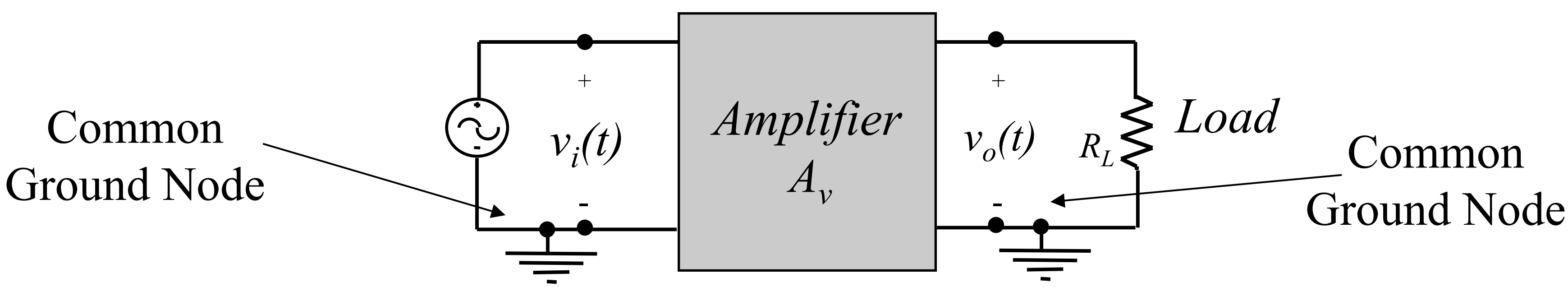
BME 301

11 - Operational Amplifiers

Basic Amplifier Types

- An amplifier produces an output signal with the same wave shape as the input signal but usually with a larger amplitude.

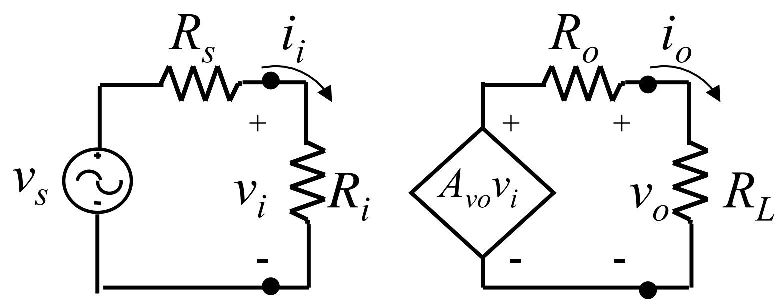
$$v_o(t)=A_v\, v_i(t)$$



- A_v is called the voltage gain and if <0 then the amplifier is inverting; otherwise non-inverting.

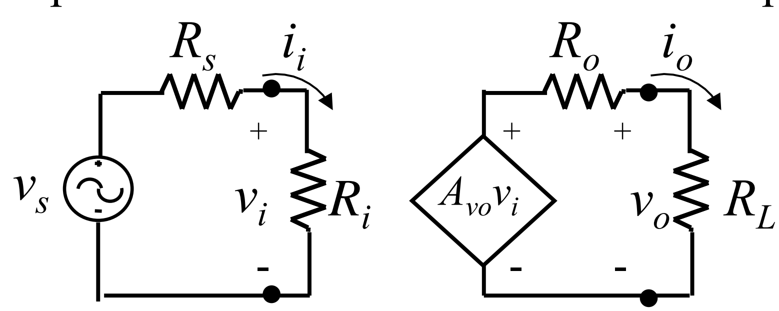
Voltage-Amplifier Model

- Circuit Parameters:
  - Starting from the left
    - v_s is the source input voltage and represents a microphone of an audio amplifier or the action potential of a muscle.
    - R_S is the resistance of the source voltage device
    - v_i is the voltage to the input of the amplifier
    - i_i is the current flowing in the input of the amplifier
    - $R_i = v_i / i_i$ is the resistance at the input to the amplifier



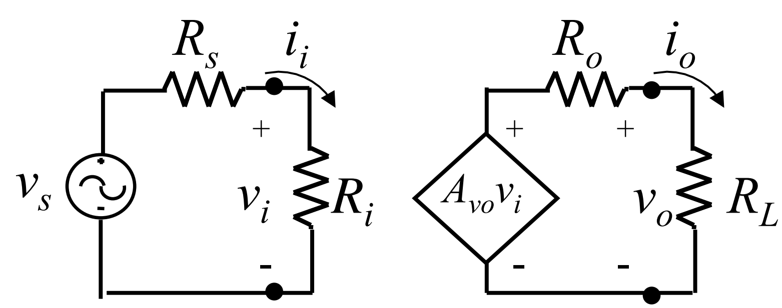
Voltage-Amplifier Model

- Circuit Parameters:
  - Moving to the right side of the model
    - We see on the right part of the model a new icon called dependent voltage source.
      - It's voltage which depends on a voltage at left side of the model, the input voltage at the amplifier
      - In particular, it's the voltage across the input resistor R_i.
      - And the gain of the amplifier when nothing is connected to it's output
    - Open Circuit Voltage Gain A_{vo}
    - R_o is the resistance at output of the amplifier
    - V_o is the voltage at the output of the ampilier
    - i_0 is the current flowing in the output of the amplifier
    - $R_L=v_o/i_o$ represents the device connect to the output of the amplifier.



Voltage-Amplifier Model

- Performance parameters:
  - Voltage Gain $A_v = v_o / v_i$ from the input of the amplifier to the output of the circuit
  - Voltage Gain $A_{vs} = v_o / v_s$ from the source of the amplifier model to the output of the circuit
  - Current Gain $A_i = i_o / i_i$
  - Power Gain $G = P_o / P_i$ from the input of the amplifier to the output of the circuit
  - Power Gain $G_S = P_o / P_s$ from the source of the amplifier model to the output of the circuit



Ideal Amplifiers

- Some calculations:

Starting from the output (right side of the model) and using voltage division:

$$v_o = \frac{R_L}{R_L + R_O} A_{vo} v_i$$

Or

$$A_v = \frac{v_o}{v_i} = \frac{R_L}{R_L + R_O} A_{vo}$$

Now

looking at the left part of the model and using voltage division:

