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Near-perfect-reconstruction low-complexity two-band IIR/FIR QMF banks with FIR phase-compensation filters

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Abstract

In this paper, we present a novel approach for the design of near-perfect-reconstruction mixed allpass-/FIR-based two-band quadrature-mirror filter banks. The proposed design method is carried out in the polyphase domain, where FIR filters are employed for compensating the non-linear phase introduced by the allpass filters. In contrast to previous approaches in literature the FIR phase-compensation filters can be designed very efficiently using analytical expressions. Furthermore, starting from a generalized two-band structure, we introduce three special cases with different properties which are based on the same design principle. In all systems the remaining amplitude and phase distortions are controllable and can be made arbitrarily small at the expense of additional system delay. Simultaneously, aliasing can be minimized or completely canceled if further delay can be tolerated.

Keywords: Low-complexity QMF bank; Allpass polyphase components; FIR phase-compensation filter

1. Introduction

Critically subsampled two-band quadrature-mirror filter banks (QMF banks) are utilized in a variety of applications, for example in audio and image compression, where most design techniques are dealing with FIR analysis and synthesis filters. On the other hand, employing IIR filters leads to very efficient filter bank realizations, which can be designed such that they have stable and causal subband filters and are free from both aliasing and amplitude distortion. However, phase distortion usually remains in the reconstructed signal.

One solution for canceling the phase distortion and obtaining a perfect reconstruction (PR) system can be achieved by using anticausal

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filtering [1] and employing a double buffering scheme as in [2,3] for the processing of infinite-length signals. However, this method leads to a computationally costly synthesis filter bank, which requires segmentation and time-reversal of the subband signals. Other approaches for designing PR filter banks with causal and stable IIR filters are based on lifting-like factorizations [4,5], which have the drawback that selective subband frequency responses are difficult to obtain due to the PR constraints being structurally imposed on the design process. In order to design low-complexity two-band near-PR systems, recent publications propose the use of very efficient allpass-based analysis filter banks [6–8] in combination with a stable FIR- or mixed IIR-/FIR-based synthesis [9–12]. The proposed work also falls into this category.

In this paper, we present novel near-PR design techniques for a class of mixed allpass-/FIR-based two-band QMF filter banks. For example, this also includes the design of a near-PR analysis–synthesis system with a low-complexity analysis or synthesis bank solely based on allpass polyphase components. All design approaches are based on FIR approximations for inverse first-order allpasses, where the remaining error only depends on the allpass parameters and the order of the FIR approximation. In contrast to similar approaches in literature, where the synthesis filters are obtained by solving an approximation problem for the ideal overall system response using linear programming [9], an FIR all-pole filter approximation based on a deconvolution approach [10,11], or a weighted least-square design method [12], here the FIR approximation filters are designed via simple analytical expressions. Employing this phase compensation technique FIR synthesis filters are obtained in such a way that the non-linear phase introduced by the allpass filters is compensated and furthermore, aliasing and linear distortions are strongly reduced or even canceled in the case of aliasing distortions. Furthermore, the presented design methods have the interesting property that an increase of the reconstruction error at the output of the synthesis filter bank can be traded for a reduction of the overall system delay and vice versa.

The outline of the paper is as follows. In Section 2 we briefly review the basic relations for the allpass-based two-band critically subsampled filter bank. The phase-compensation approach utilized in the sequel of the paper is discussed in Section 3. Section 4 then proposes a general two-band mixed IIR/FIR analysis–synthesis system and discusses three special cases emerging from this general structure. Finally, design examples are given in Section 5.

2. Classical allpass-based two-band QMF banks

The classical two-band critically subsampled filter bank structure [13,8] is depicted in Fig. 1, where the $H_k(z)$ denote the analysis subband filters and the $G_k(z)$ the synthesis subband filters for $k = 0, 1$, respectively.

The input–output relation for this system can be given as

$$\hat{X}(z) = X(z)\,T_{\text{lin}}(z) + X(-z)\,T_{\text{alias}}(z) \tag{1}$$

with the linear distortion transfer function

$$T_{\text{lin}}(z) = \tfrac{1}{2}[H_0(z)\,G_0(z) + H_1(z)\,G_1(z)]. \tag{2}$$

If $T_{\text{lin}}(z) \overset{!}{=} c \cdot z^{-D}$ with $c \in \mathbb{R}$ and D denoting the overall system delay, the filter bank has no amplitude and phase distortion. The transfer function $T_{\text{alias}}(z)$ in (1) corresponds to the aliasing distortion and can be written as

$$T_{\text{alias}}(z) = \tfrac{1}{2}[H_0(-z)\,G_0(z) + H_1(-z)\,G_1(z)]. \tag{3}$$

By choosing the synthesis filters according to

$$G_0(z) = H_1(-z) \quad \text{and} \quad G_1(z) = -H_0(-z), \tag{4}$$

aliasing can be completely canceled. Furthermore, when the analysis filters are related as $H_0(z) = H_1(-z)$, they are referred to as quadrature-mirror

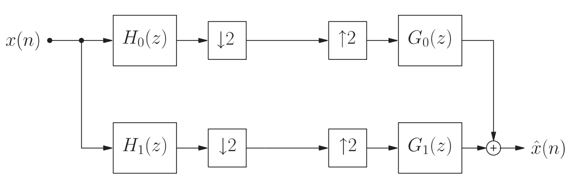

Fig. 1. Two-band critically subsampled filter bank structure.

filters. A very efficient way of representing the filter bank can be obtained by using polyphase components.

