BME 301

4-Simple Circuits

Circuits

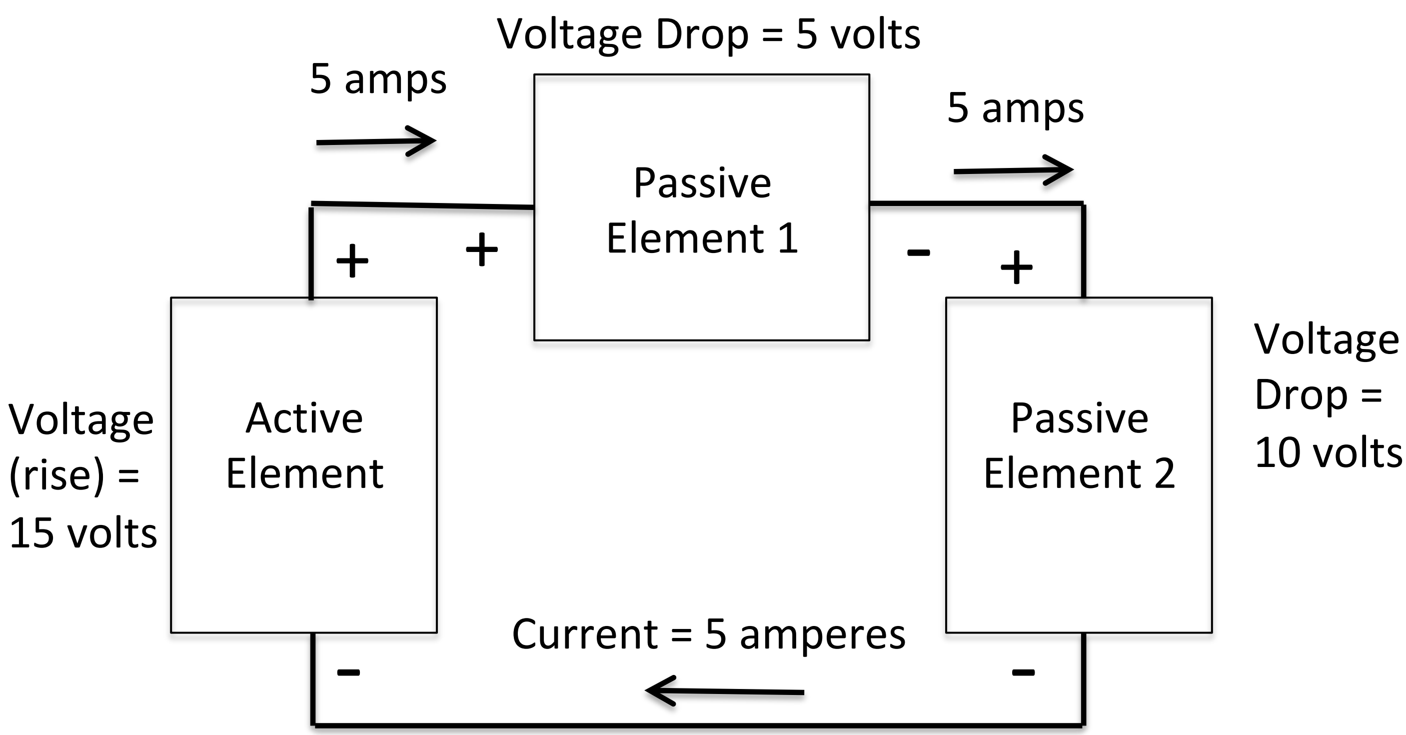
- A circuit is a grouping of passive and active elements
- Elements may be connecting is series, parallel or combinations of both

Circuits Continued

- Series Connection: Same current through the devices
  - The resultant resistance of two or more Resistors connected in series is the sum of the resistance
  - The resultant inductance of two or more Inductors connected in series is the sum of the inductances
  - The resultant capacitance of two or more Capacitors connected in series is the inverse of the sum of the inverse capacitances

Series Circuit Connection

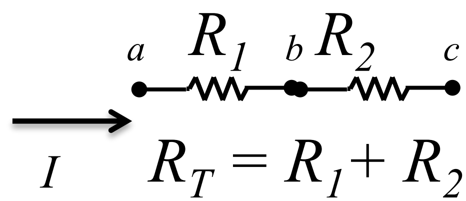
- A series connection is one where the same (identical) current flows through the elements.
- You can connect 2 or more elements in series.
- Note that passive element 1 and passive element 2 have the same identical current flowing through them and are therefore in series. In fact all three elements are in series.
- Note in this circuit although both passive elements have the same current flowing through them, they do not have the same voltage across each and therefore the voltage the active element has to be equal to the sum of these two passive elements.



Series Circuits with Passive Elements

- We can replace multiple elements of the same kind circuits with simpler circuits.

