Complex Analysis HW4

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These problems are usually from the course textbook [1], but sometimes directly from Prof. Blackmore, as is the case in this instance.

1 Problem 1 - Show that the countour integral is invariant under different parametrizations.

See original text for full conditions on the problem.

We define a mapping H such that $\tau = H(t), H : [a, b] \to [\alpha, \beta]$, noticing that H is one to one and invertible with $t = H^{-1}(\tau)$ and $H(a) = \alpha, H(b) = \beta$, We then have that:

$\psi(\tau) = \psi(H(t)) = \phi(t)$.

Via chain rule we have that:

$\frac{d\psi}{d\tau}\frac{d\tau}{dt} = \frac{d\psi}{dH}\frac{dH}{dt}$.

$\frac{d\psi}{d\tau}d\tau = \frac{d\psi}{dH}\frac{dH}{dt}dt$.

We also have that;

$\psi(H(t)) = \phi(t)$ implies that $\frac{d\psi}{dH}\frac{dH}{dt} = \frac{d\phi}{dt} = \frac{d\psi}{dH}\frac{dH}{dt}dt = \frac{d\phi}{dt}dt$.

Thus, we use these equations and we have that:

$\int_c f(z)dz = \int_\alpha^\beta f(\psi(\tau))\frac{d\psi}{d\tau}(\tau)d\tau = \int_a^b f(\psi(H(t)))\frac{d\psi}{dH}\frac{dH}{dt}dt = \int_a^b f(\phi(t))\frac{d\phi}{dt}(t)dt$.

$\square$

2 Conclusion

Thank you to Prof. Blackmore for his instruction, lectures and office hours effort. I look forward to any feedback and learning more of the material in this course.

References

[1] Mark J Ablowitz, Athanassios S Fokas, and AS Fokas. *Complex variables: introduction and applications.* Cambridge University Press, 2003.