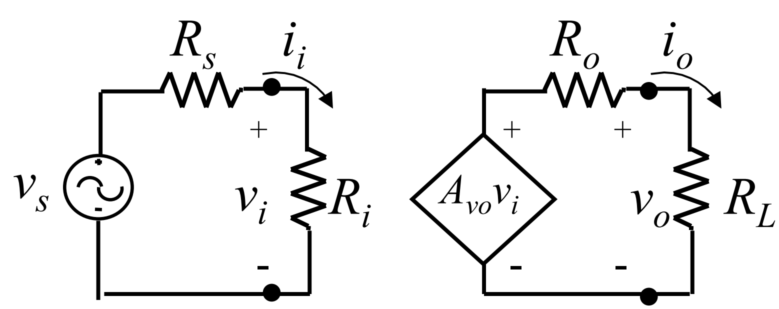
$$v_i = \frac{R_i}{R_i + R_s} v_s$$

Combining the two:

$$v_o = \frac{R_L}{R_L + R_O} A_{vo} \frac{R_i}{R_i + R_s} v_s$$

Or

$$A_{vs} = \frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} A_{vo} \frac{R_L}{R_L + R_O}$$



Ideal Amplifiers

- Some calculations:

Starting from the output (right side of the model) and using voltage division:

$$i_O = \frac{v_o}{R_L} = \frac{1}{R_L}\,\frac{R_L}{R_L + R_O} A_{vo} v_i = \frac{1}{R_L + R_O} A_{vo} v_i \text{ substituting for } v_o$$

$$i_O = \frac{1}{R_L + R_O} A_{vo} v_i = \frac{1}{R_L + R_O} A_{vo} i_i R_i = \frac{R_i}{R_L + R_O} A_{vo} i_i \text{ substituting for } v_i$$

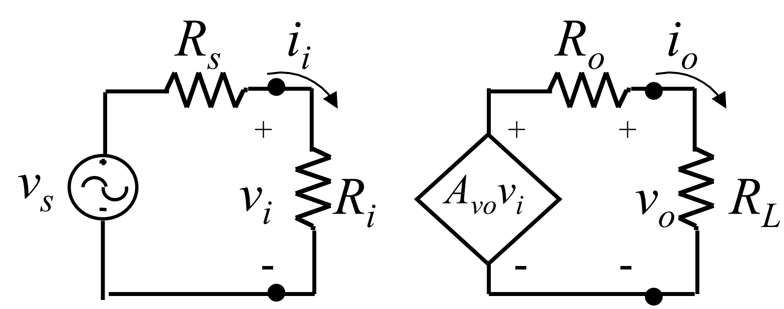
Or

$$A_i = \frac{i_o}{i_i} = \frac{R_i}{R_L + R_O} A_{vo}$$

Now the power Gain

$$G = \frac{v_o i_O}{v_i i_i} = \frac{v_o}{v_i}\,\frac{i_O}{i_i} = A_v A_i$$

$$G_S = \frac{v_o i_O}{v_s i_i} = \frac{v_o}{v_s}\,\frac{i_O}{i_i} = A_{vs} A_i$$



Ideal Amplifiers

- Performance Parameters:

Going back to the gain from the source to the output

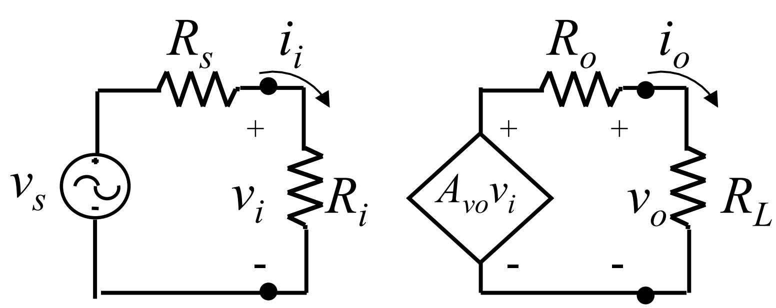
$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} A_{vo} \frac{R_L}{R_L + R_O}$$

This states the gain of the amplifier depends on the external components.

This is BAD!!!!!

However, if $R_i \to \infty$ and $R_O \to 0$; then $\frac{R_i}{R_i + R_s} \to 1$ and $\frac{R_L}{R_L + R_O} \to 1$.

Therefore, $\frac{v_o}{v_s} \to A_{vo}$ and the gain of the amplifier is independent of the external components.



Another Amplifier
The Differential Amplifier

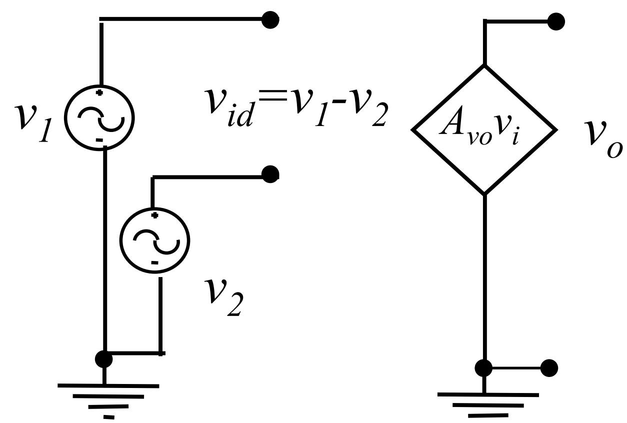
- Output of the amplifier is a function of the difference of the inputs:

$$v_o = A_d(v_{i1} - v_{i2})$$

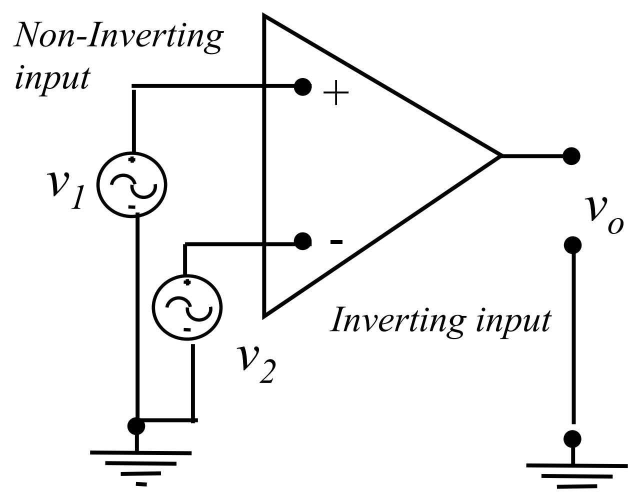
- However, most real Differential Amplifier are affected by the average of the input and we define the common mode input signal

$$v_{\text{icm}} = 1/2(v_{i1} + v_{i2})$$

- Therefore, $v_o = A_d(v_{i1} - v_{i2}) + A_m\, v_{\text{icm}}$ and the ratio of A_d to A_m is called the common mode rejection ratio