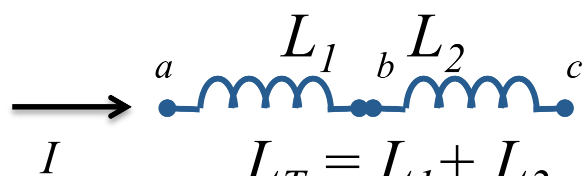
- Resistors



$$R_T = R_1 + R_2$$

$$V_{ac} = V_{ab} + V_{bc} = IR_1 + IR_2$$
$$= I(R_1 + R_2) = IR_T$$

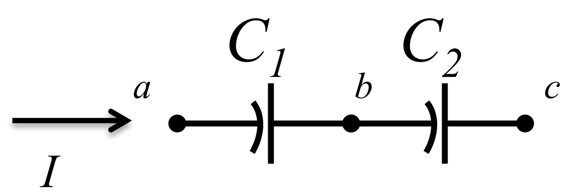
- Inductors



$$L_T = L_1 + L_2$$

$$V_{ac} = V_{ab} + V_{bc} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$
$$= (L_1 + L_2)\frac{dI}{dt} = L_T \frac{dI}{dt}$$

- Capacitors

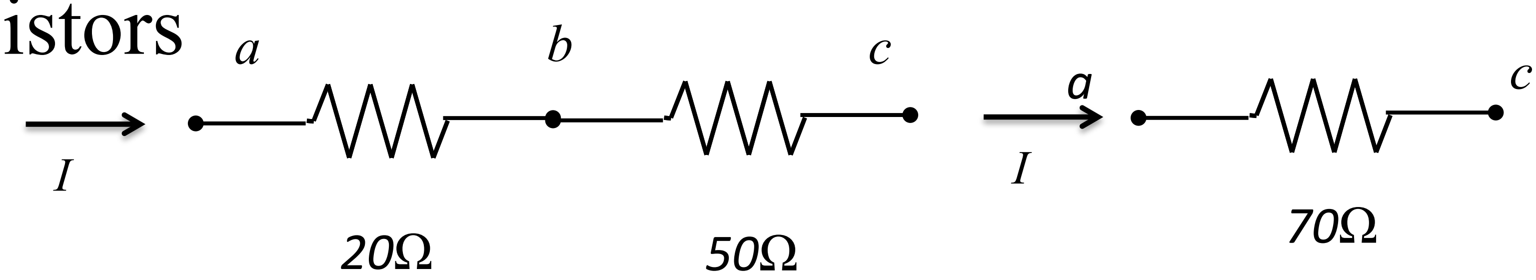


$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{C_1}\int Idt + \frac{1}{C_2}\int Idt$$
$$= (\frac{1}{C_1} + \frac{1}{C_2})\int Idt = \frac{1}{C_T}\int Idt$$

Series Circuits

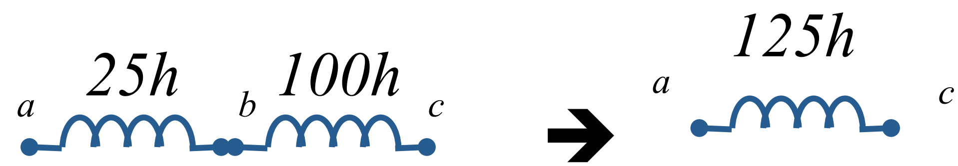
- Resistors



$$V_{ac} = V_{ab} + V_{bc} = I20 + I50$$
$$= I(20+50) = I70$$

Series Circuits

- Inductors



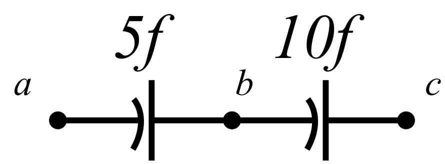
$$L_T = 25 + 100 = 125h$$

$$V_{ac} = V_{ab} + V_{bc} = 25\frac{dI}{dt} + 100\frac{dI}{dt}$$

$$= (25 + 100)\frac{dI}{dt} = 125\frac{dI}{dt}$$

Series Circuits

- Capacitors



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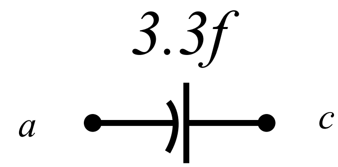


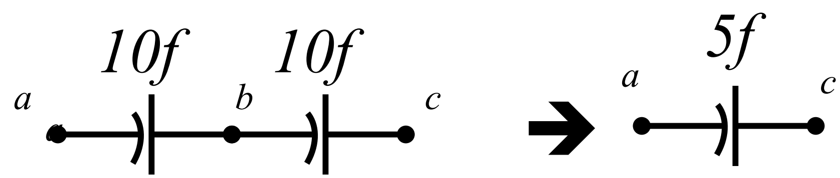
$$C_T = \frac{1}{\frac{1}{5}+\frac{1}{10}} = \frac{5\times 10}{5+10} = \frac{50}{15} = \frac{10}{3} = 3.33f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{5}\int Idt + \frac{1}{10}\int Idt$$

