Three papers in three and a third minutes each.

- By: José L. Pabón, N.J.I.T. Mathematical Sciences.

Advisors: Prof. Oza, Prof. Siegel.

Abstract – Research Plan.

- **Oza, Ristroph, Shelly model for hydrodynamic swimmers.**
- Matches experimental data well.
- But does not consider 3D.
- Also does not consider flow generated by neighbor swimmers.

+

- **Keller et al slender body potential flow results.**
- Gives us formulas for the flow generated by neighbor swimmers.
- Scaffolding to determine 3D interactions.

=

Proposal project.

Increased precision hydro-dynamic 3D model.

Agenda

- Brief discussion of the following papers:
- 'Lattices of Hydrodynamically Interacting Flapping Swimmers' by Oza, Ristroph, Shelley.
- 'Axially symmetric potential flow around a slender body' by Handelsman and Keller.
- 'Uniform Asymptotic Solutions for Potential Flow around a thin airfoil and the Electrostatic Potential about a thin conductor' by Geer and Keller.

Thanks !

- Prof. Oza, Prof. Siegel.

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- Prof. Michalopoulou, Prof. Luke, Prof. Matveev, Prof. Muratov, Prof. Blackmore, Prof. Hamfeldt, Prof. Askham, Prof. Lushi, Prof. Milojevic, Prof. Petropoulous.

Thanks !

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- Ryan (s), Rituparna, Axel, Erli, Yasser, Yichen, Connor, Lauren, Soheil, Brandon, Subhrasish, Malik, Binan, Guangyuan, Nick, Austin, Moshe, Jake, Sam, Atul, Fataou, Chyavi, Milad, Prianka, John, et al.

Thanks !

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- My family… especially my sister Maria!

Goal:

- Share some of my research literature review findings.

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- Give a glimpse of the direction of my future research.

Oza et al, Hydrodynamic swimmers

PHYSICAL REVIEW X **9**, 041024 (2019)

Lattices of Hydrodynamically Interacting Flapping Swimmers

Anand U. Oza*
Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, New Jersey 07102, USA

Leif Ristroph
Courant Institute of Mathematical Sciences, New York University, New York, New York 10012, USA

Michael J. Shelley
Courant Institute of Mathematical Sciences, New York University, New York, New York 10012, USA and Center for Computational Biology, Flatiron Institute, New York, New York 10010, USA

(Received 10 April 2019; revised manuscript received 13 August 2019; published 1 November 2019; corrected 15 November 2019)

Oza et al, Hydrodynamic swimmers.

- All media and videos are credited to source material from Ananda Oza and Sophie Ramananarivo and the Applied Mathematics Laboratory of the Courant Institute of Mathematical sciences.

