], which was based on convolutional code design techniques for block fading channels, is used in the numerical example.

III. Performance Analysis

Noting the symmetry between Node A and Node B, we focus on the error probability of packets originating at Node A, without loss of generality. Following the analysis in [8], we distinguish four possible combinations of local and relay signals for which the destination node must make a decision:

- I Node A transmits a superposition of local and relay signals; Node B is able to decode and relay A's packet.
- II Node A fails to decode B's packet and transmits only a local signal. Node B is able to decode and relay A's packet.
- III Node A transmits a superposition of local and relay signals; Node B fails to decode A and offers no help.
- IV Nodes A and B transmit only signals generated by local packets.

In this section, we calculate the probability of occurrence $\mathbb{P}_Z$ and the conditional packet error probabilities $\mathbb{P}_P(e|Z)$ for each of the above-mentioned four cases. The end-to-end packet error probability $\mathbb{P}_P(e)$ can then be bounded as a weighted sum of the conditional error probabilities [8], *i.e.*,

$$\mathbb{P}_P(e) \le \sum_{Z\in\{I,II,III,IV\}} \mathbb{P}_Z \mathbb{P}_P(e|Z). \qquad (2)$$

We calculate $\mathbb{P}_Z$ for $Z \in \{I, II, III, IV\}$ in Section III-A and derive the expressions for $\mathbb{P}_P(e|Z)$ in Section III-C.

A. Cooperation Success Rate

Node A will transmit either a superimposed signal (cooperative mode) or a signal modulated with only local bits

$$\begin{aligned} m_1(0|0) &= -m_1(1|1) = \mathbb{E}[\tilde{\ell}(r)|s=0, i=0] \\ &= \frac{4}{N_0}\left[a^2 - b^2 Q\left(\sqrt{\frac{2b^2}{N_0}}\right) + b(2a+b)Q\left(\sqrt{\frac{2}{N_0}}(2a+b)\right)\right] + \frac{2b}{\sqrt{\pi N_0}}\left(e^{-\frac{b^2}{N_0}} - e^{-\frac{(2a+b)^2}{N_0}}\right) \end{aligned} \quad (10a)$$

$$\begin{aligned} m_1(0|1) &= -m_1(1|0) = \mathbb{E}[\tilde{\ell}(r)|s=0, i=1] \\ &= \frac{4}{N_0}\left[a^2 + b(b-2a)Q\left(\sqrt{\frac{2}{N_0}}(b-2a)\right) - b^2 Q\left(\sqrt{\frac{2b^2}{N_0}}\right)\right] + \frac{2b}{\sqrt{\pi N_0}}\left(e^{-\frac{b^2}{N_0}} - e^{-\frac{(2a-b)^2}{N_0}}\right) \end{aligned} \quad (10b)$$

$$\begin{aligned} m_2(0|0) &= m_2(1|1) = \mathbb{E}[\tilde{\ell}^2(r)|s=0, i=0] \\ &= \frac{8a^2}{N_0^2}\left(2a^2+N_0\right) + \frac{8b}{\sqrt{\pi N_0^3}}\left[(b^2-2a^2-2ab)e^{-\frac{(2a+b)^2}{N_0}} + (2a^2-b^2)e^{-\frac{b^2}{N_0}}\right] \\ &+ \frac{8b}{N_0^2}(8a^3+12a^2b-2b^3+2aN_0-bN_0)Q\left(\sqrt{\frac{2}{N_0}}(2a+b)\right) - \frac{8b}{N_0^2}(4a^2b+2aN_0-2b^3-bN_0)Q\left(\sqrt{\frac{2b^2}{N_0}}\right) \end{aligned} \quad (10c)$$

$$\begin{aligned} m_2(0|1) &= m_2(1|0) = \mathbb{E}[\tilde{\ell}^2(r)|s=0, i=1] \\ &= \frac{8b}{\sqrt{\pi N_0^3}}\left[(2a^2-4ab+b^2)e^{-\frac{b^2}{N_0}} - (2a^2-2ab+b^2)e^{-\frac{(2a-b)^2}{N_0}}\right] - \frac{8b}{N_0^2}(4a^2b+2b^3-8ab^2+bN_0-2aN_0)Q\left(\sqrt{\frac{2b^2}{N_0}}\right) \\ &- \frac{8b}{N_0^2}(8a^3-12a^2b+8ab^2-2b^3+2aN_0-bN_0)Q\left(\sqrt{\frac{2}{N_0}}(b-2a)\right) + \frac{8a^2}{N_0^2}(2a^2+N_0) \end{aligned} \quad (10d)$$

