36. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

41. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$

43. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+\sqrt{n}} - \sqrt{n}\right)$

44. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n+1}}$

- MAT112 -Mr. José Pabón

*Recitation will start soon.*

We will pass this course with a great grade & meet our academic and professional goals! 10.2-10.6

- MAT112 Prof. Ro / T.A. Pabon

We will be courteous, civil to each other.
NO SUCH THING AS AN OBVIOUS QUESTION
ask ask ask any doubt to clear up

I will pass this course with a great grade!

Review (Test)

① Geometric Series Test $\sum ar^n$

$|r|<1$, then converge to $\frac{a_0}{1-r}$

$|r| \geq 1$, div

② P-Test $\sum \frac{1}{n^p}$ $p>1$ con, $p \leq 1$ div

③ Nth Term DT $\sum a_n$

$\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ div

④ Integral Test (i) Let $a_n = f(x)$ continuous, positive, decreasing

(ii) $\int_N^{\infty} f(x)dx$ con → $\sum a_n$ con

div → $\sum a_n$ div

⑤ DCT (B.C → S.C)

(S.D → B.D)

⑥ LCT (i) $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, both C or both D

(ii) $= 0$ & $\sum b_n$ con → $\sum a_n$ con

(iii) $= \infty$ & $\sum b_n$ div → $\sum a_n$ div

10.5 Absolute Convergence Test ; Ratio & Root Test

Def If $\sum |a_n|$ converges, then $\sum a_n$ converges absolutely

⑦ Thm 12 (Absolute Convergence Test)

> If $\sum |a_n|$ converges, then $\sum a_n$ converges

Ex1] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \cdots$

$\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ con by P-Test with $p = 2 > 1$

Thus, I con by ACT

3/26/R I will pass this course with a great grade!

**[Ex4]** $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n!\,n!}$ $\quad \frac{5!}{3!} = \frac{5\cdot4\cdot\cancel{3!}}{\cancel{3!}}$

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2(n+1))!}{(n+1)!\,(n+1)!}\cdot\frac{n!\,n!}{(2n)!}\right| \quad (2n+2)!$$

$$= \lim_{n\to\infty}\left|\frac{(2n+2)(2n+1)\cancel{(2n)!}}{(n+1)\cancel{n!}(n+1)\cancel{n!}}\cdot\frac{\cancel{n!}\,\cancel{n!}}{\cancel{(2n)!}}\right|$$

$$= \lim_{n\to\infty}\frac{(2n+2)(2n+1)}{(n+1)(n+1)} = \frac{4}{1} = 4 > 1$$

Thus, I div. by Ratio Test.

**Thm 13. (Ratio Test)**

Let $\sum a_n$ be any series and $r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$

Then. ① If $r<1$, then $\sum a_n$ converges (absolutely)

② If $\boxed{r>1}$ then $\sum a_n$ diverges.

③ If $r=1$, then the test fails.

$\frac{3^n}{3^{n+1}} = \frac{\cancel{3^n}}{\cancel{3^n}\,3}$

**[Ex3].** $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+5}{3^n}$ $\quad \frac{2^{n+1}}{2^n} = \frac{\cancel{2^n}\,2}{\cancel{2^n}} = 2$

$$r = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(2^{n+1}+5)}{3^{n+1}}\cdot\frac{3^n}{(2^n+5)}\right|$$

$$= \frac{1}{3}\lim_{n\to\infty}\frac{2^{n+1}+5}{2^n+5} = \frac{1}{3}\lim_{n\to\infty}\frac{2+\left(\frac{5}{2^n}\right)^{\to 0}}{1+\left(\frac{5}{2^n}\right)^{\to 0}} = \frac{2}{3} < 1$$

Thus, I con. by Ratio Test (absolutely)

4/1/W I will pass this course with a great grade!

[Ex7] $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)}}$ con < absol? / cond?

div?