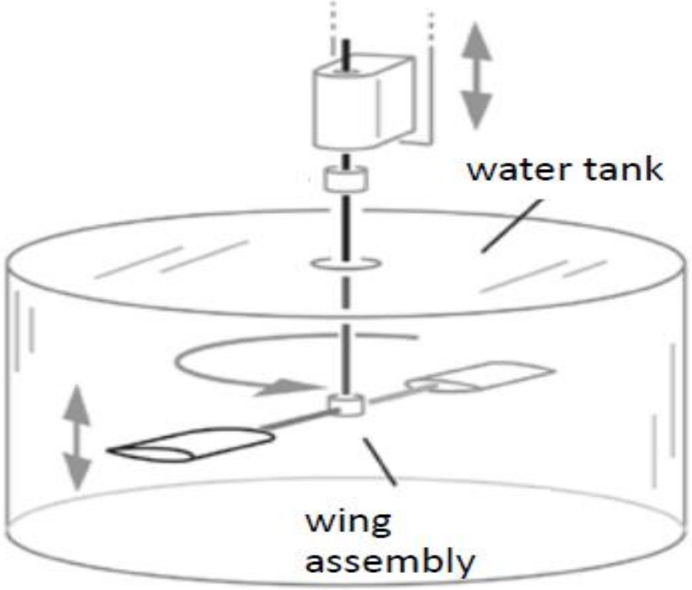
Oza et al, Hydrodynamic swimmers

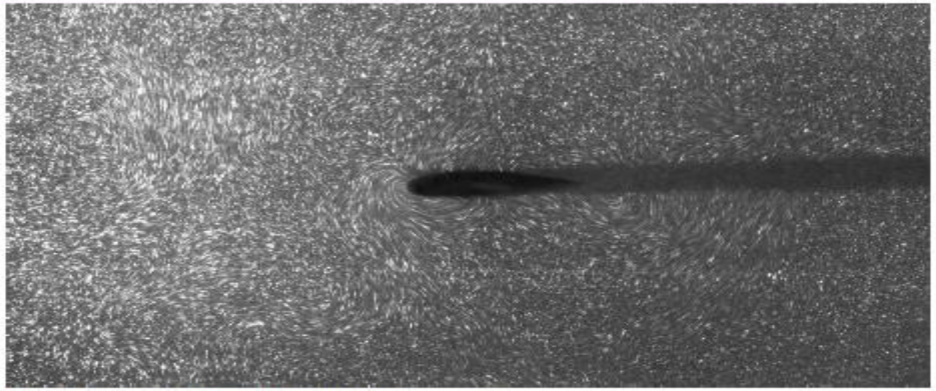
- The work centers around the complex collective dynamics of schools of fish and flocks of birds and the influence of hydrodynamics on the self organization of these.

Oza et al, Hydrodynamic swimmers.