In the following, we consider QMF banks where (4) is satisfied and where the polyphase components consist of stable and causal allpass transfer functions $A_i(z)$, $i = 0, 1$, of first order according to

$$A_i(z) = \frac{z^{-1} + a_i}{1 + a_i z^{-1}}, \quad 0 < |a_i| < 1, \quad a_i \in \mathbb{R}. \tag{5}$$

The analysis and synthesis filters then are IIR filters, have the power-symmetric property and can be given in a more compact matrix notation as follows:

$$\begin{bmatrix} H_0(z) \\ H_1(z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} A_0(z^2) \\ z^{-1} A_1(z^2) \end{bmatrix}, \tag{6}$$

$$[G_0(z)\, G_1(z)] = \frac{1}{2} \left[z^{-1} A_1(z^2)\, A_0(z^2) \right] \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \tag{7}$$

For the sake of simplicity we restrict ourselves to classical power-symmetric elliptical subband analysis filters which can be realized with allpass-based polyphase components (see, e.g., [6,7]). By using the fact that these filters have their poles on the imaginary axis of the z-plane, it can be shown that this class of filters can be derived from first-order real-valued allpass polyphase components (5) having only positive parameters a_i [8]. Higher-order filters can then be obtained by simply concatenating the first-order allpasses in (5).

By inserting the subband filters from (6) and (7) into (2) we obtain the linear distortion transfer function

$$T_{\text{lin}}(z) = \tfrac{1}{2} z^{-1} A_0(z^2)\, A_1(z^2), \tag{8}$$

which is now an allpass transfer function. Thus, amplitude distortion is eliminated and aliasing distortion is canceled since (4) is satisfied, but phase distortion persists and depends on the phase responses of the allpass filters $A_0(z)$ and $A_1(z)$. A straightforward solution to eliminate the phase distortions in (8) would be to use non-stable inverse allpasses $A_i(z^{-2})$ instead of the $A_i(z^2)$ for the synthesis filters in (7). However, an implementation of the synthesis bank is problematic due to the non-causal synthesis polyphase components.[2] In the next section we propose as one possible solution a new closed-form FIR approximation $F_i(z)$ for a non-causal allpass $A_i(z^{-1})$, where stability problems do not arise.

3. Compensation of the phase distortion

We present a novel analytical expression for an FIR filter with the transfer function $F_i(z)$, $i = 0, 1$, which approximately compensates the phase distortion caused by a first-order allpass $A_i(z)$ up to a certain error. Thus, $F_i(z)$ may also be interpreted as a causal approximation for the non-stable inverse allpass $A_i(z^{-1})$. To this end, let us consider the relation

$$A_i(z) F_i(z) = z^{-d_i} - \varepsilon(a_i, d_i) = z^{-d_i} - (-1)^{d_i} a_i^{d_i}, \quad i = 0, 1, \quad d_i \in \mathbb{R}, \tag{9}$$

where the phase compensation error is expressed by the term $\varepsilon(a_i, d_i) = (-1)^{d_i} a_i^{d_i}$. When the polynomial factorization relation

$$z^{-d_i} - (-1)^{d_i} \cdot a_i^{d_i} = (z^{-1} + a_i) \sum_{k=0}^{d_i - 1} (-1)^k \cdot a_i^k z^{-(d_i - 1 - k)} \tag{10}$$

is inserted on the right-hand side of (9) and the first-order allpass transfer function $A_i(z)$ in (5) on the left-hand side, we obtain the desired closed-form expression for the phase compensation filter $F_i(z)$ as

$$F_i(z) = (1 + a_i z^{-1}) \sum_{k=0}^{d_i - 1} (-1)^k a_i^k z^{-(d_i - 1 - k)} \tag{11}$$

$$= (-1)^{d_i - 1} a_i^{d_i - 1} + \sum_{k=1}^{d_i - 1} (-1)^{d_i - 1 - k} a_i^{d_i - 1 - k} \cdot (1 - a_i^2) z^{-k} + a_i z^{-d_i}. \tag{12}$$

[2] The synthesis bank may be implemented by applying time-reversed subband sequences to the non-causal synthesis filters [1], where this solution is only suitable for short input sequences. Additionally, for achieving PR state information for the analysis filters has to be transmitted to the synthesis side besides the subband data, which increases the overall data rate.

Since $0<|a_i|<1$, we can select the order d_i of the filter $F_i(z)$ such that the term $\varepsilon(a_i, d_i)$ in (9) can be made arbitrarily small at the expense of additional delay. Then (9) can be approximated as $A_i(z)F_i(z) \approx z^{-d_i}$.

