A Network Coding Approach to Cooperative Diversity

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***Abstract*—This paper proposes a network coding approach to cooperative diversity featuring the algebraic superposition of channel codes over a finite field. The scenario under consideration is one in which two "partners"—Node A and Node B—cooperate in transmitting information to a single destination; each partner transmits both locally generated information and relayed information that originated at the other partner. A key observation is that Node B already knows Node A's relayed information (because it originated at Node B) and can exploit that knowledge when decoding Node A's local information. This leads to an encoding scheme in which each partner transmits the algebraic superposition of its local and relayed information, and the superimposed codeword is interpreted differently at the two receivers— i.e., at the other partner and at the destination node—based on their different *a priori* knowledge. Decoding at the destination is then carried out by iterating between the codewords from the two partners. It is shown via simulation that the proposed scheme provides substantial coding gain over other cooperative diversity techniques, including those based on time multiplexing and signal (Euclidean space) superposition.**

***Index Terms*—Cooperative diversity, error control coding, fading channels, iterative decoding, network coding.**

I. Introduction

DIVERSITY techniques offer an effective countermeasure against multipath fading by providing the receiver with multiple replicas of the same information—different versions of the data transmitted with (ideally) independent channel gains [1]. In a system employing *cooperative diversity*, this robustness is obtained by allowing a node to both transmit its own information *and* to serve as a relay for the data transmitted by the other node(s). Because information is transmitted multiple times—once from the originating node and then again from the relaying node(s)—spatial diversity is effected [2]–[7].

Consider a system in which two source nodes are paired as partners for the transmission of their data to a common destination node. The two partners take turns transmitting information, and each partner transmits two kinds of information during its time slot—the *local information* it has generated and the *relayed information* it received previously from its partner, the relayed information being retransmitted to provide diversity. In previously proposed approaches, some mechanism is used to split the transmitter's resources between locally generated bits and relayed bits. For instance, [3]–[6] adopted time multiplexing in which each partner uses a portion of its time slot for local information and the rest for relayed information. In contrast, in [2] and [7], the superposition (in Euclidean space) of modulated signals is used to multiplex local and relayed bits in the context of direct sequence spread spectrum modulation and pulse amplitude modulation (PAM) signaling with unequal error protection, respectively. As a result, the resources exclusively designated for one kind of information cannot be used to facilitate decoding of the other kind. [1]

In this paper, we propose a new system design using algebraic superposition of error control codes that circumvents the explicit resource allocation in the previous designs and hence makes efficient use of resources. The proposed design is referred to as a *network coding* [9], [10] approach to cooperative diversity because it has the defining characteristic of network coding: the transmitting node constructs a linear combination (over a finite field) of the multiple data flows it wishes to convey and transmits that linear combination, rather than simply routing the data flows individually. By viewing the known information at the partner node as additional flows, the data from the transmitting partner node can be recovered as in network coding.

Regarding related work: An information-theoretic treatment of a similar problem is provided in [11]; however, the focus in [11] is on stationary discrete memoryless channels, whereas our motivation is diversity gain on fading channels. Moreover, unlike [11], we consider orthogonal source-to-relay links obtained via time-division multiple access (TDMA). More recently, a scheme combining network coding and error control coding was

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[1]The authors recently became aware of a manuscript by Yue *et al.* [8] that avoids this allocation of resources; the approach taken in [8] is similar in philosophy to the approach in this paper, albeit for a configuration in which both sources are in full-duplex mode. The explicit partially multiplexed constructions in [8] do not yield the full network coding gain provided by the constructions in this paper.

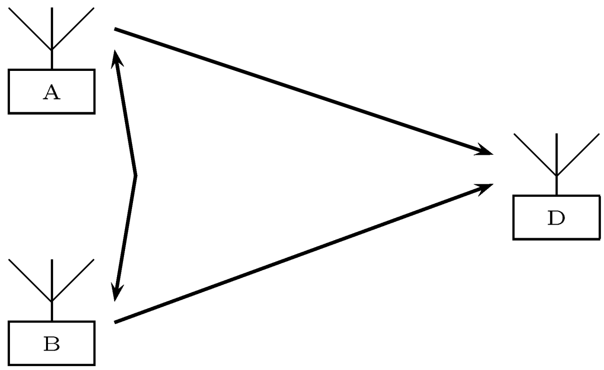

Fig. 1. In cooperative diversity, Node A and Node B work together to deliver their packets to a common destination Node D.

described in [12], but for two-way relay channels where the decoders always possess hard bits to cancel from the received codeword.

II. Cooperative Diversity and the Cooperative Dilemma

The scenario addressed in this paper is depicted in Fig. 1. Two source nodes A and B work in cooperation to deliver their packets of k bits each to a common destination node D. We refer to nodes A and B as each other's partner. During time slot t, Node A transmits in the first half slot, while Node B transmits in the second half slot. Each source node receives the frame sent by its partner node and attempts to decode its partner's information. If decoding is successful, then some of this information—the part that originated at the transmitting partner—will be relayed in a future transmission to provide the destination D with spatial diversity. If a source node fails to decode its partner's information, and therefore does not have the information to relay, then that source node will operate in a noncooperative mode.