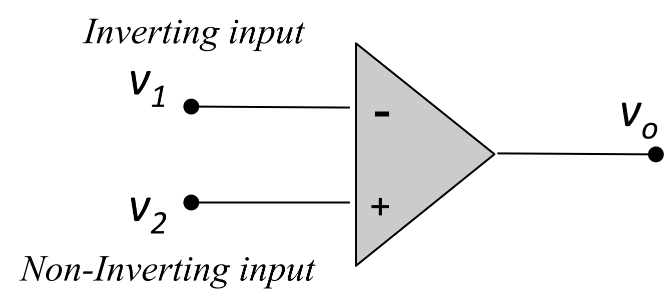
Model



Icon

Operational Amplifiers

- An operational Amplifier is an ideal differential amplifier with the following characteristics:
  - Infinite input impedance, R_i is infinite
  - Zero output impedance, R_o is zero
  - Infinite gain for the differential signal, A_d is infinite
  - Zero gain for the common-mode signal
  - Infinite Bandwidth



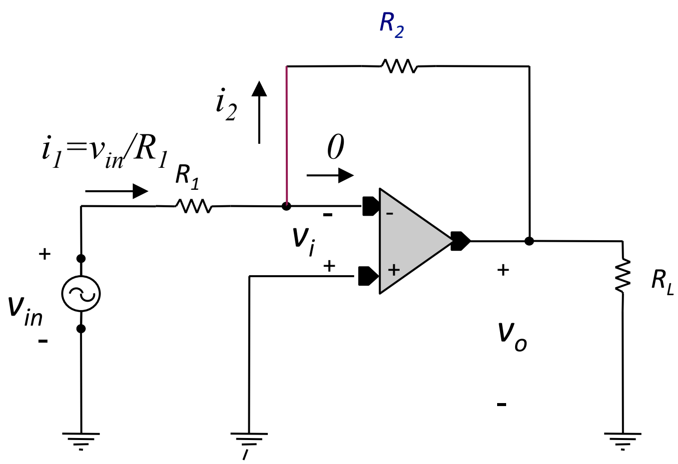
Operational Amplifier Feedback

- Operational Amplifiers are used with negative feedback
- Feedback is a way to return a portion of the output of an amplifier to the input
  - Negative Feedback: returned output opposes the source signal
  - Positive Feedback: returned output aids the source signal
- For Negative Feedback
  - In an Op-amp, the negative feedback returns a fraction of the output to the inverting input terminal forcing the differential input to zero.
  - Since the Op-amp is ideal and has infinite gain, the differential input will exactly be zero. This is called a virtual short circuit
  - Since the input impedance is infinite the current flowing into the input is also zero.
  - These latter two points are called the summing-point constraint.

Operational Amplifier Analysis Using the Summing Point Constraint

- In order to analyze Op-amps, the following steps should be followed:
  1. Verify that negative feedback is present
  2. Assume that the voltage and current at the input of the Op-amp are both zero (Summing-point Constraint
  3. Apply standard circuit analyses techniques such as Kirchhoff's Laws, etc.to solve for the quantities of interest.

Example: Inverting Amplifier



1. Verify Negative Feedback: Note that a portion of v_o is fed back via R_2 to the inverting input. So if v_i increases and, therefore, increases v_o, the portion of v_o fed back will then have the affect of reducing v_i (i.e., negative feedback).
2. Use the summing point constraint.
3. Use KVL at the inverting input node for both the branch connected to the source and the branch connected to the output

$$v_{in} = i_1 R_1 + 0 \text{ since } v_i \text{ is zero due to the summing-point constraint}$$

$$i_1 = i_2 \text{ due to the summing-point constraint}$$

$$v_0 = -i_2 R_2 + 0 \text{ since } v_i \text{ is zero}$$

$$= -\frac{R_2}{R_1} v_{in} \text{ which is independent of } R_L \text{ (note that the output is opposite to the input : inverted)}$$

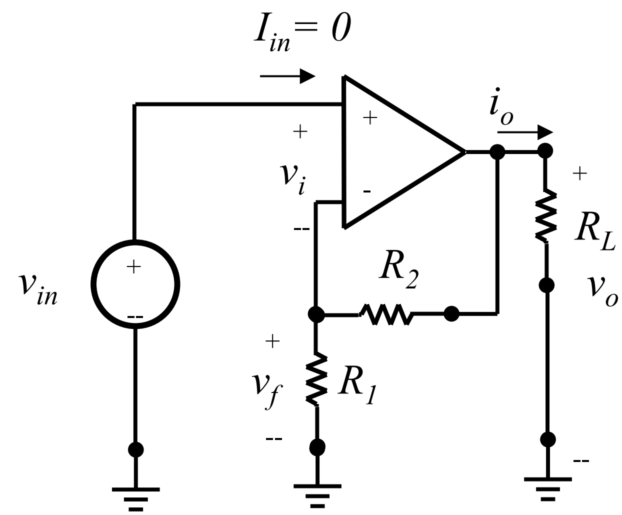
Op-amp

- Because we assumed that the Op-amp was ideal, we found that with negative feedback we can achieve a gain which is:

1. Independent of the load
2. Dependent only on values of the circuit parameter
3. We can choose the gain of our amplifier by proper selection of resistors.