$$= (\frac{1}{5}+\frac{1}{10})\int Idt = \frac{3}{10}\int Idt$$

Series Circuits

- Capacitors



$$C_T = \frac{1}{\frac{1}{10}+\frac{1}{10}} = \frac{10\times 10}{10+10} = \frac{100}{20} = \frac{10}{2} = 5f$$

$$V_{ac} = V_{ab} + V_{bc} = \frac{1}{10}\int Idt + \frac{1}{10}\int Idt$$

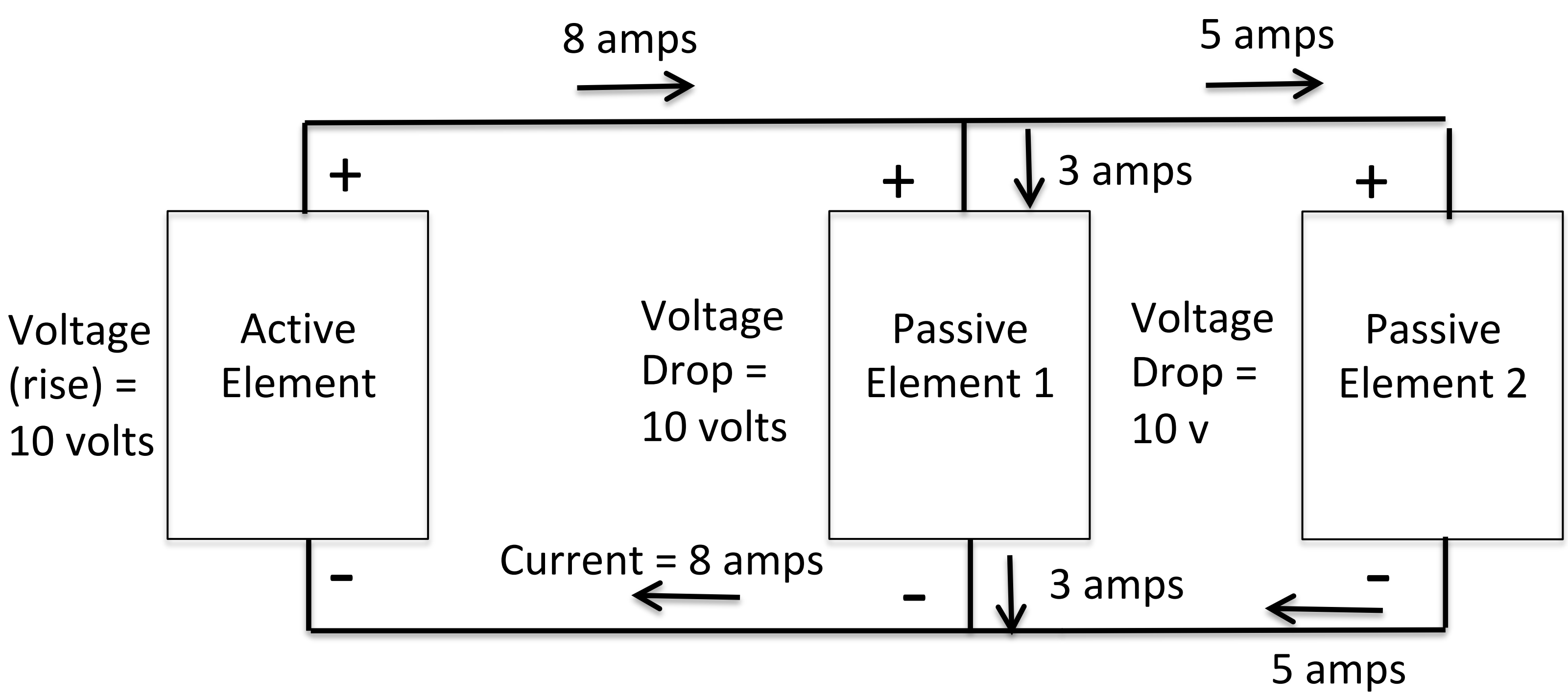
$$= (\frac{1}{10}+\frac{1}{10})\int Idt = \frac{2}{10}\int Idt = \frac{1}{5}\int Idt$$

Circuits Continued

- Parallel Connection: Same Voltage across the devices
  - The resultant resistance of two or more Resistors connected in parallel is the inverse of the sum of the inverse resistances
  - The resultant inductance of two or more Inductors connected in parallel is the inverse of the sum of the inverse inductances
  - The resultant capacitance of two or more Capacitors connected in parallel is the sum of the capacitances

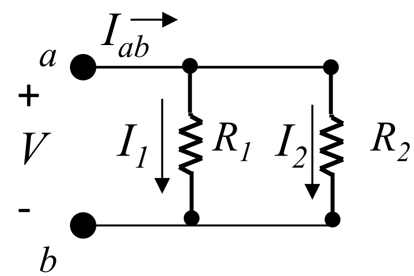
Parallel Circuit Connection

- A parallel connection is one where the same (identical) voltage appears across the elements.
- You can connect 2 or more elements in parallel.
- Note that passive element 1 and passive element 2 have the same identical voltage across them and are therefore in parallel. In fact all three elements are in parallel.
- Note in this circuit although both passive elements have the same voltage across them, they do not have the same current flowing through them and therefore the current flowing to and from the active element has to be equal to the sum of these two passive elements.