By symmetry, we can focus on packets that originate at Node A. In such a decode-and-forward system, there can be no diversity at the destination node if the packets originating at Node A are not received correctly at Node B; this illustrates the importance of minimizing $P_{A,B}$, the packet error rate for the link between A and B. Since the diversity order determines the slope of the signal-to-noise ratio (SNR) versus error rate curves on a log-log scale in the high-SNR regime [1, Ch. 14], it is desirable to have a small $P_{A,B}$ to increase the likelihood that Node A is assisted by its partner.

The frame structures of three different resource allocation schemes are shown in Fig. 2. In these graphs, the horizontal axis represents time while the vertical axis represents power. The shaded portions of each graph represent the resources dedicated to the transmission of local information from one partner to the other. It is assumed each source node generates k bits of local data per time slot and the channel resources available during a half-slot are n transmitted symbols with a transmit power of P.

- In the noncooperative configuration, Nodes A and B transmit their own information in turn during each slot. In this case, the information bits may be protected by a rate-k/n code with full power—the best level of protection a node can offer.
- In a time multiplexing cooperative scheme [3]–[6], each source node uses part of its transmission time to act as a relay for its partner. A node's relayed information, originally generated at its partner, is of no use to the partner node. Because each node must transmit $2k$ bits—k local bits and k relayed bits—the channel code must have rate $2k/n$, and full power may be used during transmission. Moreover, non-binary modulation is required if $k/n > 1/2$.
- In a cooperative scheme using signal superposition [2], [7], the modulated signal for a relayed codeword is added to the modulated signal for the locally generated codeword. The partner node, with its knowledge of the relayed bits, can subtract the portion of the signal due to the relayed codeword from the received signal and decode the locally generated information bits. Since part of the power has been used to modulate the relayed codeword, which is later subtracted at the partner node, each source node decodes its partner's information based on a rate-k/n code, but with only partial power $P_L < P$.

The preceding analysis indicates that, in the existing cooperative diversity schemes, the transmission of a source node's local information to its partner is carried out with diminished resources—with either a higher code rate or less power— compared to the noncooperative approach. Since higher code rates and lower power imply a higher probability of error, the packet error rate $P_{A,B}$ between A and B necessarily increases when resources are allocated for relay cooperation under these previous designs. Because $P_{A,B}$ is, effectively, the probability that Node B will not be able to supply diversity information about Node A's local information to the destination, we have this apparent conundrum: the extent to which a user dedicates resources to cooperation reduces the probability that the cooperation is successful.

We refer to this as the *cooperative dilemma*. This dilemma is explained by the fact that a source node must deliver both its own packet and a relayed packet to the destination node, but only its own packet is required at the partner node. When transmission time and/or power are divided, the portion of those resources allocated to the relayed information is wasted from the perspective of the partner node.

III. Design Based on Algebraic Superposition: Source Nodes

The notation used to characterize the new system design is summarized in Table I. Let $i_L^A(t)$ denote the local information vector originating at Node A for transmission during time slot t, and let $i_R^A(t)$ denote the relayed information vector transmitted by Node A during the same time slot. Similarly, for Node B we define $i_L^B(t)$ and $i_R^B(t)$. We assume each source node can determine whether it has successfully decoded the local information vector transmitted by its partner; this can be done by using a cyclic redundancy check (CRC).

Let $C^A(t)$ and $C^B(t)$ denote the n-bit codeword sent by Nodes A and B, respectively, during time slot t. The code generator matrix for the locally generated information bits is $\boldsymbol{G}_L$

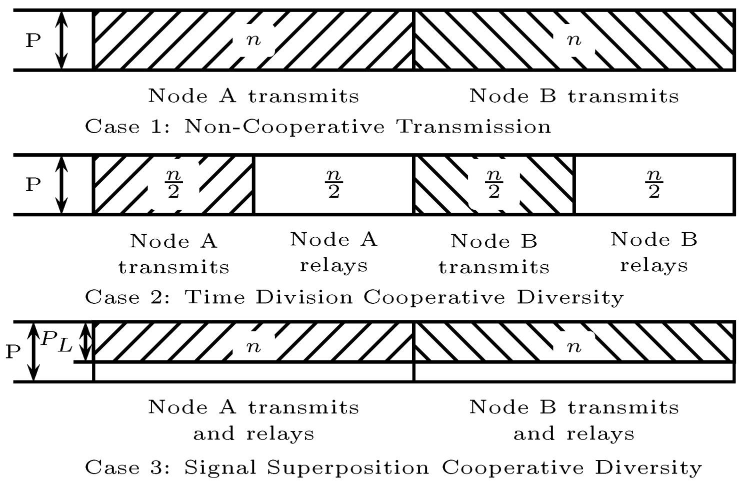

Fig. 2. The frame structure illustrates three different resource allocation schemes.

TABLE I
NOTATION SUMMARY

	Time slot t first half	Time slot t second half
Transmission	$A \rightarrow B, D$	$B \rightarrow A, D$
Codeword	$C^A(t)$	$C^B(t)$
Local Information	$i_L^A(t)$	$i_L^B(t)$
Relayed Information	$i_R^A(t) = \pi\big(i_L^B(t-1)\big)$	$i_R^B(t) = \pi\big(i_L^A(t)\big)$

and for the relayed bits is $\boldsymbol{G}_R$; both codes have rate k/n. Invoking symmetry, we now provide a detailed description of the encoding operation at Node A and the decoding operation at Node B.