Note that the equalization problem in (9) is similar to the all-pole FIR approximation problem stated in [10,11]. In the latter approach a FIR phase compensation filter is designed via the pseudo-inverse of a convolution matrix obtained from the coefficients of the truncated allpass impulse response, where depending on the choice of the design parameters similar filters as from (12) may be obtained. However, the design via the approach proposed in [10,11] has a large design complexity of order $O(d_i^3)$, which makes the design of longer FIR filters (which are required when, e.g., $A_i(z)$ has zeros close to the unit circle [11]) quite costly. In contrast, using the closed-form relation in (12) the design complexity is linear in the filter order d_i, which for example allows to change the filter bank parameters adaptively in different blocks of the input signal also for small approximation errors $\varepsilon(a_i, d_i)$ with large d_i.

An example for the proposed phase compensation approach is given in Fig. 2, where the individual group delays for the first-order allpass $A_0(z)$ with $a_0 = 0.65$, the corresponding phase-compensation filter $F_0(z)$ of order $d_0 = 20$ from (12), and the product filter $A_0(z)F_0(z)$ from (9) are depicted.

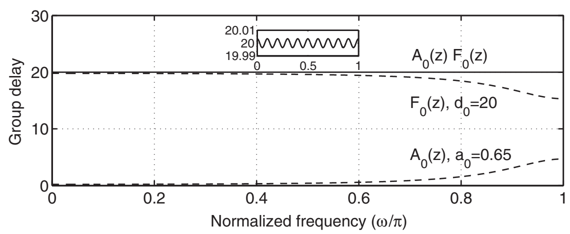

Fig. 2. Group delays for the first-order allpass $A_0(z)$ with $a_0 = 0.65$, the corresponding phase-compensation filter $F_0(z)$ with $d_0 = 20$, and the product filter $A_0(z)F_0(z)$.

4. Generalized two-band IIR/FIR filter bank

In this section, the FIR phase compensation approach derived in the previous section is applied to the generalized two-band allpass-based IIR/FIR analysis–synthesis system depicted in Fig. 3, where the $A_{a/s,i}(z)$, $i = 0, 1$, denote first-order allpass filters and the $F_{a/s,i}(z)$ FIR filters in the analysis and synthesis bank, respectively. Note that by choosing $F_{a,i}(z) = 1$ the allpass-based analysis bank from Section 2 is contained as one special case. A general expression for the linear distortion function $T_{\text{lin}}(z)$ can be given as

$$T_{\text{lin}}(z) = \tfrac{1}{4}z^{-1}(A_{a,0}(z^2)\,F_{a,0}(z^2)\,A_{s,0}(z^2)\,F_{s,0}(z^2) + A_{a,1}(z^2)\,F_{a,1}(z^2)\,A_{s,1}(z^2)\,F_{s,1}(z^2)), \quad (13)$$

and for the aliasing function $T_{\text{alias}}(z)$ we have

$$T_{\text{alias}}(z) = \tfrac{1}{4}z^{-1}(A_{a,0}(z^2)\,F_{a,0}(z^2)\,A_{s,0}(z^2)\,F_{s,0}(z^2) - A_{a,1}(z^2)\,F_{a,1}(z^2)\,A_{s,1}(z^2)\,F_{s,1}(z^2)). \quad (14)$$

In the following we discuss special cases with different properties arising from Fig. 3.

4.1. Case (i): minimization of amplitude, phase, and aliasing distortion

By choosing

$$F_{a,0}(z) = 1, \quad A_{a,0}(z) = A_0(z), \quad F_{a,1}(z) = 1, \quad A_{a,1}(z) = A_1(z), \quad (15)$$

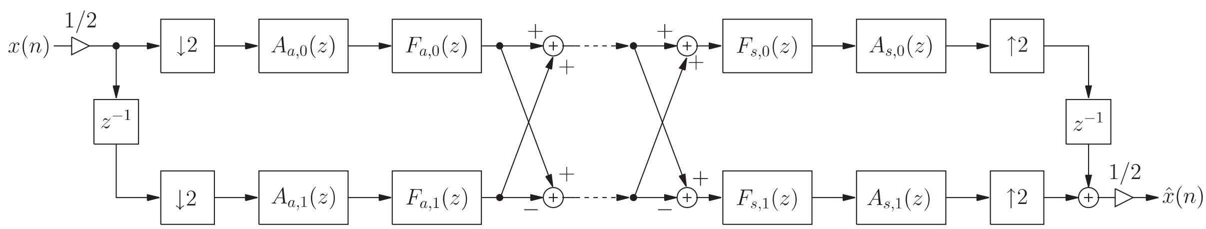

Fig. 3. Generalized two-band analysis–synthesis IIR/FIR filter bank.

we obtain the classical power-symmetric analysis filter bank with allpass polyphase components [6–8]. By canceling the phase distortion introduced by the analysis allpasses with the proposed FIR phase compensation filters the synthesis polyphase components in Fig. 3 can be given as

$$F_{s,0}(z) = 2\,F_0(z), \quad A_{s,0}(z) = z^{-(d_1-d_0)},$$
$$F_{s,1}(z) = 2\,F_1(z), \quad A_{s,1}(z) = 1. \tag{16}$$

The $F_i(z)$, $i = 0, 1$, have the order d_i and are designed according to (12). The delay term $A_{s,0}(z)$ is necessary for compensating the different lags in the polyphase branches as we will see in the following, where without loss of generality we assume that $d_1 > d_0$.