Non-inverting Amp

1. First check: negative feedback?
2. Next apply, summing point constraint
3. Use circuit analysis



$$v_{in} = v_i + v_f = 0 + v_f = v_f$$

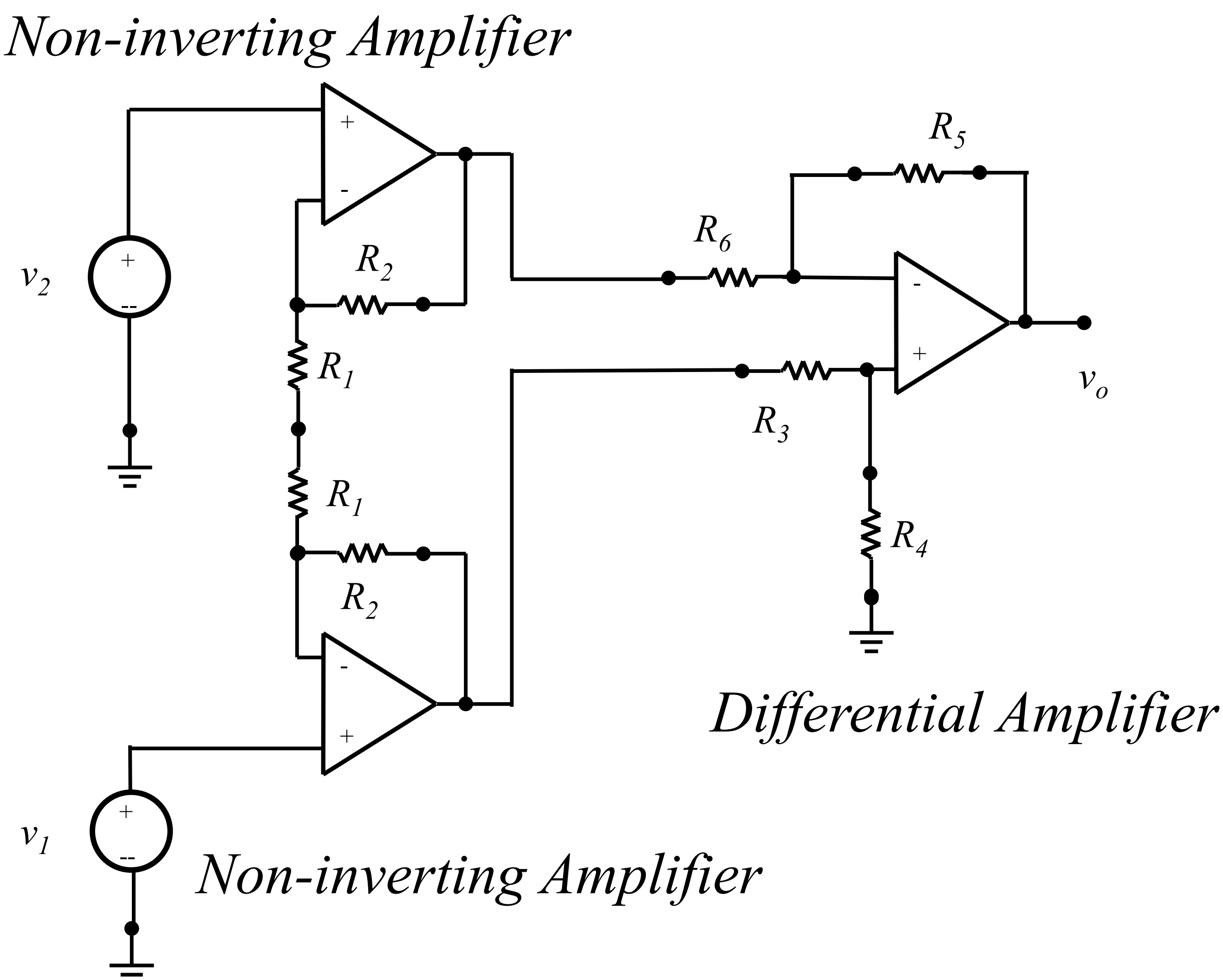
$$v_f = \frac{R_1}{R_1 + R_2} v_o = v_{in};$$

$$A_v = \frac{v_o}{v_{in}} = \frac{R_2 + R_1}{R_1} = 1 + \frac{R_2}{R_1}$$

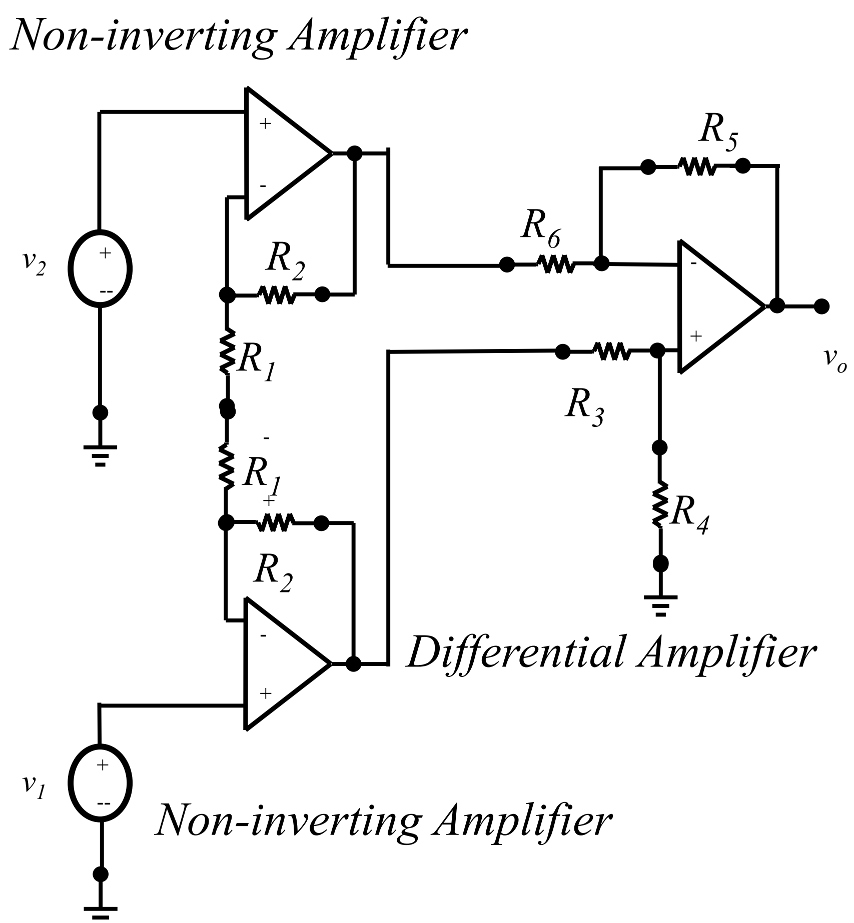
Note:

1. The gain is always greater than one
2. The output has the same sign as the input

Medical Instrumentation Amplifier

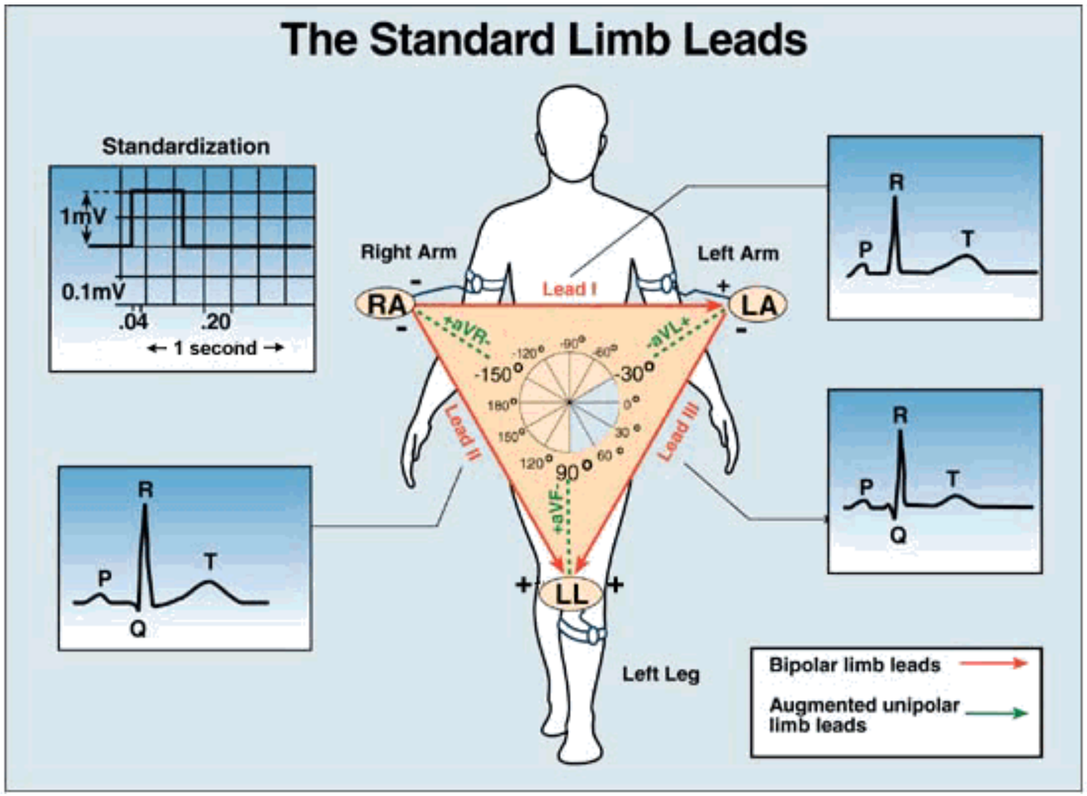


Medical Instrumentation Amplifier

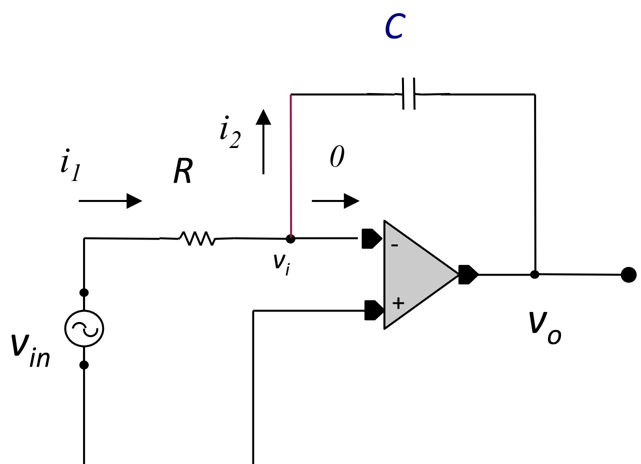


$$v_o = \frac{R_5}{R_6}(1 + \frac{R_2}{R_1})(v_1 - v_2)$$

Uses of the Differential Amplifier

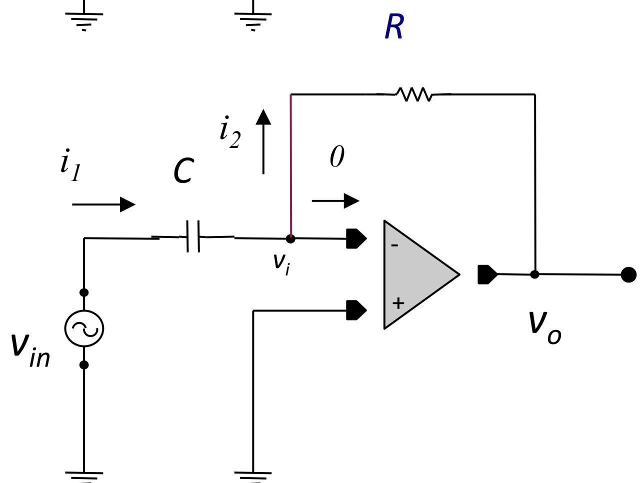


Integrators and Differentiators



$$i_1(t) = \frac{v_{in}(t)}{R} = i_2(t)$$

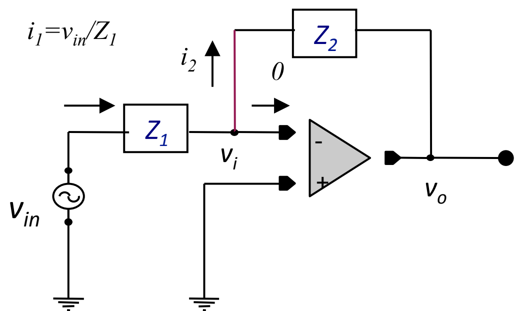
$$v_o = -\frac{1}{C}\int_0^t i_2(x)dx = -\frac{1}{RC}\int_0^t v_{in}(x)dx$$



$$i_1(t) = \frac{Cdv_{in}(t)}{dt} = i_2(t)$$

$$v_o = -i_2(t)R = -RC\frac{dv_{in}(t)}{dt}$$

Frequency Analysis



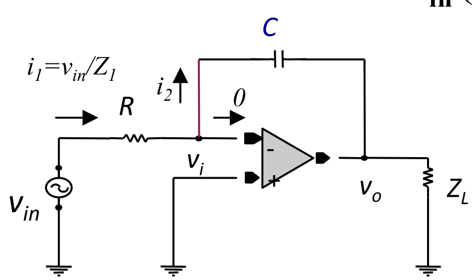
$$\mathbf{V_{in}}(j\omega) = \mathbf{I_1}(j\omega)\mathbf{Z_1}(j\omega) + 0 \text{ since } v_i \text{ is (virtually) zero}$$

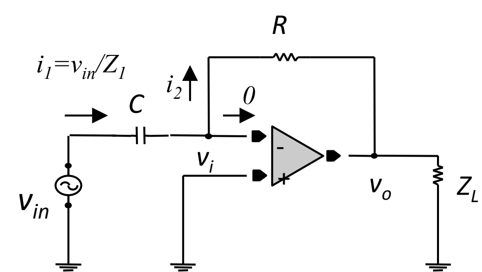
$$\mathbf{I_1}(j\omega) = \mathbf{I}_2(j\omega) \text{ due to the summing-point constraint}$$

$$\mathbf{V_o}(j\omega) = -\mathbf{I}_2(j\omega)\mathbf{Z}_2 + 0 \text{ since } v_i \text{ is (virtually) zero}$$

$$= -\frac{\mathbf{Z}_2}{\mathbf{Z_1}}\mathbf{V_{in}}(j\omega) \text{ which is independent of } \mathbf{Z_L}$$