A. The Encoder at the Source Nodes

During time slot t, Node A must convey its local information vector $i_L^A(t)$ while relaying Node B's information $i_L^B(t-1)$—assuming it has decoded $i_L^B(t-1)$ correctly.

If Node A has decoded $i_L^B(t-1)$ successfully, it first interleaves $i_L^B(t-1)$ to generate the relay information, i.e.,

$$i_R^A(t) = \pi\left(i_L^B(t-1)\right) \tag{1}$$

and the transmitted codeword is the XOR of the codeword containing Node A's local information and the codeword containing Node B's relayed bits

$$\begin{aligned} C^A(t) &= i_L^A(t)\boldsymbol{G}_L \oplus i_R^A(t)\boldsymbol{G}_R \\ &= \left[i_L^A(t) i_R^A(t)\right] \begin{bmatrix} \boldsymbol{G}_L \\ \boldsymbol{G}_R \end{bmatrix}. \end{aligned} \tag{2}$$

The pseudorandom interleaving in (1) facilitates iterative decoding at the destination node; it guarantees that the destination's decoder for Node B provides extrinsic information to its decoder for Node A that is independent of the other information available to the Node A decoder. Also note that taking the XOR of two codewords is equivalent to encoding the vectors $i_L^A(t)$ and $i_R^A(t)$ using a nested code with generator $\boldsymbol{G} \triangleq \begin{bmatrix} \boldsymbol{G}_L \\ \boldsymbol{G}_R \end{bmatrix}$, as noted in (2).

If Node A has *failed* to decode $i_L^B(t-1)$, then Node A's codeword is simply the encoded version of $i_L^A(t)$

$$C^A(t) = i_L^A(t)\boldsymbol{G}_L = \left[i_L^A(t)\vec{0}\right] \begin{bmatrix} \boldsymbol{G}_L \\ \boldsymbol{G}_R \end{bmatrix}. \tag{3}$$

We assume a flag bit is transmitted along with the codeword to alert the receivers which of the two encoding methods ((2) or (3)) was used. In an analogous manner, if Node B has decoded $i_L^A(t)$ correctly, then Node B's transmitted codeword is

$$\begin{aligned} C^B(t) &= i_L^B(t)\boldsymbol{G}_L \oplus i_R^B(t)\boldsymbol{G}_R \\ &= \left[i_L^B(t) i_R^B(t)\right] \begin{bmatrix} \boldsymbol{G}_L \\ \boldsymbol{G}_R \end{bmatrix} \end{aligned} \tag{4}$$

where $i_R^B(t) = \pi(i_L^A(t))$. Similarly, if Node B has failed to decode $i_L^A(t)$, then Node B transmits

$$C^B(t) = i_L^B(t)\boldsymbol{G}_L = \left[i_L^B(t)\vec{0}\right] \begin{bmatrix} \boldsymbol{G}_L \\ \boldsymbol{G}_R \end{bmatrix}. \tag{5}$$

B. The Decoder at the Source Nodes

The decoder at Node B first checks the flag bit. If the frame was generated using (3)—i.e., Node A failed to cooperate—then Node B uses the decoder for the code with generator $\boldsymbol{G}_L$ to obtain $i_L^A(t)$. Note that in this case Node B decodes a rate k/n code with full power.

Now suppose the frame was generated using (2), so it represents the XOR of a local codeword and a relayed codeword. For hard-decision decoding, the output of the demodulator is

$$\begin{aligned} \hat{C}^A(t) &= C^A(t) \oplus e(t) \\ &= i_L^A(t)\boldsymbol{G}_L \oplus i_R^A(t)\boldsymbol{G}_R \oplus e(t) \end{aligned} \tag{6}$$

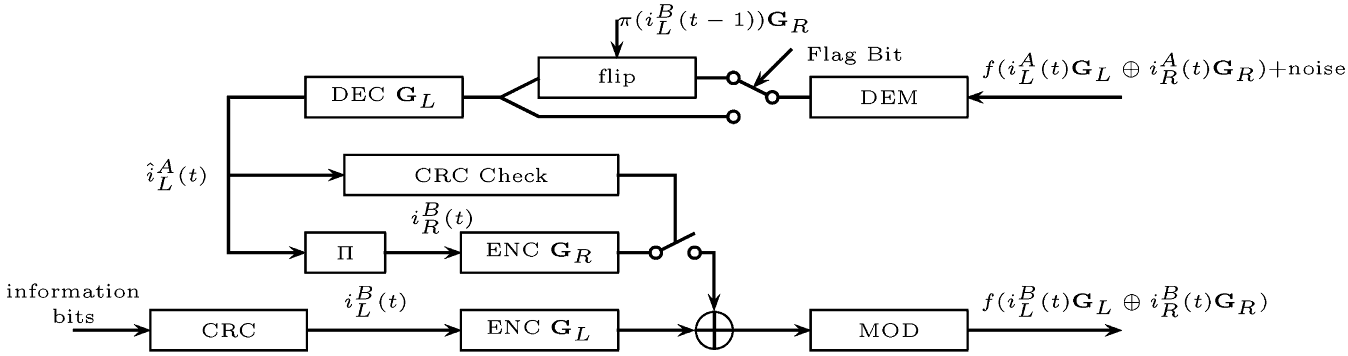

Fig. 3. The transceiver structure at source Node B. The processing at Node A is analogous.

where $e(t)$ is the binary error pattern. Note that, because Node B knows $i_R^A(t) = \pi(i_L^B(t-1))$, the codeword $i_R^A(t)\boldsymbol{G}_R$ can be stripped from the channel decoder input by forming

$$\tilde{C}^A(t) = \hat{C}^A(t) \oplus i_R^A(t)\boldsymbol{G}_R = i_L^A(t)\boldsymbol{G}_L \oplus e(t). \quad (7)$$

At this point, the hard-decision decoder for the code $\boldsymbol{G}_L$ can be used to estimate $i_L^A(t)$ from $\tilde{C}^A(t)$. Note that the *effective code rate* here is k/n.