Model system for schooling swimmers

Experimental setup



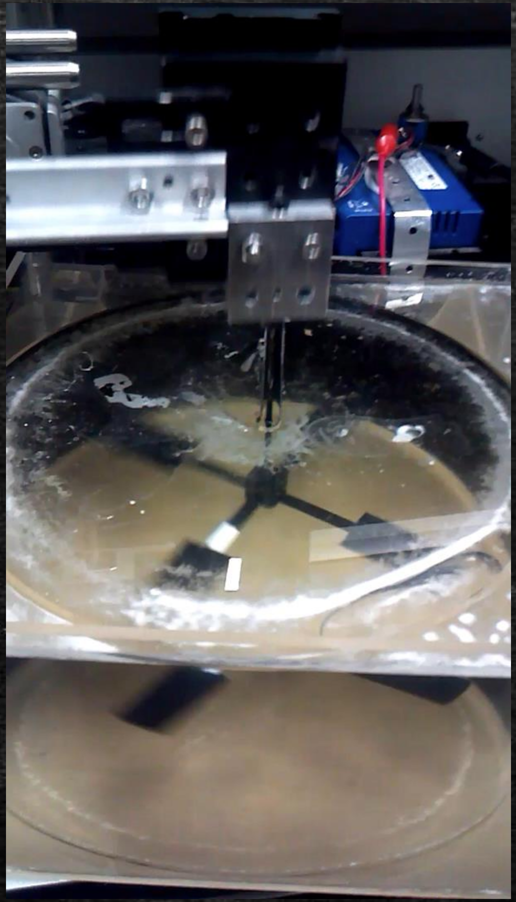


Video credit: Leif Ristroph

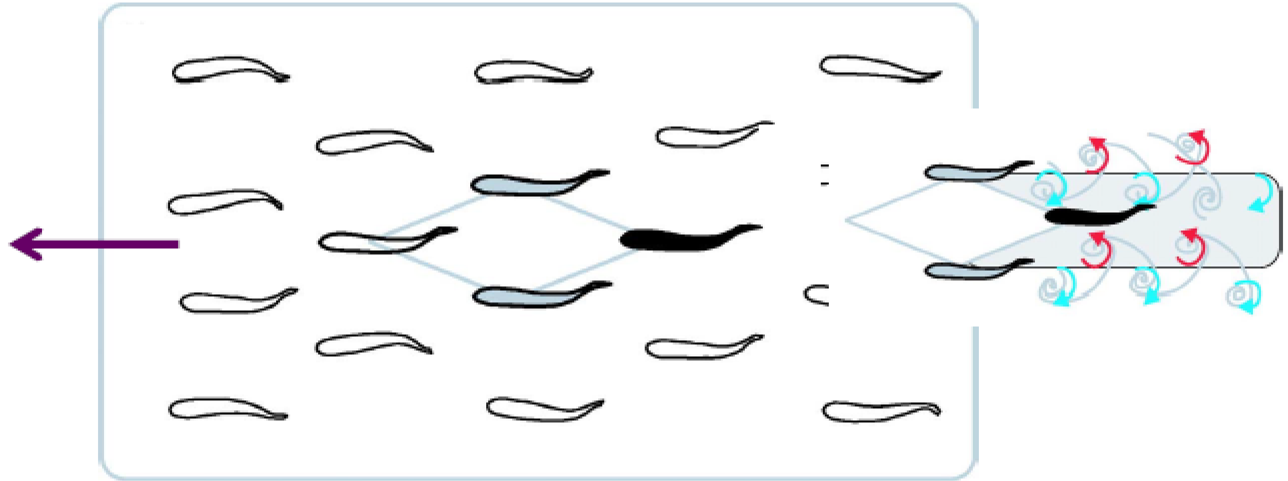
- Wings interact through their wakes.
- Imposed flapping motions, but freely swimming.

Becker, Masoud, Newbolt, Shelley & Ristroph. *Nature Comm.* (2015)

Oza et al, Hydrodynamic swimmers.



Oza et al, Hydrodynamic swimmers.



- Assume fish in first column shed reverse von Kármán vortex streets.

- Flapping swimmers extract energy from upstream vortices.
  - Claimed that the diamond lattice with the smallest lateral spacing is optimal.

Oza et al, Hydrodynamic swimmers.

- Clarifications / definitions:

Oza et al, Hydrodynamic swimmers.

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- Vortex shedding is an oscillating flow that fluid generates when it flows past a bluff body at certain velocities, depending on the size and shape of the body. On a streamlined body, as opposed to bluff body, the flow would be different.

Oza et al, Hydrodynamic swimmers.

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- Vortex shedding is an oscillating flow that fluid generates when it flows past a bluff body at certain velocities, depending on the size and shape of the body. On a streamlined body, as opposed to bluff body, the flow would be different.
- A Karman vortex street is the repeating pattern of swirling vortices created by the process of vortex shedding.

Oza et al, Hydrodynamic swimmers.

Oza et al, Hydrodynamic swimmers

- The work centers around the complex collective dynamics of schools of fish and flocks of birds and the influence of hydrodynamics on the self organization of these.
- An iterated map is formulated to describe the hydrodynamic interactions between the vortices and the downstream swimmers.

Oza et al, Hydrodynamic swimmers.

Iterated map

$$u_{n+1} = u_n + \frac{T}{m}\left(F_D(u_n) + F_T[x_n, u_n, v_n, \omega_n(z)]\right)$$

drag + thrust

$$x_{n+1} = x_n + u_{n+1}T$$

$$\omega_{n+1}(z) = \omega_n(z)\mathrm{e}^{-T/\tau} + (-1)^n\gamma \sum_{j=-1}^{1} \delta\left(z - (z_{n+1} + jL)\right)$$