(noncooperative mode). Since Node B can subtract the relay portion from the received signal prior to decoding in the former case, it can decode A's message with partial power $(1-\beta_A)$ when signal superposition is used and with full power when A is not relaying. Let η_A and η_B be the probabilities that Nodes A and B, respectively, are sending superimposed signals. Because superposition is used at Node A if and only if Node B's packet was successfully decoded at Node A (and vice versa), we have the relations

$$1-\eta_A = \eta_B \cdot P^S_{B\to A} + (1-\eta_B)\cdot P^N_{B\to A} \quad (3a)$$
$$1-\eta_B = \eta_A \cdot P^S_{A\to B} + (1-\eta_A)\cdot P^N_{A\to B}, \quad (3b)$$

where $P^S_{B\to A}$ and $P^N_{B\to A}$ are the packet (block) error probabilities of Node B's packets at Node A with superposition (partial power $(1-\beta_B)$) and without superposition (full power), respectively. Similarly $P^S_{A\to B}$ and $P^N_{A\to B}$ are the packet error probabilities of Node A's packets at Node B with SNRs $\gamma_{AB}(1-\beta_A)$ and γ_{AB}, respectively. Solving these equations for η_A and η_B, we obtain

$$\eta_A = \frac{\left(1-P^N_{B\to A}\right) - \left(1-P^N_{A\to B}\right)\left(P^S_{B\to A}-P^N_{B\to A}\right)}{1-\left(P^S_{B\to A}-P^N_{B\to A}\right)\left(P^S_{A\to B}-P^N_{A\to B}\right)} \quad (4a)$$

$$\eta_B = \frac{\left(1-P^N_{A\to B}\right) - \left(1-P^N_{B\to A}\right)\left(P^S_{A\to B}-P^N_{A\to B}\right)}{1-\left(P^S_{B\to A}-P^N_{B\to A}\right)\left(P^S_{A\to B}-P^N_{A\to B}\right)}. \quad (4b)$$

As a special case, when the channels $A \to B$ and $B \to A$ have the same statistics, and the superposition factors at A and B are also identical, then $P^N_{A\to B} = P^N_{B\to A}$, $P^S_{A\to B} = P^S_{B\to A}$, and (4) can be simplified as

$$\eta_A = \eta_B = \frac{1-P^N_{A\to B}}{1+P^S_{A\to B}-P^N_{A\to B}}. \quad (5)$$

The probability of the four cases described above can now be expressed as

$$\mathbb{P}_I = \eta_A \cdot \left(1-P^S_{A\to B}\right) \quad (6a)$$
$$\mathbb{P}_{II} = (1-\eta_A)\cdot\left(1-P^N_{A\to B}\right) \quad (6b)$$
$$\mathbb{P}_{III} = \eta_A \cdot P^S_{A\to B} \quad (6c)$$
$$\mathbb{P}_{IV} = (1-\eta_A)\cdot P^N_{A\to B}. \quad (6d)$$

For independent inter-user channels, the error probabilities $P^S_{B\to A}$, $P^N_{B\to A}$, $P^S_{A\to B}$, and $P^N_{A\to B}$ can be evaluated for block fading channels as in [11]. For inter-user channels where the channel gains from Node A to Node B and from Node B to Node A are always the same, *i.e.*, $\gamma_{AB} = \gamma_{BA}$, the error probabilities at Nodes A and B are correlated. However, conditioned on the instantaneous SNR γ_{AB}, the probabilities become independent. In this case, the probabilities $P^S_{B\to A}$, $P^N_{B\to A}$, $P^S_{A\to B}$, and $P^N_{A\to B}$ should be evaluated for AWGN channels with SNRs $\gamma_{AB}(1-\beta_B)$, γ_{AB}, $\gamma_{AB}(1-\beta_A)$, and γ_{AB}, respectively, and then the expressions in (6) should be averaged over the distribution for γ_{AB} to take the fading into account. The four case probabilities for Node B can be calculated in a similar manner.