If soft-decision decoding is to be used, a similar procedure can be carried out. Assume the demodulator generates a log-likelihood ratio (LLR) for each bit in $C^A(t) = i_L^A(t)\boldsymbol{G}_L \oplus i_R^A(t)\boldsymbol{G}_R$. Let $[C](i)$ and $L[C](i)$ denote the ith bit in the codeword C and its LLR value, respectively. Then the LLR value of $i_L^A(t)\boldsymbol{G}_L$ can be obtained by canceling the effect of $i_R^A(t)\boldsymbol{G}_R$ as a special case of the box-plus operation [13]; i.e.,

$$\begin{aligned} &L\left[i_L^A(t)\boldsymbol{G}_L\right](i) \\ &\quad= \log\frac{\mathbb{P}r\{\left[i_L^A(t)\boldsymbol{G}_L\right](i)=0\}}{\mathbb{P}r\{\left[i_L^A(t)\boldsymbol{G}_L\right](i)=1\}} \\ &\quad= \begin{cases} L[C^A(t)](i), & \text{when } \left[i_R^A(t)\boldsymbol{G}_R\right](i)=0 \\ -L[C^A(t)](i), & \text{when } \left[i_R^A(t)\boldsymbol{G}_R\right](i)=1. \end{cases} \end{aligned} \quad (8)$$

With the LLR for $i_L^A(t)\boldsymbol{G}_L$ now available, Node B can employ a soft-decision decoder for $\boldsymbol{G}_L$ to estimate $i_L^A(t)$. Once again, the *effective code rate* is k/n.

In summary, for either hard-decision or soft-decision decoding, the relayed codeword that was algebraically superimposed with the locally generated codeword can be canceled at the partner prior to decoding, and this operation is transparent to the channel decoder. No resources are wasted, and the information that is useful to Node B—namely, $i_L^A(t)$—is protected by a rate-k/n code with full power P. This efficiency comes from the network coding gain obtained by transmitting a linear combination of codewords over a finite field instead of routing both local and relayed information separately. The transceiver structure at Node B is shown in Fig. 3.

IV. Design Based on Algebraic Superposition–Destination Node

The receiver at the destination node is required to make decisions about all the packets generated by both of the partners. Unlike the partner nodes, the destination node has no *a priori* certainty of any portion of the received signal that it can use to decode at a lower effective rate; the structure of the destination decoder must exploit the redundancy provided by the channel code in a way that accommodates the algebraic superposition of the transmitted signal.

Without loss of generality, consider the decoding of $i_L^A(t)$ at Node D. Note that both $C^A(t)$ and $C^B(t)$ potentially carry information about $i_L^A(t)$: $C^A(t)$ as "local" information and $C^B(t)$ (in interleaved form) as "relayed" information. There are four possible ways that $C^A(t)$ and $C^B(t)$ could have been formed, as illustrated in Fig. 4:

1) Both $C^A(t)$ and $C^B(t)$ were constructed from local information only—i.e., $C^A(t) = i_L^A(t)\boldsymbol{G}_L$ and $C^B(t) = i_L^B(t)\boldsymbol{G}_L$. In this case, $C^B(t)$ is not helpful in estimating $i_L^A(t)$. Using a decoder for $\boldsymbol{G}_L$ to process the noisy version of $C^A(t)$ is sufficient.
2) Now assume $C^A(t)$ was constructed from both local and relayed information while $C^B(t)$ used only local information—i.e., $C^A(t) = i_L^A(t)\boldsymbol{G}_L \oplus i_R^A(t)\boldsymbol{G}_R$ and $C^B(t) = i_L^B(t)\boldsymbol{G}_L$. Once again, $C^B(t)$ is not helpful in estimating $i_L^A(t)$. However, $i_R^A(t)$ is an interleaved version of $i_L^B(t-1)$, which has already been processed; thus, we can use the extrinsic information about $i_L^B(t-1)$ obtained from $C^B(t-1)$ as *a priori* information for $i_R^A(t)$ and decode $i_L^A(t)$ using a maximum *a posteriori* probability (MAP) decoder.
3) Next assume

$$C^A(t) = i_L^A(t)\boldsymbol{G}_L$$

and

$$C^B(t) = i_L^B(t)\boldsymbol{G}_L \oplus i_R^B(t)\boldsymbol{G}_R.$$

Iterative decoding can be used to exchange extrinsic information about $i_L^A(t)$ and $i_R^B(t) = \pi(i_L^A(t))$ between the decoders for $C^A(t)$ and $C^B(t)$ using soft-in-soft-out decoders. Since an estimate of $i_L^B(t)$ will be made in the *next* decoding step, we use zero *a priori* information in the soft-decision decoding of $C^B(t)$.
4) Finally, assume

$$C^A(t) = i_L^A(t)\boldsymbol{G}_L \oplus i_R^A(t)\boldsymbol{G}_R$$

and

$$C^B(t) = i_L^B(t)\boldsymbol{G}_L \oplus i_R^B(t)\boldsymbol{G}_R.$$

Iterative decoding with a soft-in, soft-out decoder for $\boldsymbol{G}$ can be employed. Since $i_R^A(t) = \pi(i_L^B(t-1))$ has already been processed, extrinsic information obtained from $C^B(t-1)$