vortex shedding

$$\text{where} \quad z_{n+1} = 2a + \frac{x_{n+1} + x_n}{2} + i(-1)^n h_0$$

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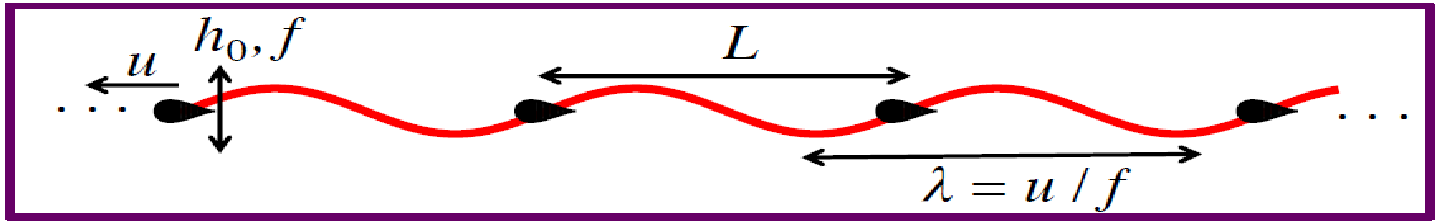
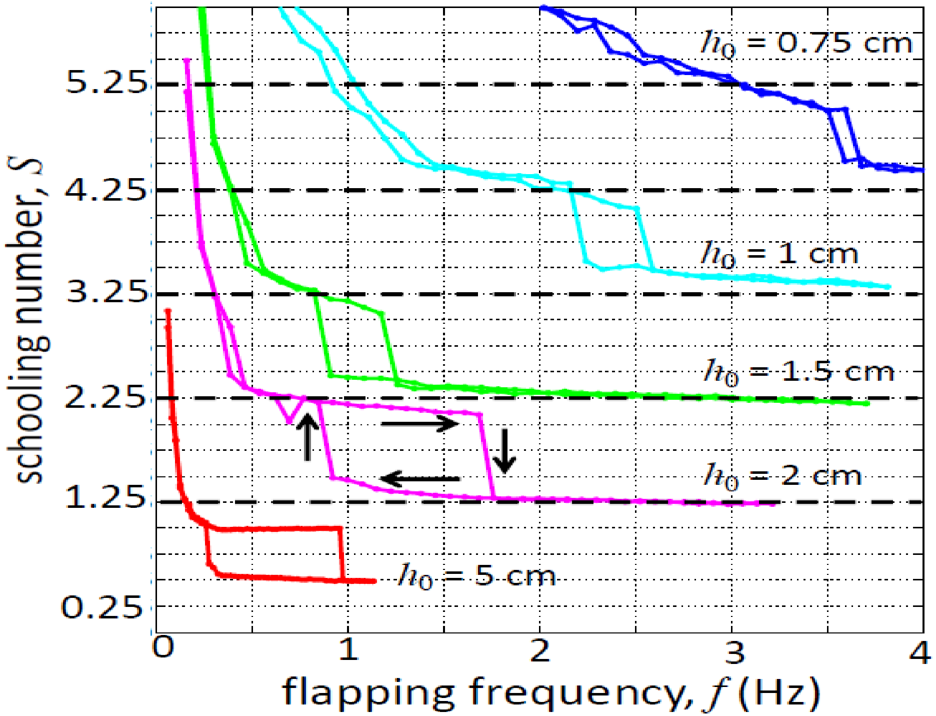
Oza et al, Hydrodynamic swimmers.

Experimental results





- Schooling number $S = L / \lambda$: strokes separating neighbors.
  - Preferred values $S \approx n$ + ¼.
- Bistability of states & **speedup** due to collective interaction.
- Numerical simulations are expensive ($\mathrm{Re} \approx 10^4$).
  - Alben *et al*. 2012, Daghooghi & Borazjani 2015, Zhu *et al*. 2014, Peng *et al*. 2018, Dai *et al*. 2018, …