The linear distortion transfer function of the overall analysis–synthesis system can be obtained by inserting (15) and (16) into (13) and applying (9) as

$$T_{\text{lin}}(z) = z^{-2\,d_1-1} + \underbrace{\tfrac{1}{2}((-1)^{d_0+1} a_0^{d_0}\, z^{-2(d_1-d_0)-1} + (-1)^{d_1+1} a_1^{d_1}\, z^{-1})}_{=:\ E_{\text{lin}}(z)}, \tag{17}$$

where $E_{\text{lin}}(z)$ denotes the transfer function of the linear distortion error. If the parameter d_i with $d_0 < d_1$ is selected such that $\varepsilon(a_i, d_i) \approx 0$ in (9), we also have $E_{\text{lin}}(z) \approx 0$. Then, $T_{\text{lin}}(z)$ has approximately linear phase and we have $|T_{\text{lin}}(e^{j\omega})| \approx 1$ for all ω, leading to an overall system delay of $D = 2\max(d_0, d_1) + 1$ samples. Likewise, by combining (15), (16), (14), and (9) the aliasing distortion transfer function can be written according to

$$T_{\text{alias}}(z) = \tfrac{1}{2}(-1)^{d_1} a_1^{d_1} z^{-1} - \tfrac{1}{2}(-1)^{d_0} a_0^{d_0} z^{-2(d_1-d_0)-1}. \tag{18}$$

Again, if the parameters a_0, a_1, d_0, and d_1 are chosen appropriately, the aliasing distortion transfer function tends to zero at all frequencies.

We can remove the delay element $A_{s,0}(z)$ in the synthesis bank at the expense of a slightly higher computational complexity by choosing the same order $d' = \max(d_0, d_1)$ for both phase compensation filters. The linear distortion error transfer function $E_{\text{lin}}(z)$ in (17) now only consists of a single delay term according to

$$E_{\text{lin}}(z) = \tfrac{1}{2}((-1)^{d'+1} z^{-1}(a_0^{d'} + a_1^{d'})) \tag{19}$$

and the new aliasing transfer function can be given as

$$T_{\text{alias}}(z) = \tfrac{1}{2} z^{-1}\,(-1)^{d'}(a_1^{d'} - a_0^{d'}). \tag{20}$$

Note that the magnitude frequency response $|T_{\text{alias}}(e^{j\omega})|$ is now a constant function in ω, that is, we have *the same* aliasing attenuation for all frequencies.

4.2. Case (ii): minimization of amplitude and phase distortion, cancelation of aliasing distortion

In addition to case (i), here aliasing is canceled completely, where the analysis polyphase components are again chosen as in the classical allpass-based QMF case in (15). For the synthesis filter bank we first define $Q_i(z) = A_i(z)F_i(z) = z^{-d_i} - (-1)^{d_i} a_i^{d_i}$ as FIR filter of order d_i. The synthesis polyphase components in Fig. 3 are then specified as

$$F_{s,0}(z) = 2F_0(z)Q_1(z), \quad A_{s,0}(z) = 1,$$
$$F_{s,1}(z) = 2F_1(z)Q_0(z), \quad A_{s,1}(z) = 1. \tag{21}$$

It is easy to verify that aliasing is completely canceled, i.e., $T_{\text{alias}}(z) = 0$. The linear distortion transfer function $T_{\text{lin}}(z)$ of the analysis–synthesis system can be derived from (9), (13), (15), and (21) as

$$T_{\text{lin}}(z) = z^{-(2d_1+2d_0+1)} + E_{\text{lin}}(z) \quad \text{with}$$
$$E_{\text{lin}}(z) = (-1)^{d_0+d_1} a_0^{d_0} a_1^{d_1} z^{-1} - (-1)^{d_0} a_0^{d_0} z^{-(2d_1+1)} - (-1)^{d_1} a_1^{d_1} z^{-(2d_0+1)}. \tag{22}$$

Clearly, the remaining linear distortion error $e_{\text{lin}}(n) = \mathscr{Z}^{-1}\{E_{\text{lin}}(z)\}$ represents the only distortion in the reconstructed signal. The aliasing cancelation approach leads to an increased system delay of $D = 2d_1 + 2d_0 + 1$ samples compared to case (i).

4.3. Case (iii): almost linear-phase analysis and synthesis filters

Two-band QMF filter banks with linear-phase analysis and synthesis filters are desired in some applications, for example in image processing and compression. By modifying the structure from case (ii) above, it is possible to reduce the group delay deviations of the analysis and synthesis filters, and to obtain approximately linear-phase subband filters. In the analysis filter bank of Fig. 3 the polyphase components are chosen according to

$$A_{a,0}(z) = 1, \quad F_{a,0}(z) = Q_0(z), \quad A_{a,1}(z) = A_1(z), \\ F_{a,1}(z) = F_0(z). \tag{23}$$

Likewise we have

$$A_{s,0}(z) = 1, \quad F_{s,0}(z) = 2Q_1(z), \quad A_{s,1}(z) = A_0(z), \\ F_{s,1}(z) = 2F_1(z), \tag{24}$$

in the synthesis filter bank. Now, the modified analysis filters can be given as

$$\begin{aligned} H'_0(z) &= \tfrac{1}{2}(Q_0(z^2) + z^{-1}A_1(z^2)F_0(z^2)) \\ &= H_0(z)F_0(z^2), \\ H'_1(z) &= \tfrac{1}{2}(Q_0(z^2) - z^{-1}A_1(z^2)F_0(z^2)) \\ &= H_1(z)F_0(z^2). \end{aligned} \tag{25}$$

These filters have a smaller deviation from a linear-phase response compared to the pure allpass-based QMF analysis bank, which will be shown in the following for the filter $H'_0(z)$.