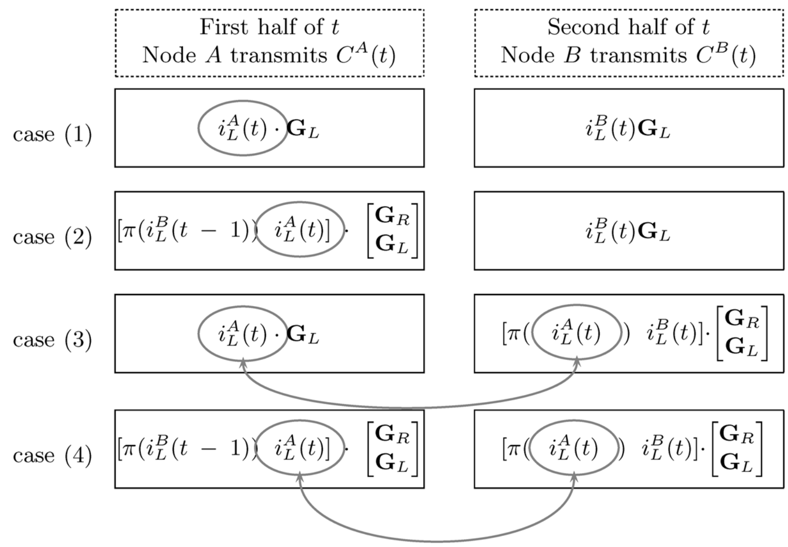

Fig. 4. The decoding operation at Node D for $i_L^A(t)$. In cases 1) and 2), $C_L(t)$ does not contain information about $i_L^A(t)$ and is not used in decoding. In cases 3) and 4), iterative decoding is adopted to exploit diversity, represented by the double arrows. The extrinsic information about $i_L^B(t-1)$ received from $C^B(t-1)$ is used as a priori information in cases 2) and 4).

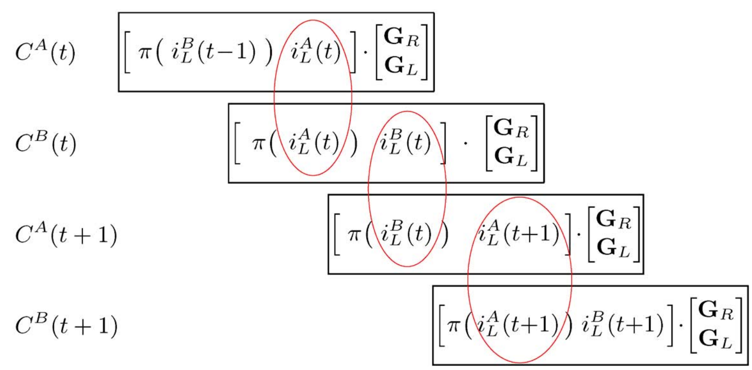

Fig. 5. Multiple codeword blocks can be viewed as a chain.

can be used as *a priori* information in processing $C^A(t)$, while zero *a priori* information is used for $i_L^B(t)$. The soft decoders processing $C^A(t)$ and $C^B(t)$ exchange extrinsic information about $i_L^A(t)$.

Cooperative diversity is obtained with iterative decoding when Node B has relayed $i_L^A(t)$. The decoding of B's packet $i_L^B(t)$ is similar and makes use of both $C^B(t)$ and $C^A(t+1)$.

This approach can be improved by extending the iterative processing to more than just two codewords. Take the decoding of $i_L^A(t)$ as an example. Instead of processing only $C^A(t)$ and $C^B(t)$—the two codewords containing $i_L^A(t)$—we can also make use of $C^A(t+1), C^B(t+1),$, and so on. The structure of the codewords can be viewed as a chain, as shown in Fig. 5. (Here we assume for simplicity that all codewords result from encoding two information vectors—one local and one relayed.) Consider a decoder in which the iterative processing occurs over a window of W received codewords. (In Fig. 5, $W = 4$.) The window begins with the codeword in which the desired information vector is local and extends over the next $W-1$ codewords. (In Fig. 5, the desired information vector is $i_L^A(t)$.) The codewords are processed in a back-and-forth manner. The extrinsic information from the codewords immediately before and after a given codeword is used in processing that codeword, with the exception of the last codeword in the window, where there is zero *a priori* information about the local information vector. After a fixed number of iterations, the decoder makes a decision about the desired information vector and the sliding window is advanced by one codeword. Information-theoretic aspects of window decoding methods can be found in [14] for multiple-access channel with generalized feedback.

