# Complex Analysis MAT656 August 2019.

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May, 2020

These problems are from the pdf received from Prof. Blackmore. The course textbook is Ablowitz and Fokas [1].

## 1 August 2019 Problem 6. - REDACTED

$$f(z) = 1 + \frac{1}{z} + \frac{1}{2!z^2} + ... + \frac{1}{n!z^n}$$

### 1.1 a -

$$f(z) = \frac{1}{2\pi i}\int \frac{1}{f(z)} f^{'}(z)dz$$

#### 1.1.1 Solution:

By the argument principle, this is the number of zeros of f minus the number of poles of f (counted with multiplicity) inside the disk of radius r

### 1.2 b - REDACTED

$r$?

To make the notation clearer, we redefine $f$ as $f_n$ then we have that $f_n(z) = \sum_{k=0}^{n} z^{-k}/k!$ Note that $f_n(z) \longrightarrow e^{1/z}$ as $n \longrightarrow \infty$ for each z (this is just the Taylor series for $e^{1/z}$). By uniform convergence, the limit as $n \longrightarrow \infty$ of the integral of $\frac{1}{f_n} f_n^{'}$ is the same as the integral of the limit as $n \longrightarrow \infty$ of $\frac{1}{f_n} f_n^{'}$. Since $f_n(z) \longrightarrow e^{1/z}$, we find that $f_n^{'}(z) \longrightarrow -\frac{1}{z^2}e^{\frac{1}{z}}$, so $\frac{1}{f_n(z)} f_n^{'}(z) \longrightarrow -\frac{1}{z^2}$. In other words, $lim_{n\to\infty} \frac{1}{(2\pi i)}\int_{|z|=r} \frac{1}{f_n(z)} f_n^{'}(z)dz = \frac{1}{(2\pi i)}\int_{|z|=r} -\frac{1}{z^2}dz$. This integral can be evaluated by parameterizing the curve $|z| = r$, and we find it is 0.

### 1.3 REDACTED

For fixed r, the integral approaches 0 as n approaches infinity. Since the integral always takes integral values, we find that the integral is exactly 0 for all n sufficiently large. Thus $f(z)$ has no zeroes or poles for large n.

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## 2 Conclusion

Thank you to Prof. Blackmore for his instruction, lectures and office hours effort. It's been an honor and a privilege to be your student. I look forward to any feedback and learning more of the material in this course and beyond.

## References

[1] Mark J Ablowitz, Athanassios S Fokas, and AS Fokas. *Complex variables: introduction and applications*. Cambridge University Press, 2003.