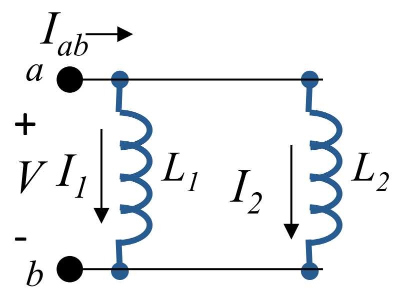
Parallel Circuits

- Resistors $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$



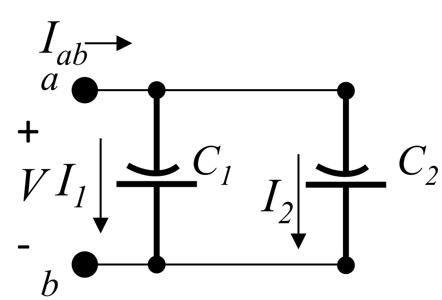
$$I_{ab} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$
$$= (\frac{1}{R_1} + \frac{1}{R_2})V = \frac{V}{R_T}$$

- Inductors $L_T = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2}} = \dfrac{L_1 L_2}{L_1 + L_2}$



$$I_{ab} = I_1 + I_2 = \frac{1}{L_1}\int V dt + \frac{1}{L_2}\int V dt$$
$$= (\frac{1}{L_1} + \frac{1}{L_2})\int V dt = \frac{1}{L_T}\int V dt$$

- Capacitors $C_T = C_1 + C_2$



$$I_{ab} = I_1 + I_2 = C_1\frac{dV}{dt} + C_2\frac{dV}{dt}$$
$$= (C_1 + C_2)\frac{dV}{dt} = C_T\frac{dV}{dt}$$

Some Rules

	Resistors	Inductors	Capacitors
Series	Simple Addition	Simple Addition	Reciprocal Addition
Parallel	Reciprocal Addition	Reciprocal Addition	Simple Addition

Simple Addition

$$X_T = X_1 + X_2$$

Reciprocal Addition

$$X_T = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2}} \text{ or } \frac{1}{X_T} = \frac{1}{X_1} + \frac{1}{X_2}$$

Shortcut for 2 resistors (or inductors) in parallel.

$$X_T = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2}} \rightarrow X_T = \frac{X_1 X_2}{X_1 + X_2}$$

For multiple elements

Simple Addition

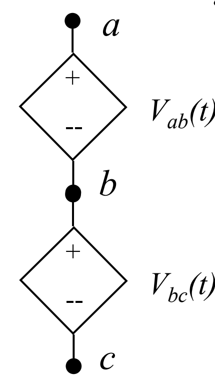
$$X_T = X_1 + X_2 + \cdots + X_N$$

Reciprocal Addition

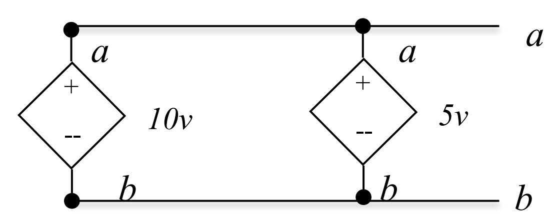
$$X_T = \frac{1}{\frac{1}{X_1} + \frac{1}{X_2} + \cdots + \frac{1}{X_N}} \text{ or } \frac{1}{X_T} = \frac{1}{X_1} + \frac{1}{X_1} + \cdots + \frac{1}{X_N}$$

Series and parallel connections of active sources

- Voltage Sources
  - The resultant voltage of two or more Voltage Sources connected in series is the sum of the voltages
  - $Vac(t)=Vab(t)+Vbc(t)$
  - This is ok.

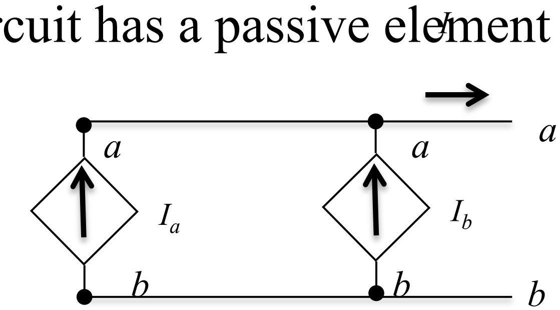


  - Two of more Voltage Sources can not be connected in parallel
  - What is *Vab*? 5 or 10?
  - This can't work.