V. Code Search and Simulation Results

A. Code Search

We constrain the local and relay codes described by $\boldsymbol{G}_L$ and $\boldsymbol{G}_R$ to be binary convolutional codes for two reasons. First, the soft-in, soft-out maximum-likelihood (ML) and MAP decoders for convolutional codes are simple to implement using the Bahl–Cocke–Jelinek–Raviv (BCJR) algorithm [15], [16]. Second, convolutional codes can be used as building blocks for capacity-achieving codes.

The generator matrices $\boldsymbol{G}_L$ and $\boldsymbol{G}_R$ should, ideally, be optimized to minimize the overall error probability. However, an exact performance analysis quickly becomes intractable due to

TABLE II
TABLE OF GOOD CODES

Constraint Length	$\mathbf{G}_L$	free distance $\mathbf{G}_L$	$\mathbf{G}_R$	free distance $\mathbf{G}$
Rate $1/3$ $\mathbf{G}_L$ and Rate-$2/3$ $\mathbf{G}$				
2	$[1,\ 3,\ 3]_8$	5	$[1,\ 1,\ 0]_8$	2
3	$[5,\ 7,\ 7]_8$	8	$[7,\ 3,\ 0]_8$	4
4	$[15,\ 13,\ 17]_8$	10	$[02,\ 07,\ 15]_8$	6
5	$[25,\ 33,\ 37]_8$	12	$[32,\ 37,\ 05]_8$	8
6	$[47,\ 53,\ 75]_8$	13	$[62,\ 15,\ 05]_8$	8
Rate $1/4$ $\mathbf{G}_L$ and Rate-$1/2$ $\mathbf{G}$				
2	$[1,\ 1,\ 3,\ 3]_8$	6	$[2,\ 1,\ 1,\ 0]_8$	4
3	$[5,\ 5,\ 7,\ 7]_8$	10	$[6,\ 3,\ 6,\ 1]_8$	7
4	$[13,\ 13,\ 15,\ 17]_8$	13	$[15,\ 06,\ 16,\ 03]_8$	9
5	$[25,\ 27,\ 33,\ 37]_8$	16	$[13,\ 36,\ 34,\ 13]_8$	11
6	$[45,\ 53,\ 67,\ 77]_8$	18	$[36,\ 73,\ 12,\ 01]_8$	12

the nature of iterative decoding and the multiple channel SNR parameters. Therefore, a more *ad hoc* approach is taken based on the following observations.

- The partner-to-partner link is protected by the code with generator $\boldsymbol{G}_L$.
- The partner-to-destination link is protected by the code with generator $\boldsymbol{G} = \begin{bmatrix}\boldsymbol{G}_L\\ \boldsymbol{G}_R\end{bmatrix}$. Under normal operation, each information vector is encoded twice using this generator—once as local information and once as relayed information. However, if the source-to-destination link goes into a deep fade, it may be required to recover both the information vectors based on a single received codeword. This suggests that the code with generator matrix $\boldsymbol{G} = \begin{bmatrix}\boldsymbol{G}_L\\ \boldsymbol{G}_R\end{bmatrix}$ should be a strong code.
- The code with generator $\boldsymbol{G}_R$ is never decoded by itself; therefore, $\boldsymbol{G}_R$ can be used as a free parameter in optimizing $\boldsymbol{G} = \begin{bmatrix}\boldsymbol{G}_L\\ \boldsymbol{G}_R\end{bmatrix}$.

Based on these observations, $\boldsymbol{G}_L$ is selected to be the best convolutional code for a given rate and constraint length—i.e., from tables such as those in [17]. Then an exhaustive search is carried out among all appropriate choices of $\boldsymbol{G}_R$ with constraint length no larger than that of $\boldsymbol{G}_L$ to find the nested code with generator $\boldsymbol{G}$ yielding the largest free distance and fewest number of nearest neighbors among all noncatastrophic encoders. The algorithm in [18], [19] is used to calculate the free distance, and the bidirectional search proposed in [20] is used to calculate the distance spectrum. The resulting generators are tabulated in Table II for codes with rates $1/3$ and $1/4$—i.e., the effective rate of the partner-to-partner code is $1/3$ (respectively, $1/4$) while that of the partner-to-destination code is $2/3$ (respctively, $1/2$). Finally, the convolutional encoders should be converted to recursive form to improve convergence behavior.

For the simulation results presented in the later subsection, the optimum eight-state encoder $\boldsymbol{G}_L = [1, \frac{13}{15}, \frac{17}{15}]_8$ was used, and a computer search found that the eight-state encoder $\boldsymbol{G}_R = [\frac{02}{15}, \frac{07}{15}, 1]_8$ optimized $\boldsymbol{G}$ in the sense discussed above. The two partner nodes employ eight-state convolutional encoders and decoders, while a 64-state BCJR decoder is required at the destination node to exploit diversity.

B. A Lower Bound on the Packet Error Rate

In this subsection, we derive a simple lower bound on packet error rate for any cooperative diversity system employing TDMA and a fixed transmit power constraint.