B. The Soft Demodulator

When the two source nodes cooperate, the destination node must calculate the LLR of each bit from a received

$$P_I(d|\gamma_{AD},\gamma_{BD}) = \sum_{j=0}^{d_1}\sum_{k=0}^{d_2} 2^{-(d_1+d_2)}\binom{d_1}{j}\binom{d_2}{k} Q\Bigg(\Big(2j\rho_1\left(\gamma_{AD}(1-\beta_A),\gamma_{AD}\beta_A\right)+2(d_1-j)\rho_2\left(\gamma_{AD}(1-\beta_A),\gamma_{AD}\beta_A\right) + 2k\rho_1\left(\gamma_{BD}\beta_B,\gamma_{BD}(1-\beta_B)\right)+2(d_2-k)\rho_2\left(\gamma_{BD}\beta_B,\gamma_{BD}(1-\beta_B)\right)\Big)^{\frac{1}{2}}\Bigg) \tag{12a}$$

$$P_{II}(d|\gamma_{AD},\gamma_{BD}) = \sum_{k=0}^{d_2} 2^{-d_2}\binom{d_2}{k} Q\left(\sqrt{2d_1\gamma_{AD}+2k\rho_1\left(\gamma_{BD}\beta_B,\gamma_{BD}(1-\beta_B)\right)+2(d_2-k)\rho_2\left(\gamma_{BD}\beta_B,\gamma_{BD}(1-\beta_B)\right)}\right) \tag{12b}$$

$$P_{III}(d_1|\gamma_{AD}) = \sum_{j=0}^{d_1} 2^{-d_1}\binom{d_1}{j} Q\left(\sqrt{2j\rho_1\left(\gamma_{AD}(1-\beta_A),\gamma_{AD}\beta_A\right)+2(d_1-j)\rho_2\left(\gamma_{AD}(1-\beta_A),\gamma_{AD}\beta_A\right)}\right) \tag{12c}$$

$$P_{IV}(d_1|\gamma_{AD}) = Q\left(\sqrt{2d_1\gamma_{AD}}\right) \tag{12d}$$

$$\mathbb{P}_P(e|Z) \leq \begin{cases} 1-\int_0^\infty\int_0^\infty\left(1-\min\left[1,\sum_{d=d_f}^{\infty} a(d)P_Z(d|\gamma_{AD},\gamma_{BD})\right]\right)^K p_{AD}(\gamma_{AD})p_{BD}(\gamma_{BD})\mathrm{d}\gamma_{AD}\mathrm{d}\gamma_{BD} & Z=I,II \\ 1-\int_0^\infty\left(1-\min\left[1,\sum_{d_1=d_f'}^{\infty} a'(d_1)P_Z(d_1|\gamma_{AD})\right]\right)^K p_{AD}(\gamma_{AD})\mathrm{d}\gamma_{AD} & Z=III,IV \end{cases} \tag{13}$$

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superimposed signal of the form

$$r = a(-1)^s + b(-1)^i + n, \tag{7}$$

where s is the desired bit (either relayed or local), i is the superimposed interfering bit, n is zero mean Gaussian noise with variance $N_0/2$, and a and b are the amplitudes of the desired signal and the interfering signal, respectively, whose values are determined by the channel gain and the superposition factor β_A or β_B.

To provide a meaningful analysis for such a system, a model for the soft demodulator is required. The ideal soft demodulator output for bit s is

$$\ell(r) = \log\frac{\exp(-\frac{(r-a+b)^2}{N_0})+\exp(-\frac{(r-a-b)^2}{N_0})}{\exp(-\frac{(r+a+b)^2}{N_0})+\exp(-\frac{(r+a-b)^2}{N_0})}. \tag{8}$$

A good approximation that also simplifies the analysis can be obtained by taking only the dominant terms in the numerator and denominator – i.e., the Jacobian logarithm:

$$\begin{aligned} \tilde{\ell}(r) &= \frac{1}{N_0}\Big\{\min[(r+a+b)^2,(r+a-b)^2] \\ &\quad -\min[(r-a+b)^2,(r-a-b)^2]\Big\} \\ &= \begin{cases} \frac{4a}{N_0}(r+b) & r<-a \\ \frac{4(a-b)}{N_0}r & -a\leq r\leq a \\ \frac{4a}{N_0}(r-b) & r>a. \end{cases} \end{aligned} \tag{9}$$