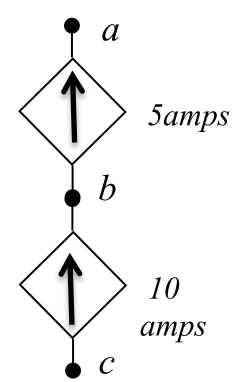


Series and parallel connections of active sources

– Current Sources
  - The resultant current of two or more Current Sources connected in parallel is the sum of the currents
  - $I = I_a + I_b$ once this circuit has a passive element connected across terminals a-b
  - This is ok.



  - Two of more Current sources can not be connected in series
  - Is the current 5 or 10 amps?
  - This doesn't work



Kirchhoff Law’s

- There are two Kirchhoff Law’s
- Kirchhoff Voltage Law: The sum of the voltages around a loop must equal zero.
- Kirchhoff Current Law: The sum of the currents leaving (entering) a node must equal zero.

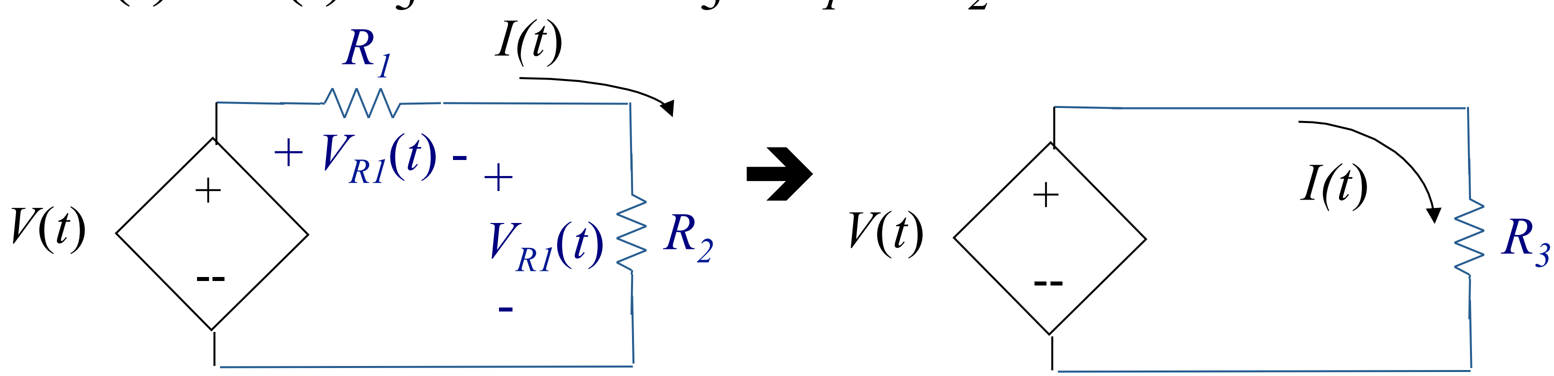
Combining Elements
Series Circuit

- We can use KVL or KCL to write and solve an equation associated with the circuit.
  - Example: a series Resistive Circuit using KVL

    $V(t)=V_{R1}(t)+V_{R2}(t)$ and Ohm's law $V=IR$

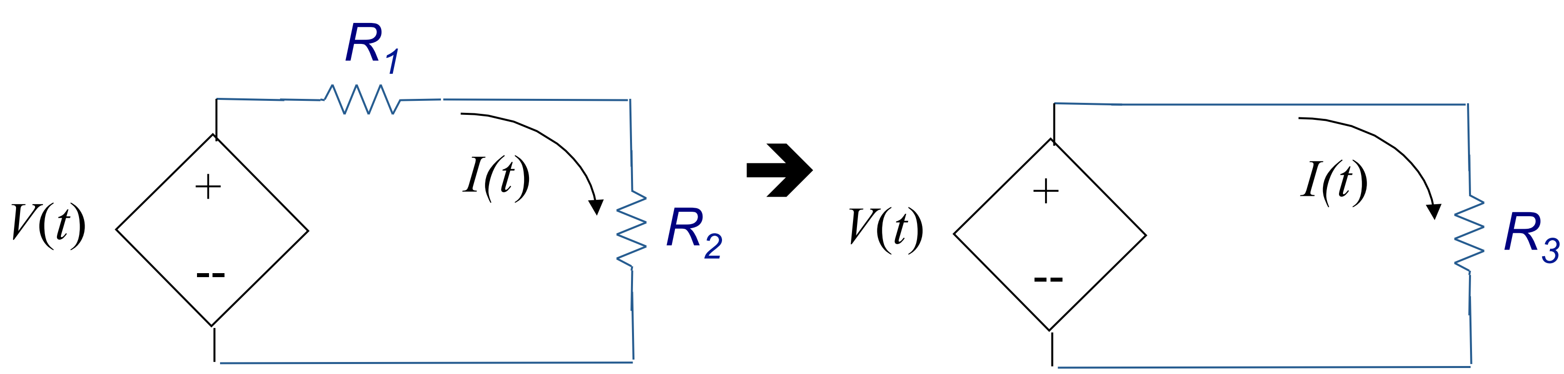
$V(t) = I(t)R_1 + I(t)R_2$ ➔ $V(t) = I(t)(R_1 + R_2)$