Using relation (9) the filter $F_0(z^2)$ in (25) can be stated as

$$F_0(z^2) = A_0(z^{-2})\,(z^{-2d_0} - (-1)^{d_0} a_0^{d_0}). \tag{26}$$

Note that this definition still refers to a stable system since the two poles of the inverse allpass $A_0(z^{-2})$ are canceled by two zeros of $(z^{-2d_0} - (-1)^{d_0} a_0^{d_0})$. Now, by inserting (26) into (25) we can write the lowpass analysis filter $H'_0(z)$ according to

$$H'_0(z) = H_0(z)A_0(z^{-2})\;(z^{-2d_0} - (-1)^{d_0} a_0^{d_0}). \tag{27}$$

In the following, we only consider the remaining group delay of the system

$$V(z) = H_0(z)A_0(z^{-2}) = \tfrac{1}{2}(1 + z^{-1}A_1(z^2)\,A_0(z^{-2})) \tag{28}$$

in (27). This restriction is justified when we assume that the term $(-1)^{d_0} a_0^{d_0}$ is sufficiently small, which then leads to an approximately linear-phase response for the term $(z^{-2d_0} - (-1)^{d_0} a_0^{d_0})$. By inserting (5) into (28) it can be shown that the numerator polynomial $N_V(z)$ of the rational transfer function $V(z) = N_V(z)/D_V(z)$ has the linear-phase property, such that it suffices to consider only the group delay contribution of the denominator polynomial $D_V(z) = a_0 + (a_0 a_1 + 1)z^{-2} + a_1 z^{-4}$.

According to [14, Chapter 4.2.5] the group delay of an all-pole system $H(e^{j\omega}) = |H(e^{j\omega})|\,e^{-jb(\omega)}$ with N poles $z_{\infty_v} = \rho_{\infty_v} e^{j\psi_{\infty_v}}$, $v = 1, 2, \ldots, N$, can be expressed as

$$\begin{aligned} \tau_H(\omega) &= \frac{\mathrm{d}\,b(\omega)}{\mathrm{d}\,\omega} \\ &= \sum_{v=1}^{N} \frac{1 - \rho_{\infty_v}\cos(\omega - \psi_{\infty_v})}{1 - \rho_{\infty_v}\cos(\omega - \psi_{\infty_v}) + \rho_{\infty_v}^2}. \end{aligned} \tag{29}$$

Since the poles $z_{\infty_{1,2}} = \pm j/\sqrt{a_0}$ and $z_{\infty_{3,4}} = \pm j\sqrt{a_1}$ of $V(z)$ are all on the imaginary axis of the z-plane the maximal deviation from a constant group delay occurs at $\omega = \pm\pi/2$. The corresponding group delay $\tau_V(\pm\pi/2)$ can be obtained from (29) and the poles of $V(z)$ after some simplifications as

$$\tau_V\left(\pm\frac{\pi}{2}\right) = \frac{2}{1 - 1/a_0} + \frac{2}{1 - a_1} + C, \tag{30}$$

where the constant $C = -1.5$ contains the group delay contributions of both the linear-phase numerator $N_V(z)$ and the fourfold zero at $z = 0$ of the system $1/D_V(z)$. Likewise, we can obtain an expression for the group delay of the original analysis filter $H_0(z)$ in (6) by inserting the pole locations $z_{\infty_{1,2}} = \pm j\sqrt{a_0}$ and $z_{\infty_{3,4}} = \pm j\sqrt{a_1}$ into (29):

$$\tau_{H_0}\left(\pm\frac{\pi}{2}\right) = \frac{2}{1 - a_0} + \frac{2}{1 - a_1} + C, \tag{31}$$

where the constant C has the same value and meaning as in (30). In order to achieve a smaller maximal deviation from the constant group delay C for the modified analysis filter $H_0'(z)$ (under the constraint that $a_0^{d_0}$ tends to zero) we thus require

$$\left|\frac{2}{1-1/a_0}\right| \overset{!}{<} \left|\frac{2}{1-a_0}\right| \quad \text{for all} \quad 0<a_0<1. \tag{32}$$

Since $|1-1/a_0|>|1-a_0|$ for all $a_0 \in]0,1[$ condition (32) is satisfied and we can conclude that the maximal group delay deviation $\Delta\tau_{H_0',\max} = |\tau_{H_0'}(\pm\pi/2) - C - 2d_0|$ is always smaller than the one for the original analysis filter $H_0(z)$.