$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}}$$

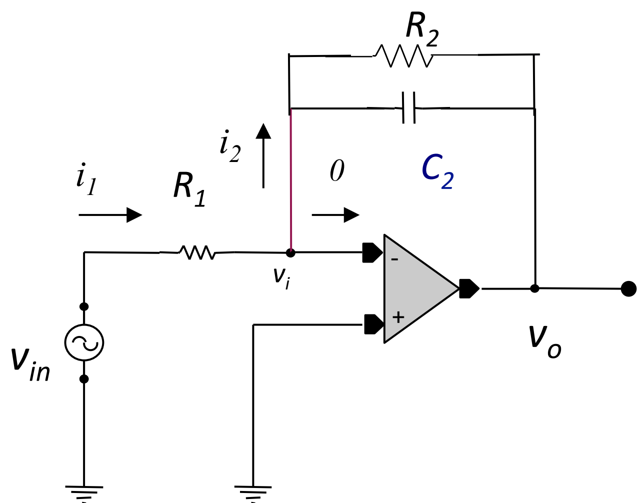




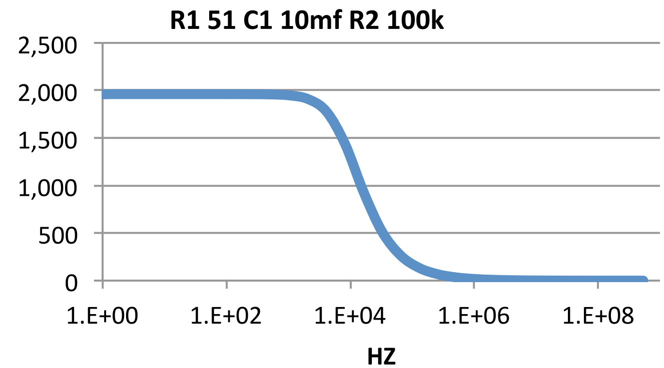
$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}} = -\frac{1}{j\omega RC} \text{ an integrator}$$

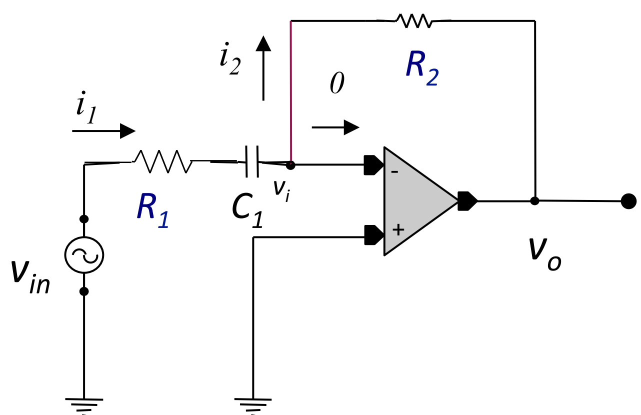
$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}} = -j\omega RC \text{ a differeniator}$$

Frequency Response

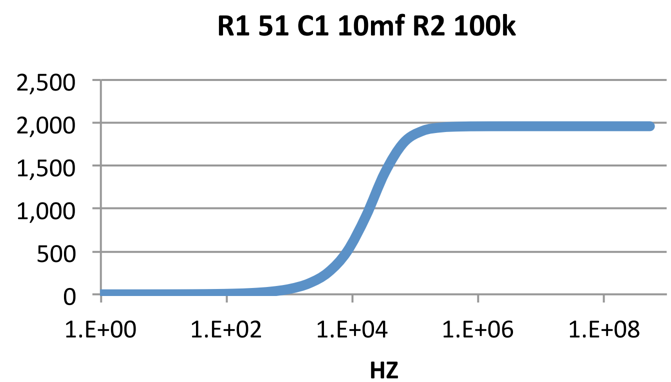


$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\frac{1}{(1+j\omega C_2 R_2)} = \frac{R_2}{R_1}\frac{1}{\sqrt{1+(\omega C_2 R_2)^2}}\angle\pi - \tan^{-1}(\omega C_2 R_2)$$