By the A.C.T

$$\sum_{n=2}^{\infty}\left|\frac{(-1)^n}{n\sqrt{\ln(n)}}\right| = \sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln(n)}}$$

by Integral Test

Let $a_n = f(x) = \frac{1}{x\sqrt{\ln(x)}}$

① continuous ② positive

③ decreasing on $[2,\infty)$

$$\int_2^{\infty}\frac{1}{x\sqrt{\ln(x)}}dx = \lim_{R\to\infty}\int_2^R\frac{1}{x\sqrt{\ln x}}dx = \lim_{R\to\infty}\int_{\ln 2}^{\ln R}\frac{1}{\sqrt{u}}du$$

$u = \ln x$

$du = \frac{1}{x}dx$

$$= \lim_{R\to\infty} 2\sqrt{u}\Big|_{\ln 2}^{\ln R} = 2\lim_{R\to\infty}\sqrt{\ln R}-\sqrt{\ln 2} = \infty \text{ div}$$

$\sum_{2}^{\infty}\frac{1}{n\sqrt{\ln n}}$ div by Integral Test

By the A.S.T

$a_n = \frac{1}{n\sqrt{\ln n}}$ ① Positive ② decreasing ③ $\lim_{n\to\infty}\frac{1}{n\sqrt{\ln n}} = 0$ ] → True ↳ $\Sigma$ converge conditionally

by A.S.T & A.C.T

4/1/W I will pass this course with a great grade!

① Geometric Series Test

$\sum ar^n$, $|r|<1$, con to $\frac{a_n}{1-r}$

$|r| \geq 1$, div

② Nth Term DT "$\sum a_n$"

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ div

③ P-Test

$\sum \frac{1}{n^p}$ ① $P>1$, con ② $P\leq 1$, div

Alternating P Test

$\sum \frac{(-1)^n}{n^p}$ ① $P>1$, con absol ② $0<P\leq 1$, con condit

④ Integral Test $\sum a_n$

i) let $a_n = f(n)$ – continuous, positive, decreasing

ii) $\int_1^\infty f(n)dx$, con → $\sum a_n$ con; div → $\sum a_n$ div

⑤ DCT

⑥ LCT

i) $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, both con or both div

ii) $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ & $\sum b_n$ con, then $\sum a_n$ con

iii) $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ & $\sum b_n$ div, then $\sum a_n$ div

⑦ ACT

If $\sum |a_n|$ con, then $\sum a_n$ con (absolutely)

⑧ AST $\sum (-1)^n a_n$

Let $a_n$ positive, decreasing, $\lim_{n\to\infty} a_n = 0$

Then $\Sigma$ con

⑨ Ratio Test

$r = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

i) $r<1$, con (absolutely)

ii) $r>1$, div

iii) $r=1$, inconclusive

⑩ Root Test

$\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

i) $\rho<1$ con (absol)

ii) $\rho>1$ div

iii) $\rho=1$, inconcl

### THEOREM 15—The Alternating Series Test

The series

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots$$

converges if the following conditions are satisfied:

1. The $u_n$'s are all positive.
2. The $u_n$'s are eventually nonincreasing: $u_n \geq u_{n+1}$ for all $n \geq N$, for some integer $N$.
3. $u_n \to 0$.

## Summary of Tests to Determine Convergence or Divergence

We have developed a variety of tests to determine convergence or divergence for an infinite series of constants. There are other tests we have not presented which are sometimes given in more advanced courses. Here is a summary of the tests we have considered.

1. **The *n*th-Term Test for Divergence:** Unless $a_n \to 0$, the series diverges.
2. **Geometric series:** $\sum ar^n$ converges if $|r| < 1$; otherwise it diverges.
3. ***p*-series:** $\sum 1/n^p$ converges if $p > 1$; otherwise it diverges.
4. **Series with nonnegative terms:** Try the Integral Test or try comparing to a known series with the Direct Comparison Test or the Limit Comparison Test. Try the Ratio or Root Test.
5. **Series with some negative terms:** Does $\sum |a_n|$ converge by the Ratio or Root Test, or by another of the tests listed above? Remember, absolute convergence implies convergence.
6. **Alternating series:** $\sum a_n$ converges if the series satisfies the conditions of the Alternating Series Test.