Similar considerations hold for both the analysis highpass filter and the synthesis filters, which can be derived from (7) and Fig. 3 as

$$G_0'(z) = 2G_0(z)F_1(z^2) \quad \text{and}$$
$$G_1'(z) = 2G_1(z)F_1(z^2). \tag{33}$$

Note that the magnitude frequency responses $|H_0'(e^{j\omega})|$ and $|H_1'(e^{j\omega})|$ of the filters in (25) are almost identical to $|H_0(e^{j\omega})|$ and $|H_1(e^{j\omega})|$ for the pure allpass-based analysis filter bank in (6), since the compensation filter $F_0(z)$ has approximate allpass behavior:

$$|H_k'(e^{j\omega})| = |F_0(e^{j2\omega})||H_k(e^{j\omega})|$$
$$\approx |H_k(e^{j\omega})|, \quad k=0,1. \tag{34}$$

A similar relation can be obtained for the synthesis filters $G_0'(z)$ and $G_1'(z)$ in (33). Furthermore, it is straightforward to verify that the same system delay and the same reconstruction errors as in case (ii) hold here.

4.4. Summary and implementation complexity

Table 1 summarizes the properties of the discussed filter bank systems and also states the implementation complexity of these systems. We can see that the smallest implementation complexity is given by case (i). The additional cancelation of the aliasing components in case (ii) only requires a slightly higher complexity of the synthesis bank, however, at the expense of a larger system delay compared to case (i). By reducing the deviation from a constant group delay in case (iii) a higher complexity is needed, where in

Table 1
Properties of the special cases derived from the general structure in Fig. 3 (see Section 4) (ALD/AMD/PHD: Aliasing/amplitude/phase distortion, MPS/APS: Multiplications/additions per input sample)

	Case (i)	Case (ii)	Case (iii)
Analysis polyphase components (cmp. Fig. 3)	$A_{a,0}(z)=A_0(z)$ $F_{a,0}(z)=1$ $A_{a,1}(z)=A_1(z)$ $F_{a,1}(z)=1$	$A_{a,0}(z)=A_0(z)$ $F_{a,0}(z)=1$ $A_{a,1}(z)=A_1(z)$ $F_{a,1}(z)=1$	$A_{a,0}(z)=1$ $F_{a,0}(z)=Q_0(z)$ $A_{a,1}(z)=A_1(z)$ $F_{a,1}(z)=F_0(z)$
Synthesis polyphase components (cmp. Fig. 3)	$A_{s,0}(z)=z^{-(d_1-d_0)}$ $F_{s,0}(z)=2F_0(z)$ $A_{s,1}(z)=1$ $F_{s,1}(z)=2F_1(z)$	$A_{s,0}(z)=1$ $F_{s,0}(z)=2F_0(z)Q_1(z)$ $A_{s,1}(z)=1$ $F_{s,1}(z)=2F_1(z)Q_0(z)$	$A_{s,0}(z)=1$ $F_{s,0}(z)=2Q_1(z)$ $A_{s,1}(z)=A_0(z)$ $F_{s,1}(z)=2F_1(z)$
Remaining distortions	ALD minimized: (18), (20) AMD minimized: (17), (19) PHD minimized	ALD canceled AMD minimized: (22) PHD minimized	ALD canceled AMD minimized: (22) PHD minimized
Phase responses of $H_k(z)$ and $G_k(z)$, $k=0,1$	Nonlinear	Nonlinear	Approx. linear in pass- and stopband
Complexity, analysis bank (first-order allpasses)	1 MPS, 3 APS [8]	1 MPS, 3 APS [8]	$0.5(d_0+3)$MPS, $0.5(d_0+5)$ APS
Complexity, synthesis bank	$0.5(d_0+d_1+2)$ MPS, $0.5(d_0+d_1+4)$ APS	$0.5(d_0+d_1+4)$ MPS, $0.5(d_0+d_1+6)$ APS	$0.5(d_1+3)$ MPS, $0.5(d_1+7)$ APS
System delay (samples)	$D=2\max(d_0,d_1)+1$	$D=2d_1+2d_0+1$	$D=2d_1+2d_0+1$

The quantity $Q_i(z)$, $i=0,1$, is defined as $Q_i(z)=A_i(z)F_i(z)$. The analysis and synthesis subband filters may be exchanged without changing the behavior of the filter bank.

comparison to case (i) and (ii) both analysis and synthesis bank approximately require the same number of operations.

Note that the analysis and synthesis subband filters may be exchanged without changing the overall behavior of the analysis–synthesis system such that case (i) and (ii) are also well suited for applications where computing power is a critical resource at the receiver.