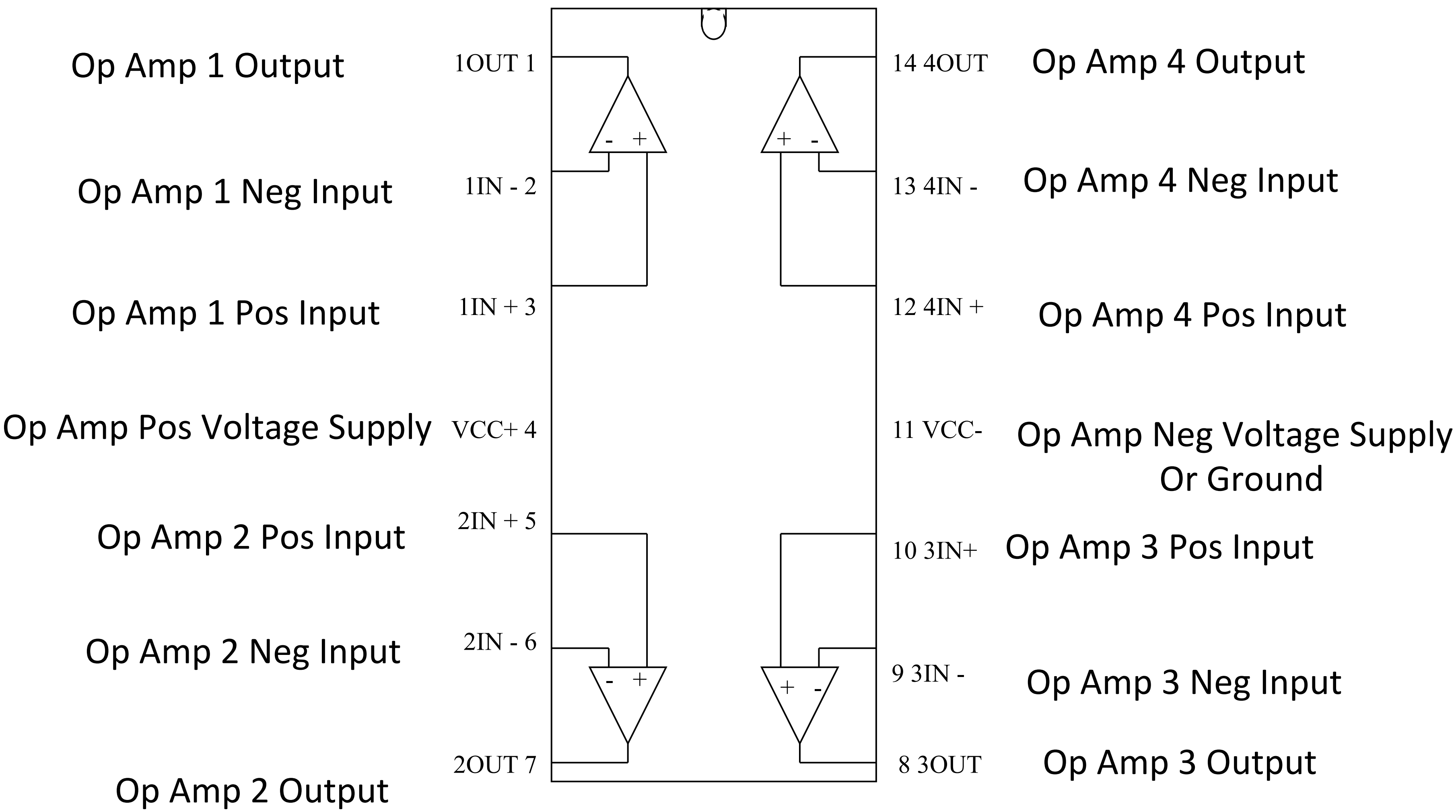




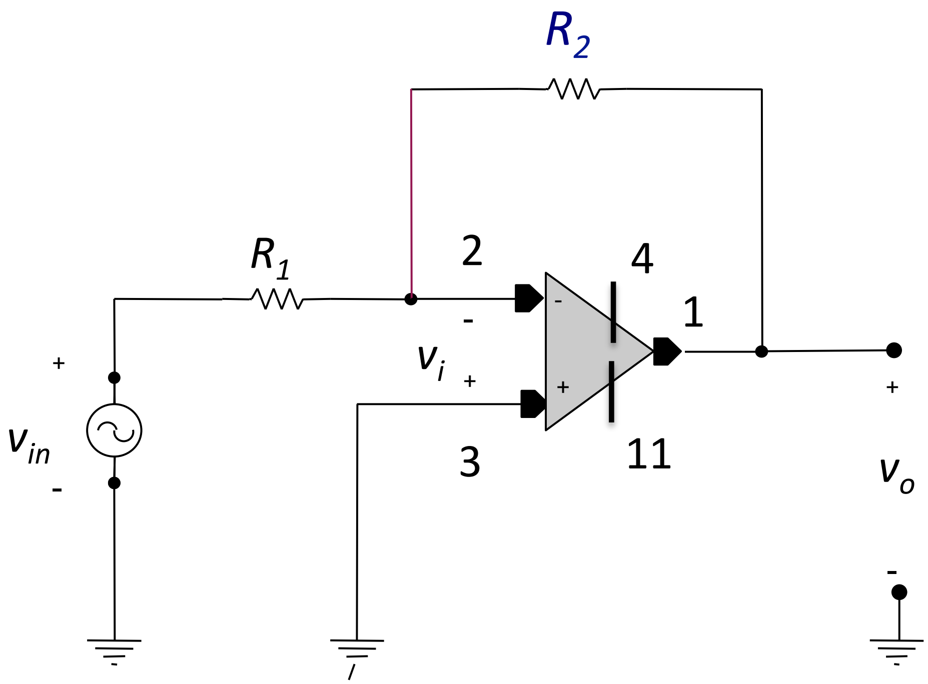
$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\frac{j\omega C_1 R_1}{(1+j\omega C_1 R_1)} = \frac{R_2}{R_1}\frac{\omega C_1 R_1}{\sqrt{1+(\omega C_1 R_1)^2}}\angle -\frac{\pi}{2} - \tan^{-1}(\omega C_1 R_1)$$