$V(t) = I(t)R_3$ where $R_3=R_1 + R_2$



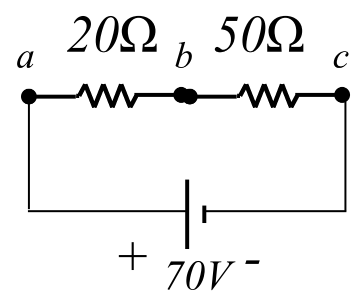
Combining Elements
Series Circuit

- Or by recognizing that R_1 and R_2 are in series and replacing this series combination
  - Since R_1 and R_2 are in series we replace them with R_3 which is $R_3 = R_1 + R_2$



Series Circuits

- Resistors



$$R_T = 20 + 50 = 70\Omega$$

$$V_{ac} = V_{ab} + V_{bc} = I20 + I50$$
$$= I(20+50) = I70 = 70V$$
$$I = 1A$$

Combining Elements
Parallel Circuit

- We can use KVL or KCL to write and solve an equation associated with the circuit.
  - Example: a parallel Resistive Circuit using KCL

$I_1(t)+I_2(t)+I_3(t)=0$ and Ohm's Law

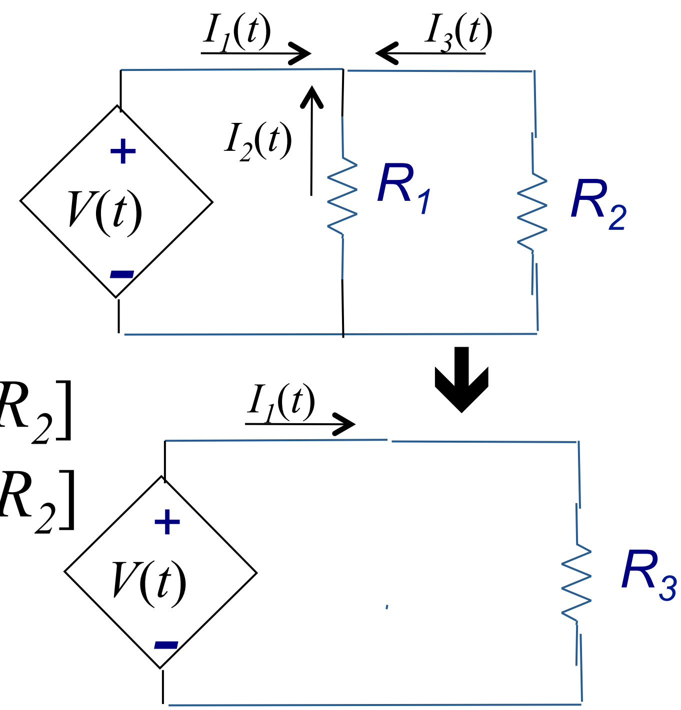
$I_2(t)=-V(t)/R_1$; $I_3(t)=-V(t)/R_2$;

Substituting we get

$I_1(t) - V(t)/R_1 - V(t)/R_2 = 0$

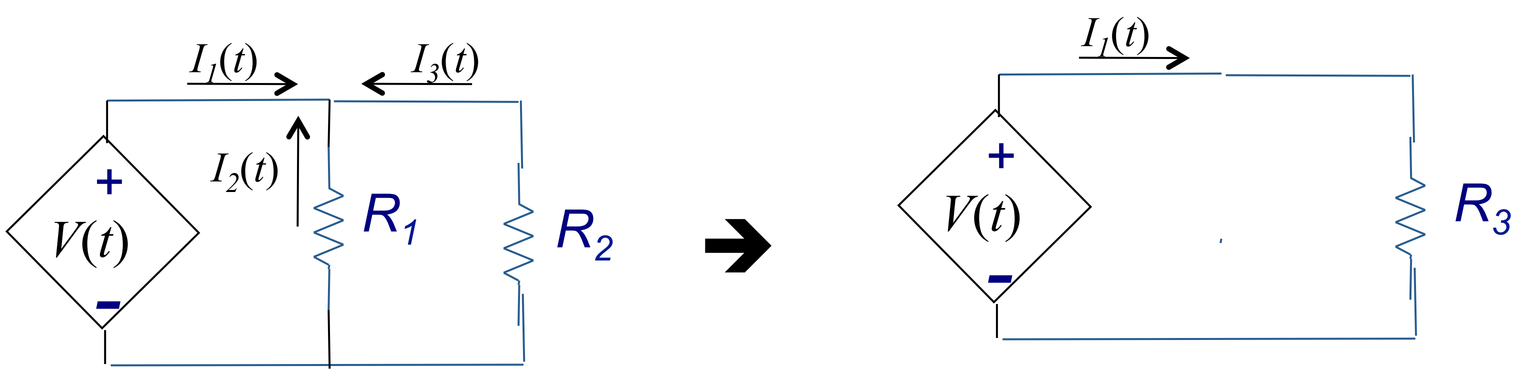
$I_1(t)=V(t)/R_1+V(t)/R_2=V(t)[1/R_1+1/R_2]$