The mean $m_1(s|i)$ and second moment $m_2(s|i)$ of $\tilde{\ell}(r)$ can be calculated as shown in (10), where the channel observation r in (7) is a Gaussian random variable for a given combination of s and i. Since the LLRs at the output of the demodulator represent the soft-inputs of the channel decoder, we now define two equivalent SNRs (defined as $\frac{2m_1^2}{m_2-m_1^2}$)[1]

$$\gamma_{eq,1} = \frac{2m_1^2(0|0)}{m_2(0|0)-m_1^2(0|0)} \triangleq \rho_1\left(\frac{a^2}{N_0},\frac{b^2}{N_0}\right) \tag{11a}$$

$$\gamma_{eq,2} = \frac{2m_1^2(0|1)}{m_2(0|1)-m_1^2(0|1)} \triangleq \rho_2\left(\frac{a^2}{N_0},\frac{b^2}{N_0}\right), \tag{11b}$$

where the quantities $\frac{a^2}{N_0}$ and $\frac{b^2}{N_0}$ can be viewed as the SNRs of the desired bit and the superimposed interfering bit, respectively. The functions $\rho_1(\cdot,\cdot)$ and $\rho_2(\cdot,\cdot)$ characterize how the equivalent SNR depends on the SNRs of the desired bit and the superimposed bit, given that the two bits have the same or opposite values.

C. Conditional Error Probabilities

In the absence of cooperation, each source node transmits its own BPSK-modulated symbols, and thus the LLRs at the demodulator output have a Gaussian distribution. When superposition is applied, however, both the exact and simplified approximate LLRs in (8) and (9) are no longer Gaussian-distributed. We adopt a Gaussian approximation for these LLRs for two reasons. First, the LLRs are piecewise linear functions of Gaussian random variables, as seen in (9), and hence have Gaussian-like tails. Also, an error event in a coded system involves several LLRs, and thus a central limit theorem argument can be invoked.

Assuming that linear codes are used, we only need to consider the pairwise error probability between the all zero codeword and an erroneous codeword with Hamming weight d to obtain a union bound. Note that, for the LLR of a

[1]The factor of two comes from the fact that SNR is defined as E_s/N_0 and the noise variance is $\frac{N_0}{2}$.

superimposed bit, the Gaussian approximation is conditioned on the value of the interfering bit; thus, the equivalent SNR in (11) depends on whether the two bits are equal or not. Hence, for an erroneous codeword with Hamming weight d', there are $2^{d'}$ possible combinations of the two equivalent SNRs $\rho_1(\cdot,\cdot)$ and $\rho_2(\cdot,\cdot)$, and the multiplicity of combinations where k out of d' bits have value one is given by $\binom{d'}{k}$. It follows that we can formulate the pairwise error probabilities for each superposition case as shown in (12), where d_1 is the Hamming weight contributed to the erroneous codeword by the local codeword $\mathbf{c}^t_{A,L}$ generated at Node A, d_2 is the Hamming weight contributed to the same erroneous codeword by the relay codeword $\mathbf{c}^t_{A,R}$ regenerated at Node B, and $d_1+d_2=d$ is the Hamming weight of the erroneous codeword in the overall low rate code that combines $\mathbf{c}^t_{A,L}$ and $\mathbf{c}^t_{A,R}$.