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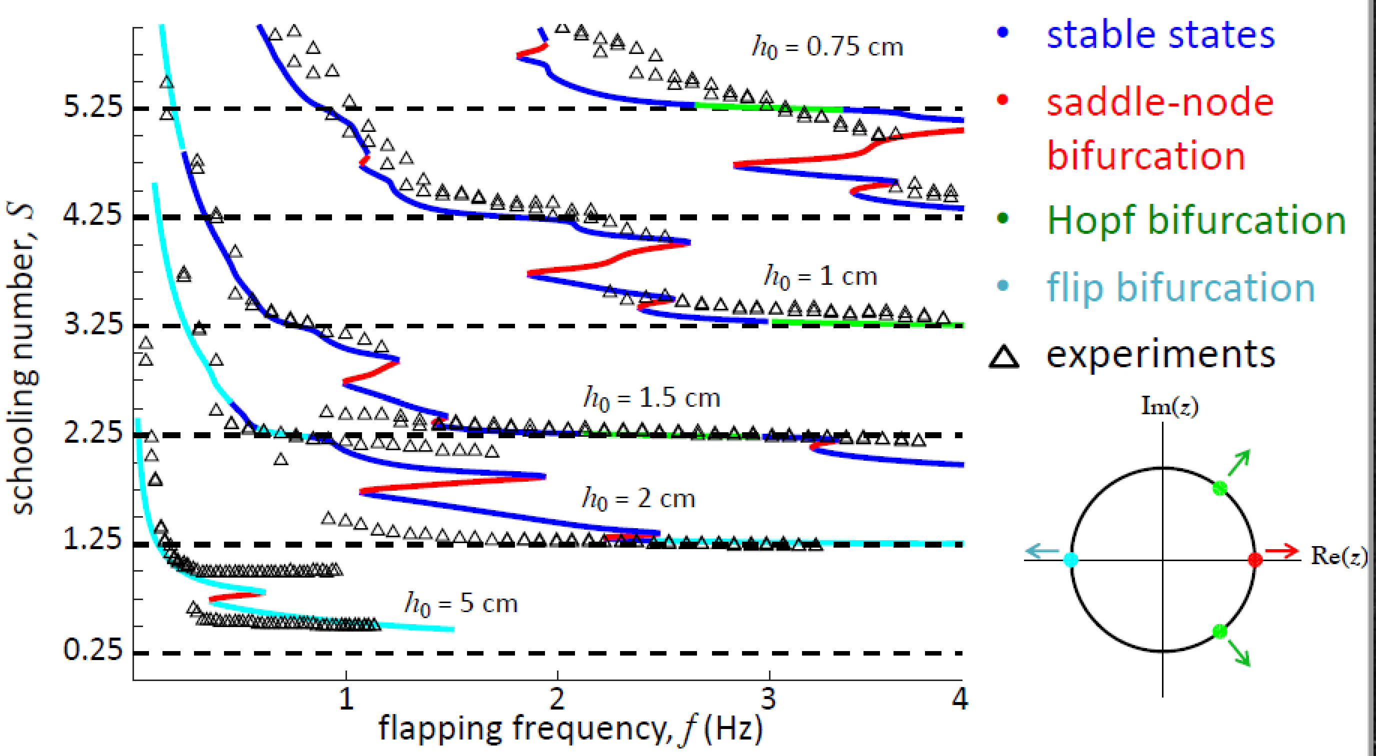
- Thanks to the kind folks at C.I.M.S., we have experimental results.
- The results exhibit good agreement with previously published experimental data.

Oza et al, Hydrodynamic swimmers.

Comparison with experiment



Three parameters - vortex decay time, vortex strength & drag coefficient - fit data across **all** flapping frequencies and amplitudes.

Oza et al, Hydrodynamic swimmers.

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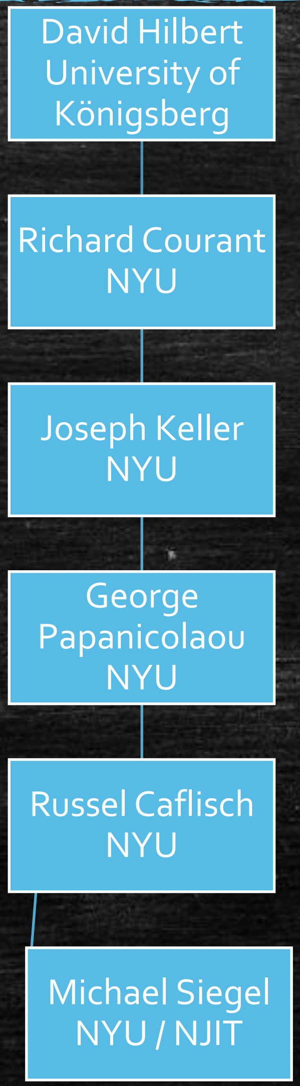
- Open questions remain…

Oza et al, Hydrodynamic swimmers.

- The work presents outstanding results.
- Open questions remain…
- One open problem question is – what are the hydrodynamics of the swimmers situation when the flow generated by neighbors is considered?

Joseph Keller – Professor Emeritus.