5. Design examples and comparison

Analysis filter bank

In order to design the IIR analysis filters in (6) we use a direct optimization approach, where the coefficients a_0 and a_1 of the first-order allpass filters are obtained via nonlinear optimization under minimization of the stopband energy. The resulting magnitude frequency responses for the analysis filters with the values $a_0 = 0.1806$ and $a_1 = 0.6485$ are depicted in Fig. 4, where the stopband edge frequency for the lowpass filter was chosen as $\omega_s = 0.64\pi$. These design parameters will be used for the corresponding allpasses $A_0(z)$ and $A_1(z)$ in all following examples:

Case (i): Here we choose the synthesis polyphase components according to (16), where the filters $F_i(z)$ are designed via (12). By using the above design for the analysis polyphase components in (15) and the delay parameters $d_0 = 6$, $d_1 = 22$ in the synthesis bank, we obtain the amplitude distortion error depicted in Fig. 5(a) with the dashed line for the overall analysis–synthesis system. The magnitude aliasing distortion $|T_{\text{alias}}(e^{j\omega})|$ for this set of delay parameters is

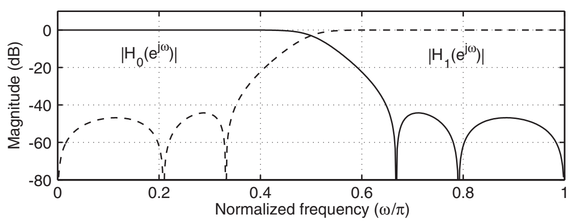

Fig. 4. Magnitude frequency responses for the allpass-based IIR QMF analysis bank.

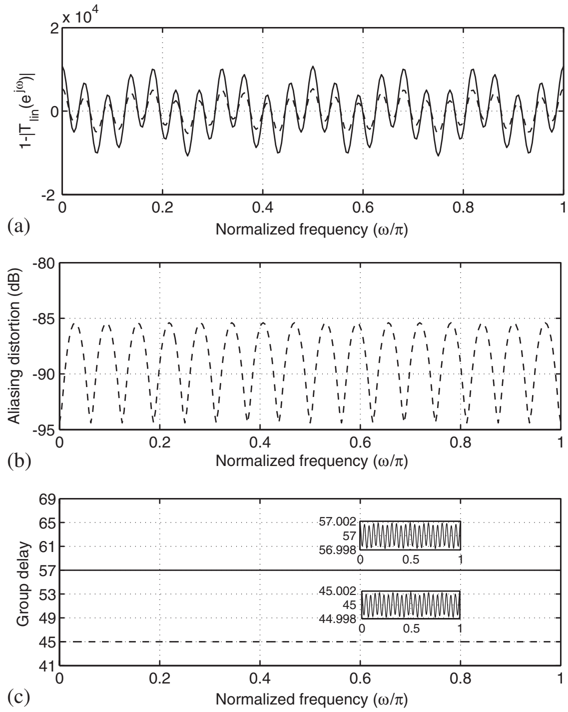

Fig. 5. Overall analysis–synthesis system, synthesis delay parameters $d_0 = 6$, $d_1 = 22$ (dashed line: case (i), solid line: case (ii)/(iii)): (a) Amplitude distortion error; (b) aliasing distortion for case (i) (for case (ii)/(iii) $T_{\text{alias}}(z) = 0$); (c) group delay.

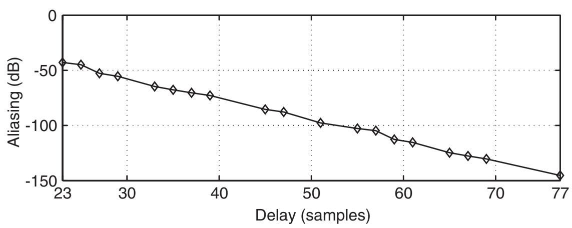

Fig. 6. Average magnitude aliasing distortion vs. system delay for case (i).

shown in Fig. 5(b), and the group delay is given with the dashed line in Fig. 5(c). We can see that the overall linear distortion function $T_{\text{lin}}(z)$ has approximately linear phase, leading to an average system delay of $D = 45$ samples.

Note that the choice of d_0 and d_1 is arbitrary and reflects a compromise between low complexity and low delay, and the minimization of aliasing and linear distortions, which can be also observed from Eqs. (17) and (18). The relation between average magnitude aliasing distortion and system delay is visualized in Fig. 6.

Case (ii): In this case the synthesis polyphase components are given in (21), which ensures that the aliasing distortion is exactly zero at all frequencies. As delay parameters we again choose $d_0 = 6$ and $d_1 = 22$. This leads to the amplitude distortion error shown in Fig. 5(a), and the almost constant group delay depicted in Fig. 5(c), both shown with solid lines.

Case (iii): Here, all distortions are the same as in case (ii) (see Figs. 5(a) and (c)) for the delay parameters $d_0 = 6$ and $d_1 = 22$. Fig. 7(a) depicts the magnitude frequency responses of the synthesis filters $G_0'(z)$ and $G_1'(z)$, which are similar to the analysis filters from Fig. 4 for the pure allpass-based case (except the amplification by factor two). Due to relation (34) this similarity also holds for the frequency responses of the analysis bank. Finally, in Fig. 7(b) the group delays for the analysis and synthesis subband filters are shown. It can be observed that all subband filters have approximately linear phase both in the passband and the stopband.