## Conditional Convergence

If we replace all the negative terms in the alternating series in Example 3, changing them to positive terms instead, we obtain the geometric series $\sum 1/2^n$. The original series and the new series of absolute values both converge (although to different sums). For an absolutely convergent series, changing infinitely many of the negative terms in the series to positive values does not change its property of still being a convergent series. Other convergent series may behave differently. The convergent alternating harmonic series has infinitely many negative terms, but if we change its negative terms to positive values, the resulting series is the divergent harmonic series. So the presence of infinitely many negative terms is essential to the convergence of the alternating harmonic series. The following terminology distinguishes these two types of convergent series.

> DEFINITION A series that is convergent but not absolutely convergent is called **conditionally convergent**.

The alternating harmonic series is conditionally convergent, or **converges conditionally**. The next example extends that result to the alternating *p*-series.

**EXAMPLE 4** If $p$ is a positive constant, the sequence $\{1/n^p\}$ is a decreasing sequence with limit zero. Therefore, the alternating $p$-series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots, \quad p > 0$$

converges.

If $p > 1$, the series converges absolutely as an ordinary $p$-series. If $0 < p \leq 1$, the series converges conditionally by the alternating series test. For instance,

$$\text{Absolute convergence } \left(p = 3/2\right)\text{:} \quad 1 - \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} - \frac{1}{4^{3/2}} + \cdots$$

$$\text{Conditional convergence } \left(p = 1/2\right)\text{:} \quad 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots$$

■

We need to be careful when using a conditionally convergent series. We have seen with the alternating harmonic series that altering the signs of infinitely many terms of a conditionally convergent series can change its convergence status. Even more, simply changing the order of occurrence of infinitely many of its terms can also have a significant effect, as we now discuss.

43. $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$

## Absolute and Conditional Convergence

Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

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**43.** $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)$

43. diverges by the $n$th-Term Test since $\lim_{n\to\infty}\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)=\lim_{n\to\infty}\left[\left(\sqrt{n+\sqrt{n}}-\sqrt{n}\right)\left(\frac{\sqrt{n+\sqrt{n}}+\sqrt{n}}{\sqrt{n+\sqrt{n}}+\sqrt{n}}\right)\right]$

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$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+\sqrt{n}}+\sqrt{n}}=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{\sqrt{n}}}+1}=\frac{1}{2}\neq 0$$

$$44. \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n+1}}$$

## Absolute and Conditional Convergence

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Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

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## Absolute and Conditional Convergence

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$$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}} \text{ converges; but } \lim_{n\to\infty} \frac{\left(\frac{1}{\sqrt{n}+\sqrt{n+1}}\right)}{\left(\frac{1}{\sqrt{n}}\right)} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}} = \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$$

**44.** $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n+1}}$

## Absolute and Conditional Convergence

Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

44. converges conditionally since $\left\{\frac{1}{\sqrt{n}+\sqrt{n+1}}\right\}$ is a decreasing sequence of positive terms converging to 0 $\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$ converges; but $\lim_{n\to\infty} \frac{\left(\frac{1}{\sqrt{n}+\sqrt{n+1}}\right)}{\left(\frac{1}{\sqrt{n}}\right)} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+\sqrt{n+1}} = \lim_{n\to\infty} \frac{1}{1+\sqrt{1+\frac{1}{n}}} = \frac{1}{2}$ so that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ which is a divergent $p$-series

**36.** $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

### Absolute and Conditional Convergence

Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

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but $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges

**41.** $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$

### Absolute and Conditional Convergence

Which of the series in Exercises 15–48 converge absolutely, which converge, and which diverge? Give reasons for your answers.

converges conditionally since $\frac{\sqrt{n+1}-\sqrt{n}}{1}\cdot\frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}=\frac{1}{\sqrt{n+1}+\sqrt{n}}$ and $\left\{\frac{1}{\sqrt{n+1}+\sqrt{n}}\right\}$ is a decreasing sequence of

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$$=\lim_{n\to\infty}\frac{1}{\sqrt{1+\frac{1}{n}}+1}=\frac{1}{2}$$

- Questions?
We’re here to help.
Remember the tutoring center is open!
Study hard, best of luck!