The destination node receives $2n$ symbols—n transmitted from Node A with SNR γ_{AD} and n transmitted from Node B with SNR γ_{BD}. Let y^{2n} denote these received symbols. Furthermore, let $C(\gamma)$ denote the capacity of a point-to-point link with SNR γ, and let $r = k/n$ be the rate of each encoder. Then, the mutual information about i_L^A and i_L^B available at the destination, $I(i_L^A, i_L^B; y^{2n})$, is bounded by $n(C(\gamma_{AD}) + C(\gamma_{BD}))$. Consider the event $\{C(\gamma_{AD}) + C(\gamma_{BD}) < 2r\}$,—i.e., the event that n times the sum capacity of the channels delivering information to the destination falls below $2k$, the entropy of the two packets (assuming equally likely realizations). Clearly, when this event occurs, the cooperative system will be in outage, and it is impossible for Nodes A and B to reliably communicate all $2k$ bits to Node D with *any* form of cooperation. The outage probability is thus lower-bounded by

$$P_O \geq \mathbb{P}\mathrm{r}\{C(\gamma_{AD}) + C(\gamma_{BD}) < 2r\}. \quad (9)$$

Since the packets generated at Nodes A and B are independent, the mutual information about the two packets available at the destination is no less than the corresponding sum, i.e.,

$$I(i_L^A; y^{2n}) + I(i_L^B; y^{2n}) \leq I(i_L^A, i_L^B; y^{2n}) \leq n(C(\gamma_{AD}) + C(\gamma_{BD})).$$

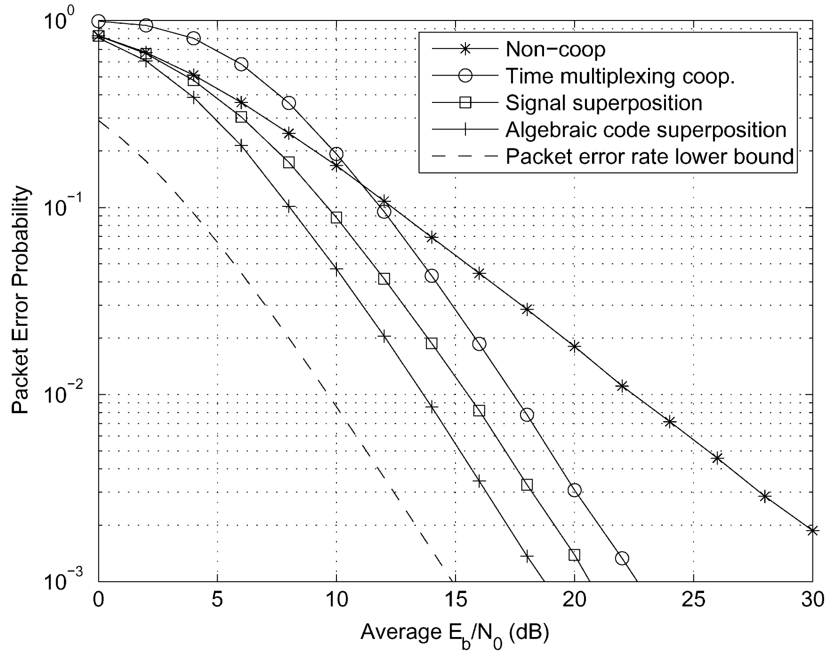

Fig. 6. Packet error probabilities for one noncooperative reference system and three cooperative systems.

When the event $\{C(\gamma_{AD})+C(\gamma_{BD}) < 2r\}$ occurs, at least one of the packets is represented by fewer than k bits at the destination. Define P_E as the smallest packet error rate possible when the mutual information about a packet is less than the packet entropy k. (P_E will approach one for large k.) Then the average packet error rate of the cooperative system is lower-bounded by

$$P_F \geq \frac{P_E}{2}\mathbb{P}\mathrm{r}\{C(\gamma_{AD})+C(\gamma_{BD}) < 2r\}. \tag{10}$$

Assuming binary phase-shift keying (BPSK) modulation and Rayleigh fading, the probability $\mathbb{P}\mathrm{r}\{C(\gamma_{AD})+C(\gamma_{BD}) < X\}$ can be evaluated using numerical integration as

$$\begin{aligned}&\mathbb{P}\mathrm{r}\{C(\gamma_{AD})+C(\gamma_{BD}) < X\}\\ &\quad = \int_0^{\frac{(J^{-1}(X))^2}{8}} \frac{1}{\Gamma_{AD}}\exp\left(-\frac{\gamma_{AD}}{\Gamma_{AD}}\right)\\ &\qquad \times \left[1-\exp\left(-\frac{\left[J^{-1}(X-J(\sqrt{8\gamma_{AD}}))\right]^2}{8\Gamma_{BD}}\right)\right]\mathrm{d}\gamma_{AD}\end{aligned} \tag{11}$$

where Γ_{AD} and Γ_{BD} are the average SNRs per coded bit for the two source–destination links, and the definition of $J(\cdot)$ and its inverse $J^{-1}(\cdot)$, together with closed-form approximations, can be found in [21].

C. Simulation Results

Computer simulations were carried out to compare the performance of four different approaches by which a pair of source nodes can convey information to a common destination: noncooperative transmission, cooperative transmission based on time multiplexing [4], cooperative transmission based on signal superposition [7], and the network coding cooperative system proposed in this paper. Each packet consisted of $k = 500$ bits, and a CRC-12 code was used in the cooperative systems to identify decoding failures. All systems used BPSK modulation, with the exception of the cooperative system based on signal superposition, which used 4-PAM with unequal error protection (see [7]). All channels were subject to additive white Gaussian noise (AWGN) plus block Rayleigh fading changing independently over each time slot with the same average power. The channels between different pairs of nodes were assumed to be independent. Finally, the following codes were incorporated into the simulations.