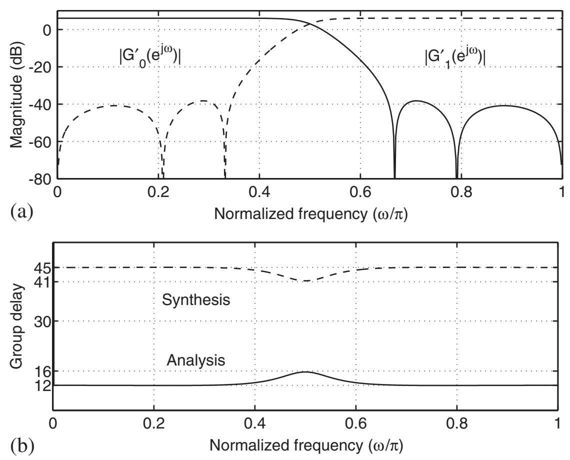

Fig. 7. Case (iii): (a) Magnitude frequency responses of the synthesis filters; (b) group delays for both low- and highpass filters in the analysis and synthesis filter bank, resp.

Comparison

The above design examples are summarized in Table 2, which also contains results for FIR-based two-band systems with identical analysis and synthesis complexities. Especially, we consider a PR two-band paraunitary lattice structure [15] and a classical near-PR QMF design [8] with linear-phase filters. Both resulting filter banks have overall system delays being identical (except small deviations due to the non-ideal phase compensation) to the delays in the above examples for case (i)–(iii). However, note that a comparison between the mixed allpass-/FIR-based systems and the classical FIR-based filter banks is quite problematic since for the allpass-based approaches the choice of the parameters d_0 and d_1 and thus also of the amount of the remaining distortions is in principle arbitrary. Nevertheless, the examples in

Table 2
Comparison of the proposed mixed IIR/FIR systems with classical FIR-based approaches (ALD/AMD: Aliasing/amplitude distortion, MPS/APS: Multiplications/additions per input sample)

System	Stopband ampl. (max)	ALD	AMD	Group delay	MPS/APS	
					Analy.	Synth.
Case (i) ($d_0 = 6$, $d_1 = 22$)	-46 dB	-85 dB	$\pm 5 \cdot 10^{-5}$	$45 \pm 2 \cdot 10^{-3}$	1/3	15/16
Paraunitary lattice [15]	-47 dB	cancelation	≈ 0	45	24/23	24/24
Case (ii) ($d_0 = 6$, $d_1 = 22$)	-46 dB	cancelation	$\pm 10^{-4}$	$57 \pm 2 \cdot 10^{-3}$	1/3	16/17
Paraunitary lattice [15]	-54 dB	cancelation	≈ 0	57	30/29	30/30
Case (iii) ($\approx$ lin. phase filter, $d_0 = 6$, $d_1 = 22$)	-46 dB	cancelation	$\pm 10^{-4}$	$57 \pm 2 \cdot 10^{-3}$	4.5/5.5	12.5/14.5
QMF [8] (lin. phase filt.)	-62 dB	cancelation	$\pm 10^{-4}$	57	29/29	29/29
IIR/FIR approach [9, case 1]	-34 dB	cancelation	± 0.044	56	1/3	18/19

Table 2 give a good overview of the performance for the proposed mixed IIR/FIR systems.

We can see from Table 2 for case (i) and (ii) that they generally show a much lower overall complexity than the PR paraunitary lattice banks at the expense of small distortions in the reconstructed signal. However, in many applications, where modifications of the subband signals are carried out, small additional distortions as introduced by the proposed near-PR systems may be tolerated. When comparing the approaches having linear-phase subband filters, we observe that the strongly reduced complexity for case (iii) is achieved at the expense of a higher stopband amplification and thus, of less selective subband filters compared to the near-PR QMF bank for the same overall system delay.

Table 2 also contains results for the allpass-based IIR/FIR system from [9], where the analysis lowpass has a slightly smaller stopband edge frequency of $\omega_s = 0.6\pi$ leading to a smaller stopband attenuation. The system from [9] achieves a constant group delay at the expense of a larger amplitude distortion, while the compexity is comparable to case (i) and (ii).

6. Conclusion

We have proposed a novel approach for the near-PR design of critically subsampled two-band mixed IIR/FIR QMF banks, where FIR filters are employed for phase compensation in the polyphase domain. These phase-compensation filters can be designed via analytical expressions for their impulse responses. We have shown that all distortions at the output of the synthesis filter bank are controllable and can be made arbitrarily small at the expense of additional system delay. Especially, it is also possible to cancel the aliasing components completely and to design subband filters with approximately linear phase if further delay can be tolerated. Despite we have only explicitly addressed the case of first-order allpass polyphase components in the analysis bank, the proposed phase-compensation approach can be extended to higher order (elliptic) subband filters by concatenating first-order allpasses in combination with higher order FIR compensation filters.

Furthermore, it is straightforward to exchange the analysis and synthesis filters in the generalized structure of Fig. 3. This leads to a low-complexity synthesis, where now a phase "precompensation" is carried out on the analysis side. This may be advantageous for two-way coding and transmission applications with a mobile and a fixed part. The mobile part would then contain the low-complexity versions of the analysis and synthesis banks for both transmitting directions, resp., whereas the fixed part would contain the corresponding higher complexity analysis/synthesis banks (see Table 1).

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