$I_1(t)=V(t)/R_3$ where $1/R_3=[1/R_1+1/R_2]$



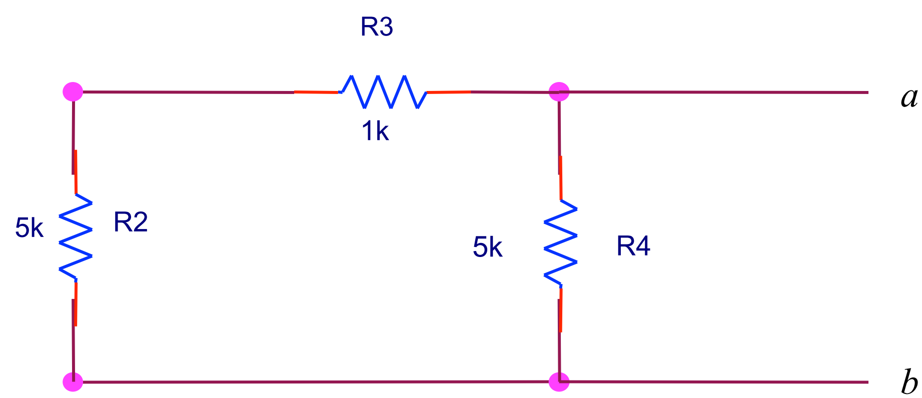
Combining Elements
Parallel Circuit

- Or recognizing that R_1 and R_2 are in parallel and replacing the parallel combination.
  - Since R_1 and R_2 are in parallel we replace them with R_3 which is $1/R_3 = [1/R_1 + 1/R_2]$



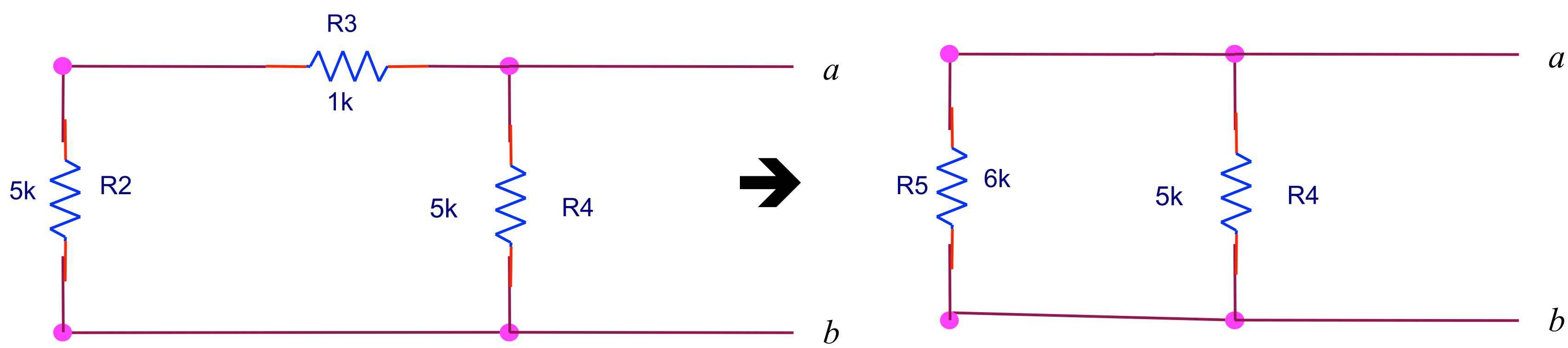
Series/Parallel Circuits
Circuit Reduction

- This problem want to know what is the resistance between terminals *a* and *b*, R_{ab}.
- We then start from the opposite side and add up the resistors approaching the terminals.



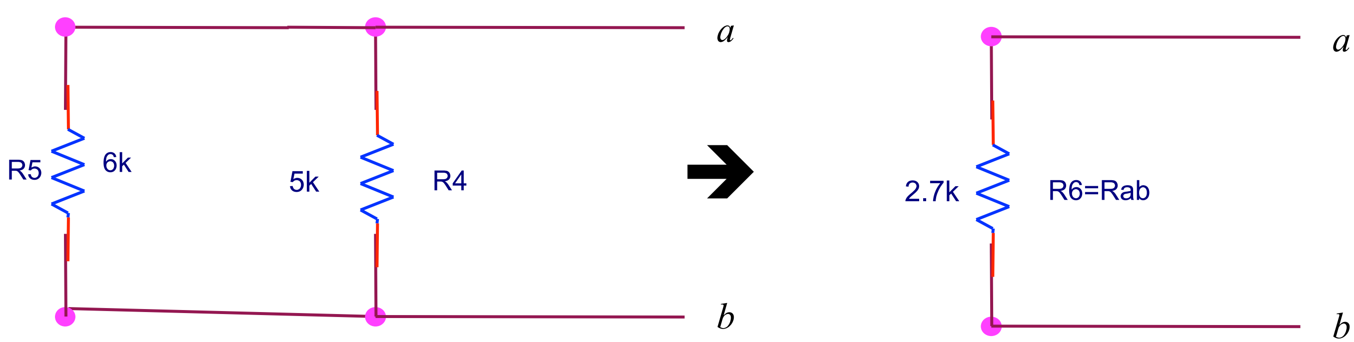
Series/Parallel Circuits
Circuit Reduction