For each case except case IV, the pairwise probability is obtained as a weighted sum of Q-functions over all possible combinations of the equivalent SNRs, where the weights correspond to the total probability of each combination. The arguments of the Q-functions depend on the sum of the SNRs associated with those bits that contribute Hamming weight to the erroneous path. For the LLRs corresponding to signal superposition, the equivalent SNR for the transmission of local information from Node A can be either $\rho_1(\gamma_{AD}(1-\beta_A), \gamma_{AD}\beta_A)$ or $\rho_2(\gamma_{AD}(1-\beta_A), \gamma_{AD}\beta_A)$. Similarly, the equivalent SNR for the transmission of relay information from Node B is either $\rho_1(\gamma_{BD}\beta_B, \gamma_{BD}(1-\beta_B))$ or $\rho_2(\gamma_{BD}\beta_B, \gamma_{BD}(1-\beta_B))$. Then the conditional packet error probability can be evaluated, using the limit before averaging technique described in [11], as shown in (13), where d_f and d'_f are (respectively) the free distances of the overall low rate convolutional code that combines $\mathbf{c}^t_{A,L}$ and $\mathbf{c}^t_{A,R}$ and the high rate local convolutional code. Additionally, $a(d)$ represents the multiplicity of combined codewords with Hamming weight $d = d_1 + d_2$, and $a'(d_1)$ is the multiplicity of local codewords with Hamming weight d_1. Finally, K is the number of information bits in one packet. (The conditional bit error probabilities can also be bounded using the pairwise error probabilities given in (12) in a straightforward manner [8], [11].)

IV. Numerical Results and Optimization

As an example, we use the cooperative convolutional code design derived from a block fading channel perspective in [4]. The local codewords $\mathbf{c}^t_{A,L}$ and $\mathbf{c}^t_{B,L}$ are generated by $[15,17]_8$ and the relay codewords $\mathbf{c}^t_{A,R}$ and $\mathbf{c}^t_{B,R}$ are generated by $[13,\ 15]_8$, both rate $1/2$ codes. The destination node uses the decoder corresponding to the rate $1/4$ generator $[15,\ 17,\ 13,\ 15]_8$ to exploit diversity when a relayed signal is available. In evaluating the union bound, terms corresponding to distance greater than $d_f + 15$ are dropped. The Hamming distances and their corresponding multiplicities can be efficiently calculated using a slightly modified BEAST algorithm [12], and they are listed in Table I ($d_f = 13$ for this example). All channels, including the two channels between Nodes A and B, are assumed to be independent and identically distributed with block Rayleigh fading, and the average channel SNRs are assumed to be the same, *i.e.*, $\Gamma_{AB} = \Gamma_{BA} = \Gamma_{AD} = \Gamma_{BD}$. The packets contain 500 bits each, and an extra 12 CRC bits are appended prior to convolutional encoding to detect decoding failures at the partner nodes. (Since the CRC is neglected in our analysis, we do not distinguish between information bits and CRC bits and use $500 + 12 = 512$ bit packets in the numerical calculation.)

The numerical results, plotted in Figure 4 using superposition factors $\beta = \beta_A = \beta_B = 10\%$ and $\beta = \beta_A = \beta_B = 35\%$, show very good agreement between the derived bound and the simulation results, despite employing a union bound and a Gaussian approximation on the LLRs. For example, at a packet error probability of 10^{-2}, the numerical prediction is only about 0.3 dB away from the simulation curve, and the same level of agreement is consistent over the entire FER range of interest. (This consistency is due to the averaging effect of the fading distribution.)

It is evident from Figure 4 that the superposition factor β plays an important role in the error performance, and hence β should be chosen to minimize the packet error probability bound. The effect of $\beta = \beta_A = \beta_B$ on the packet error probability is shown in Figure 5 for average SNR's of 20 dB and 30 dB.

In Figure 6, we plot the optimal $\beta = \beta_A = \beta_B$ as a function of the average SNR. The optimality is in the sense of minimizing the packet error probability bound and the optimal superposition factors are found by searching over the interval $(0, 0.5)$. The optimum β is found to be zero in the low SNR region. This degenerates the signal superposition system into a noncooperative system, which performs better than the signal superposition cooperative system in the low SNR region. In the medium SNR regime, protecting the vulnerable inter-user communication is a priority to achieve diversity, and thus a low β is preferred. In the high SNR regime, on the other hand, the system is cooperative most of the time and it is beneficial to use a larger β. The optimum β for SNR's above 10 dB falls into the 10% to 15% range found empirically in [6] for a repetition based configuration. It is also observed from Figure 6 that the optimum β approaches a constant for high SNR ($\beta = 0.127$ for SNR larger than 30 dB in our example), where the optimum β depends on the given channel model and convolutional code. Results might differ for different channel models and different code generators.