- In the noncooperative, point-to-point reference system, the rate-$1/3$ convolutional code with generator matrix $[15, 13, 17]_8$ was used.
- The same $[15, 13, 17]_8$ code was used as the mother code for the time multiplexing cooperative scheme. To leave half of the transmission period available for relaying, this code was punctured to rate $2/3$ for the transmission of local information. If the decoding of this information was successful at the partner node, the relayed information was constructed by re-encoding the recovered packet with the same rate-$1/3$ code and then using the puncturing pattern complementary to the one used at the originating node—i.e., transmitting the coded bits that were punctured at the originating node. The destination node D thus decoded either a rate-$1/3$ mother code or a punctured rate-$2/3$ code, depending on whether a relayed copy was received or not.
- In the cooperative system based on signal superposition, the locally generated bits were encoded using a recursive convolutional code with generator matrix $[1, \frac{13}{15}, \frac{17}{15}]_8$. If the partner node's information was successfully decoded, these bits were interleaved by an s-random interleaver and re-encoded with the same code. The relay was allocated 15% of the total transmission power, as suggested in [7]. Iterative decoding with ten iterations was adopted at the

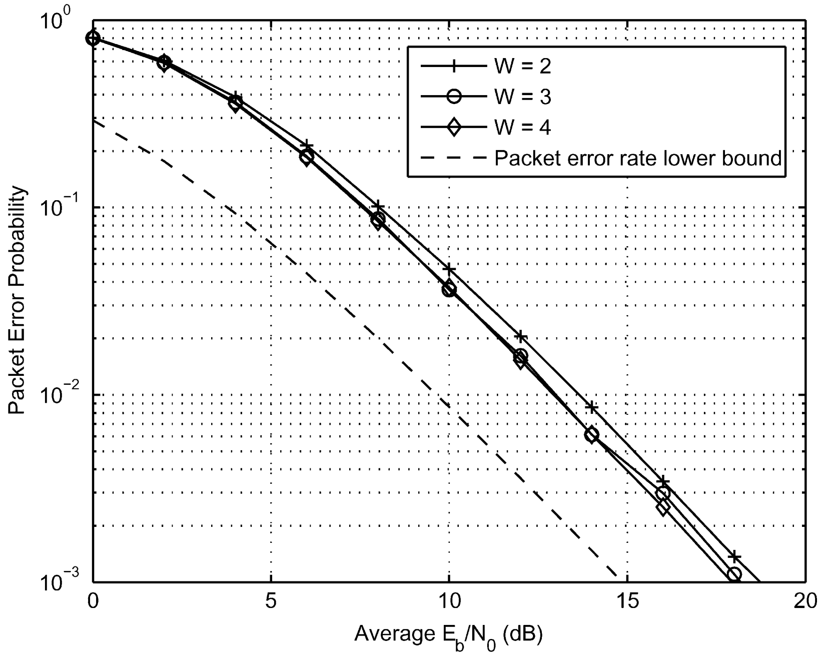

Fig. 7. Packet error probabilities for decoding over two, three, and four codeword blocks in cooperative diversity based on algebraic code superposition.

destination node to exploit diversity if a relayed copy was received.

- As noted above, the new scheme uses $\boldsymbol{G}_L = [1, \frac{13}{15}, \frac{17}{15}]_8$ and $\boldsymbol{G}_R = [\frac{02}{15}, \frac{07}{15}, 1]_8$. Moreover, an s-random interleaver was used, and ten iterations were carried out at the destination node. Note that a total of $2 + 2(W - 2)$ BCJR algorithm calculations per iteration are required for a sliding window of W codeword blocks. When comparing the new approach to the other cooperative diversity schemes, we considered only decoding over $W = 2$ codewords.

Fig. 6 shows the packet error rates for all four systems. Although all three cooperative diversity schemes have the same error curve slope at high average SNR, the merit of the new design is clear. It outperforms cooperative diversity based on signal superposition and time multiplexing by 2 and 4 dB, respectively, at a packet error rate of 10^{-3}. The lower bound on frame-error rate (FER) in (10) can be formulated in terms of the average SNR per information (uncoded) bit, and an approximation to the bound can be obtained by setting $P_E = 1$. The SNR gap of approximately 4 dB to the FER bound is on the one hand due to the short block length of $k = 500$ bits and on the other hand due to the fact that the FER bound in (10) is very conservative. For comparison, the average SNR is normalized by the number of information bits in the plots.

The effect of increasing the decoder window length W is shown in Fig. 7. It is observed that by increasing W from two to three (four BCJR calculations per iteration rather than two), an SNR improvement of about 0.5 dB can be obtained at medium to high SNR; this means that the 2–4-dB improvement over time multiplexing and signal superposition seen in Fig. 6 can be increased to 2.5–4.5 dB, albeit at the cost of additional complexity. For the case of $W = 4$, however, little additional gain is obtained compared to the $W = 3$ case, indicating that there is a point of diminishing returns.

VI. Conclusion

A novel network coding approach to cooperative diversity that makes efficient use of available resources has been proposed. By transmitting the algebraic superposition of the locally generated codeword and the relayed codeword—thereby facilitating different effective decoding rates at the partner node and the destination node—the proposed scheme works in a collaborative mode more often than in previously proposed designs. The result is a significant coding gain compared with other approaches that have been proposed in the literature.

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