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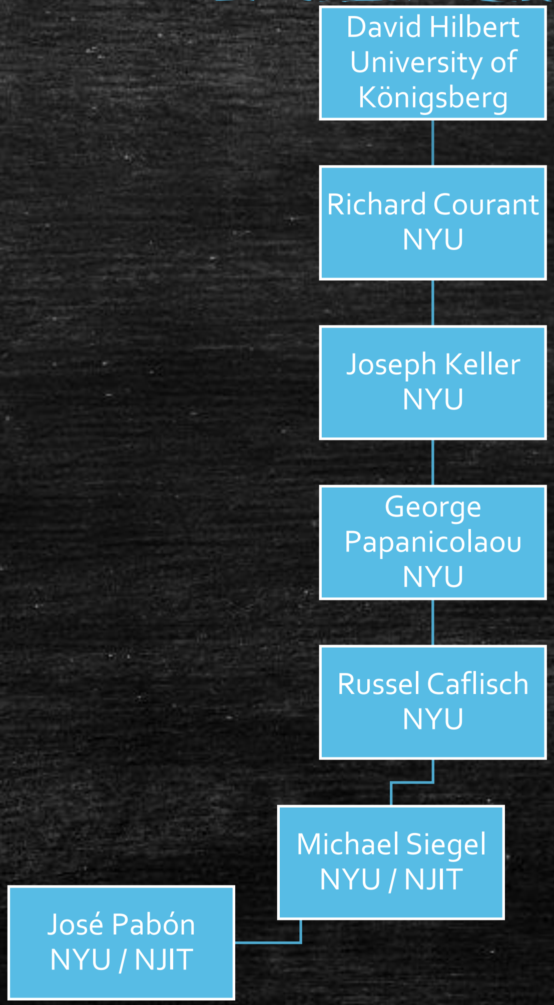

Keller et al, Slender body potential flow.

- The potential due to the body is represented as a superposition of potentials of point sources distributed along a segment of the axis inside the body.

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- The potential due to the body is represented as a superposition of potentials of point sources distributed along a segment of the axis inside the body.
- The source strength distribution satisfies a linear integral equation.
- A uniform asymptotic expansion of its solutions is obtained with respect to the slenderness ratio: the maximum radius of the body divided by its length.

Keller et al, Slender body potential flow.

- The potential due to the body is represented as a superposition of potentials of point sources distributed along a segment of the axis inside the body.

- The paper defines the potential of an irrotational axis-symmetric flow of an incompressible, inviscid fluid past a slender body of revolution as Φ, with $\Phi = \Phi_0 + \Phi_b$.
- Φ_0 is the given function for the potential of the incident flow.
- Φ_b is the potential of the incident flow due to the presence of the body which is the quantity to be determined and vanishes at infinity.
- Both Φ_0, Φ_b are harmonic functions in the exterior of the body.
- The surface of the body is assumed to be fixed and the normal derivative of Φ must vanish.

Keller et al, Slender body potential flow.

- The paper goes on to frame the situation using cylindrical coordinates (r, θ, x) with the origin at the body's nose and the x-axis along its symmetry axis.
- The axial symmetry of the flow make Φ_0 and Φ_b independent of θ and analytic in r^2, x.
- The work formulates equations for the profile curves and the cross sectional areas, which we do not share out of a concession to the time available for this talk.
- The authors represent Φ_b as a superposition of point source potentials distributed along a segment of the x-axis inside the body with unknown strength $f(x, \epsilon)/$ unit length. ϵ is the slenderness ratio, defined to be the maximum radius of the body relative to its length.

Keller et al, Slender body potential flow.

- The equation derived is:
- $$\Phi_b = \Phi(xr^2, \epsilon) = \Phi_0(x, r^2) - \frac{1}{4\pi}\int_{\alpha(\epsilon)}^{\beta(\epsilon)} \frac{1}{(x-\xi)^2 + r^2)^{0.5}} f(x, \epsilon) d\xi.$$
- The Stokes stream function Ψ is related to Φ via :
- $$\Psi_x = -r\Phi_r, \Psi_r = r\Phi_x.$$
- After some calculations, the authors find that:
- $$\Psi_0(x, \epsilon S(x)) = \frac{1}{4\pi}\int_{\alpha(\epsilon)}^{\beta(\epsilon)} \frac{1}{(x-\xi)^2 + (\epsilon S(x))^2)^{0.5}} (x-\xi) f(\xi, \epsilon) d\xi.$$
- In the above equation, $\epsilon S(x) = \frac{1}{\pi} A(x)$ defines $A(x)$ as the cross-sectional area of the body at x.