V. Conclusion

We have developed an analytical performance bound for a coded cooperative deversity signal superposition system under rather general assumptions. Numerical calculations based on the bound track computer simulation results very closely. Minimizing the bound shows the dependency of the optimum superposition factor on the operating SNR. The analysis provides a useful tool for performance prediction and design parameter optimization for signal superposition based cooperative diversity systems. It also applies to two dimensional signals with asymmetric 4-PAM on in-phase and quadrature components.

TABLE I

DISTANCE SPECTRUM OF CONVOLUTIONAL CODE $[15,\ 17,\ 13,\ 15]_8$

d_1	d_2	$d = d_1 + d_2$	$a(d)$
7	6	13	2
6	8	14	1
8	8	16	3
7	10	17	1
10	8	18	4
9	10	19	8
8	12	20	2
12	8	20	2
11	10	21	15
10	12	22	16
9	14	23	3
13	10	23	15
12	12	24	45
11	14	25	32
15	10	25	8
10	16	26	5
14	12	26	68
13	14	27	117
17	10	27	2

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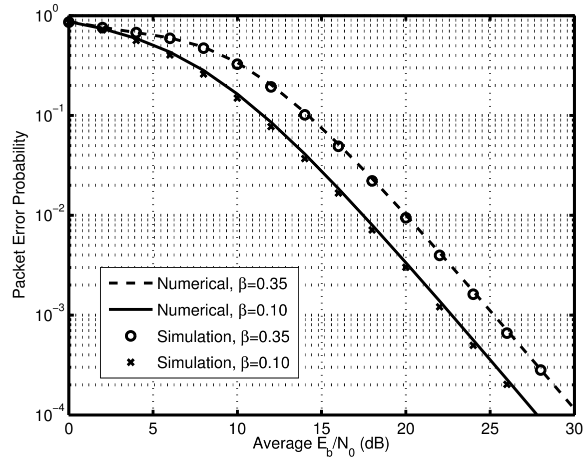

Fig. 4. Packet error probability for Rayleigh fading channels with independent inter-user channels and $\beta = \beta_A = \beta_B$.

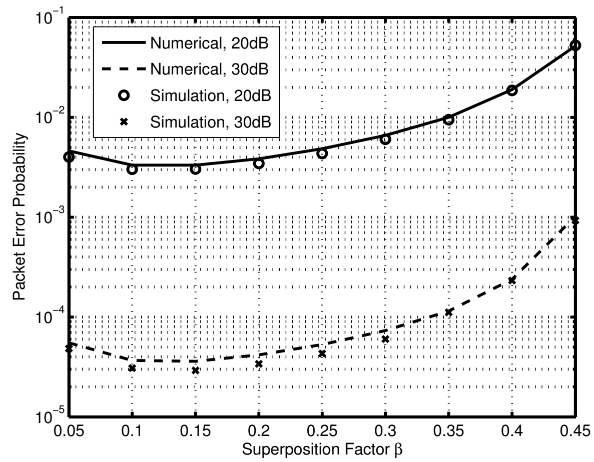

Fig. 5. Packet error probabilities as functions of the superposition factor β at an average E_b/N_0 of 20 dB and 30 dB.

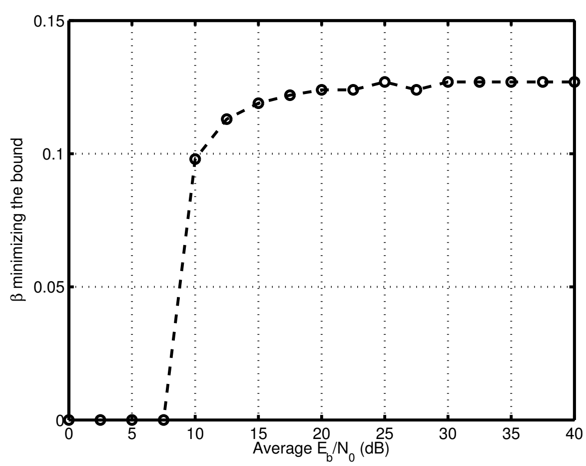

Fig. 6. Values of $\beta = \beta_A = \beta_B$ that minimize the analytical bound at different SNR's.