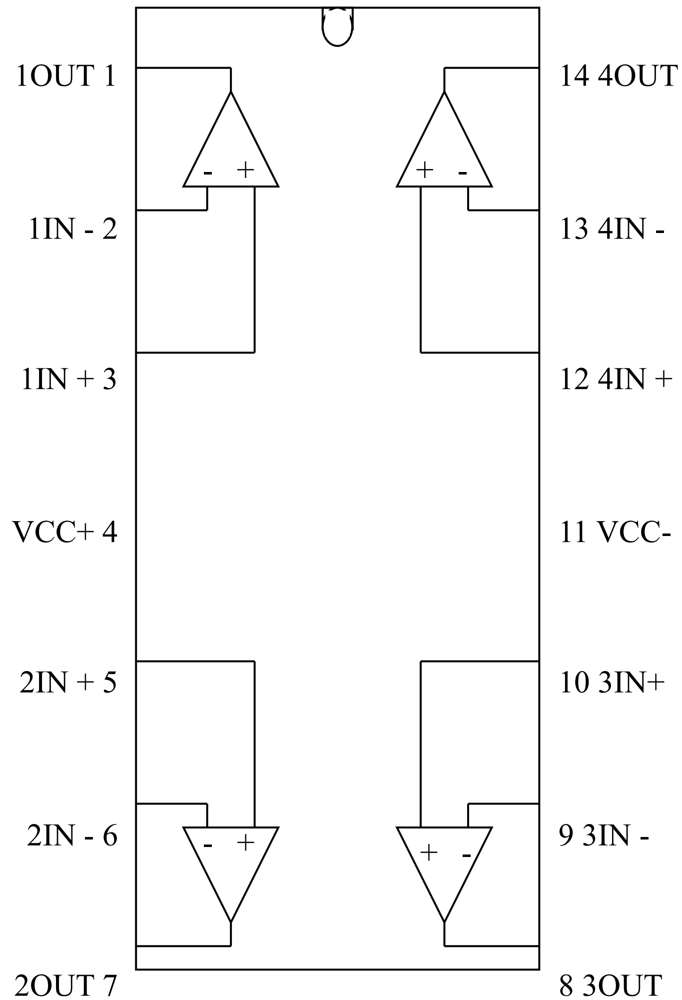


Connecting the Our OpAmp LM324

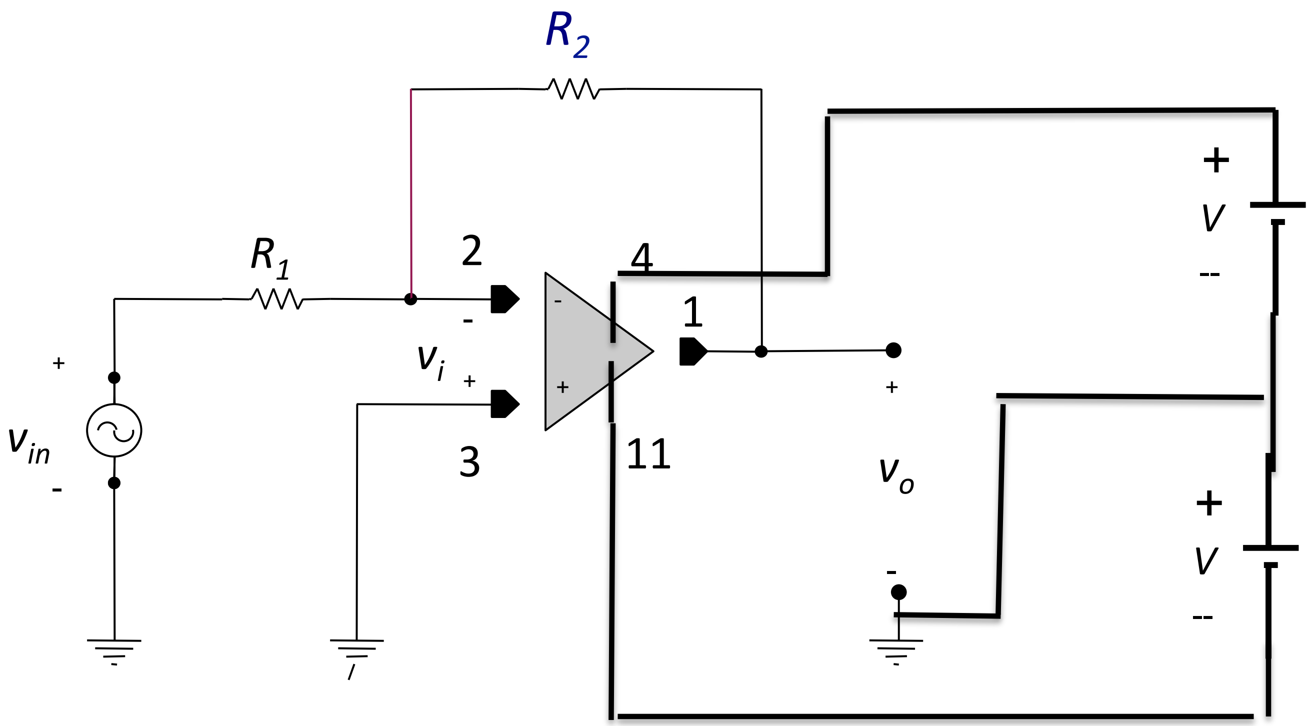


Connecting the our OpAmp



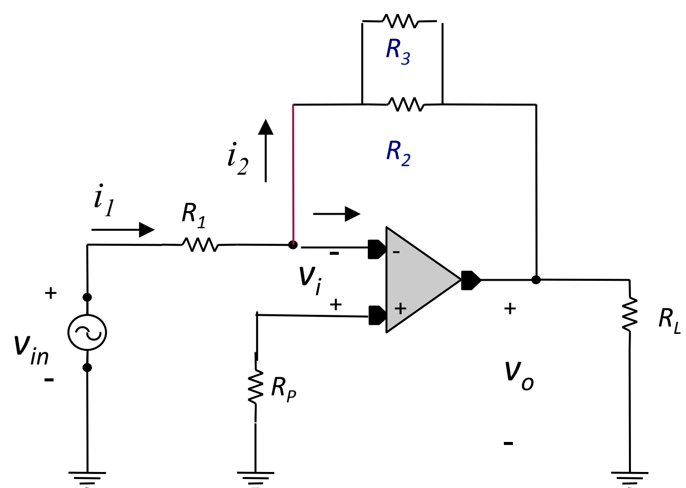


Powering the our OpAmp



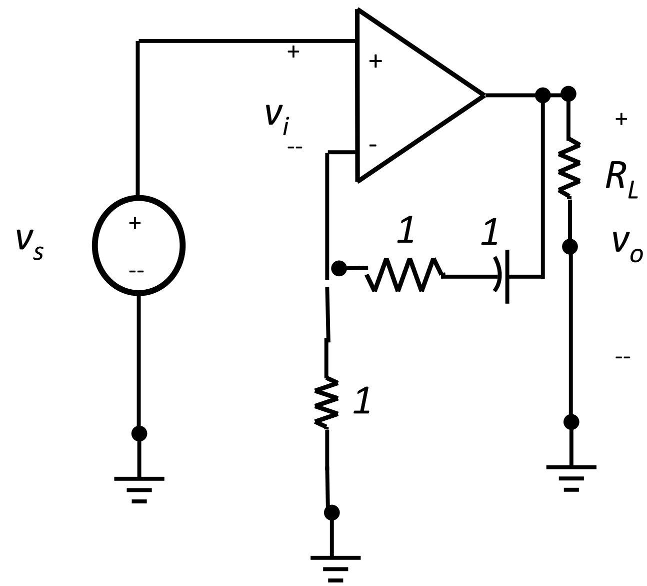
Homework

1. What is the summing point constraint?
2. Calculate the gain for this amplifier (in terms of R1, R2, and R3.



Homework

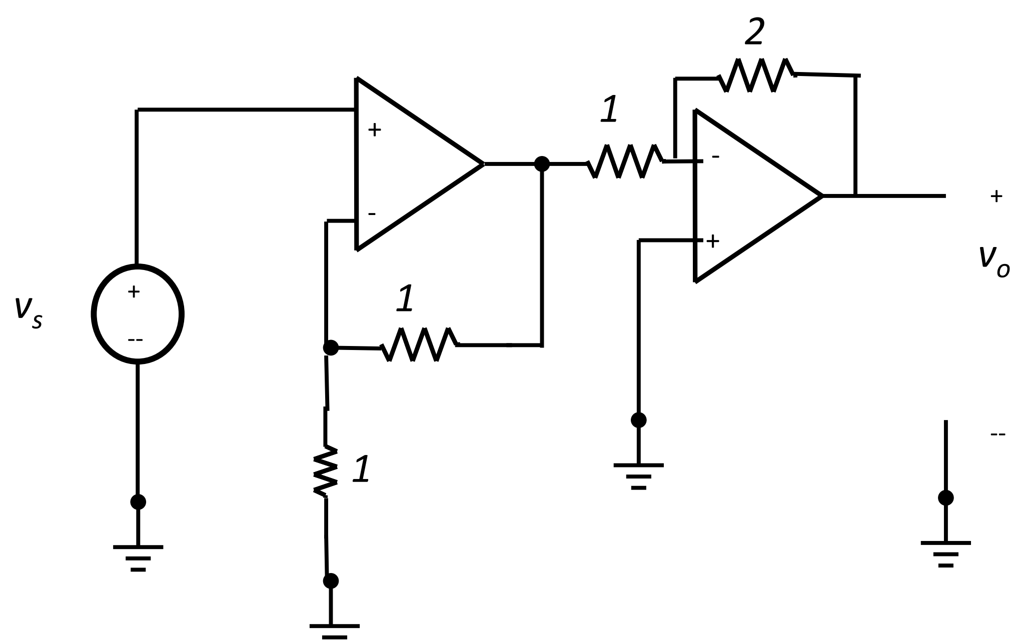
3. Calculate and plot the voltage gain of the following circuit as function of frequency, ω.



Homework

4. HONORS STUDENTS ADD THE FOLLOWING

   Calculate voltage gain of the following circuit.



Homework

5. HONORS STUDENTS ADD THE FOLLOWING

   The criteria for a proper negative feedback opamp circuit is the summing point constraint. What would it be for a proper positive feedback circuit?

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Calculate and plot the gain of this circuit. What type of filter is this?