Keller et al, Slender body potential flow.

- The potential due to the body is represented as a superposition of potentials of point sources distributed along a segment of the axis inside the body.
- The source strength distribution satisfies a linear integral equation.
- A uniform asymptotic expansion of its solutions is obtained with respect to the slenderness ratio: the maximum radius of the body divided by its length.
- Results include: expansions of the potential, the virtual mass and the dipole moment are obtained, as well as the flow about the body.

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- The method of analysis involved a technique for the asymptotic solution of integral equations.

Keller et al, Slender body potential flow.

- Much more information to be found in the literature… moving on!

Keller et al, Uniform Asymptotic Solutions.

- This work considers the two-dimensional irrotational flow of an incompressible inviscid fluid past a thin cylindrical airfoil and the two-dimensional field in the exterior of a thin cylindrical conductor.

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- A superposition of potentials of point sources distributed along a segment of the axis inside C are used to find the overall potential.

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- The work defines closed curve $C := y = \pm\epsilon(S(x))^{\frac{1}{2}}$ for $0 \leq x <\leq 1, S(0) = S(1) = 0, \max S(x) = 1$.
- Keller et al again define a function Φ, with $\Phi = \Phi_0 + \Phi_b$.
- Φ_0 is the given function for the potential of the incident flow.
- Φ_b is the disturbance potential due to the presence of a rigid body bounded by the closed curve C and that vanishes at infinity.
- Both Φ_0, Φ_b are harmonic functions in a neighborhood of C.
- The authors calculate that:

$$\Phi(x, y^2, \epsilon) = \Phi_0(x, y^2) - \frac{1}{4\pi}\int_{\alpha(\epsilon)}^{\beta(\epsilon)} \log\{(x-\xi)^2 + y^2\} f(\xi, \epsilon) d\xi.$$

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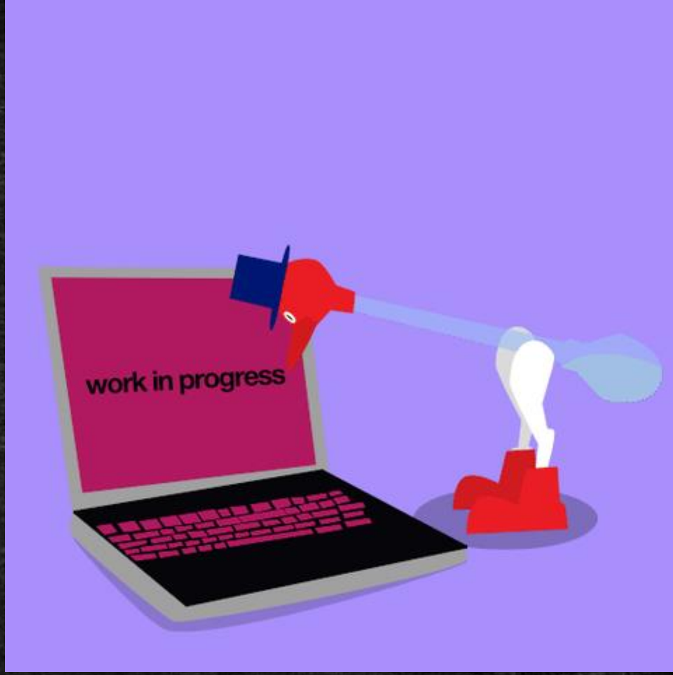
- Uniform asymptotic expansions of these two potentials are obtained with respect to the slenderness ratio of the profile curve C of the cylinder.
- A superposition of potentials of point sources distributed along a segment of the axis inside C are used to find the overall potential.
- The source strength distribution satisfies a linear integral equation.
- Complete expansions of the virtual mass, the polarizability, and the lift of the cylinder are obtained as results.

Thanks for listening! Questions / comments are warmly welcome… the work in progress.



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