- So for this circuit we start with R2 and R3 and note that they are in series and we can replace them with R5 the series addition of R2 and R3.
- So we add them up using simple addition. R5=R2+R3=5k+1k=6k.



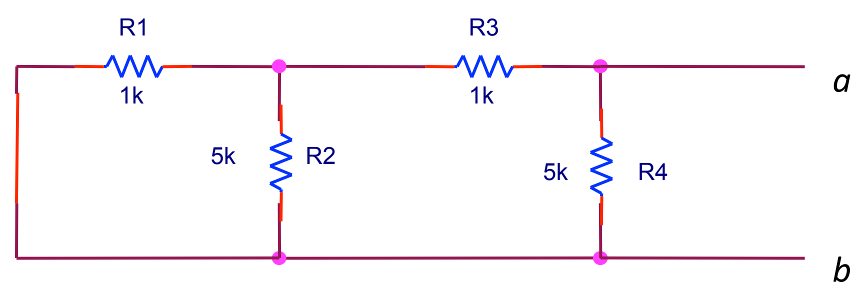
Series/Parallel Circuits
Circuit Reduction

- After redrawing the circuit with R5 in place of R2 and R3, we now see that R5 is in parallel with R4 and we can replace them with R6 the parallel addition of R5 and R4.
- So we add them up using Reciprocal addition. 1/R6=1/R5+1/R4 R6=R5*R4/(R5+R4)=30k/11=2.7k=R_{ab}



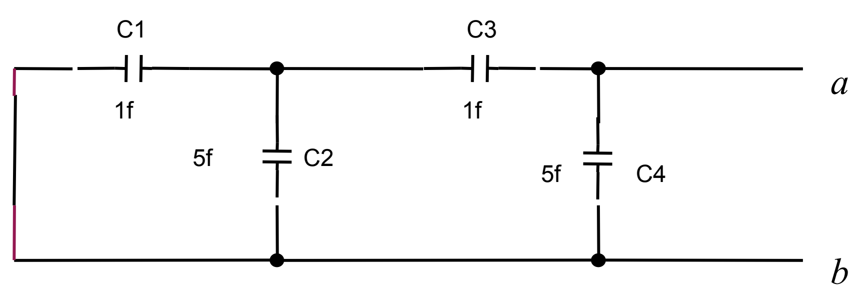
Now try this one

- R1 and R2 are in parallel and equals 5/6k.
- This 5/6k resistor is in series with R3 and equals 1.83k
- This 1.83k is in parallel with R4 and equals 1.3k.



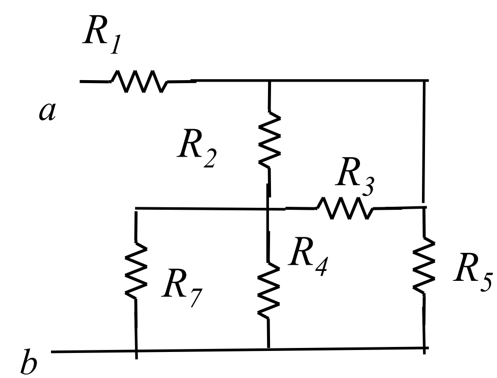
And this one

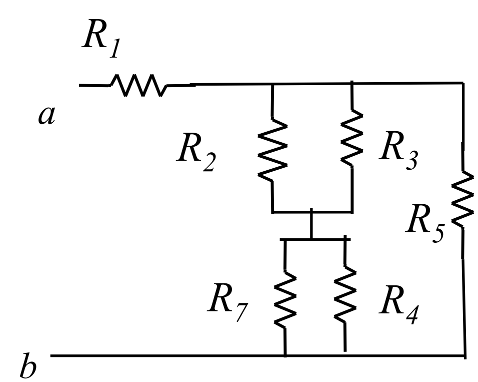
- Same rules apply except this circuit uses capacitors: start at the opposite side to the terminals and move forwards to get Cab.
- C1 is in parallel with C2 and equals 6f
- This 6f capacitor is in series with C3 and equals 0.86f
- This 0.86f capacitor is in parallel with C4 and equals 5.86f which is Cab.



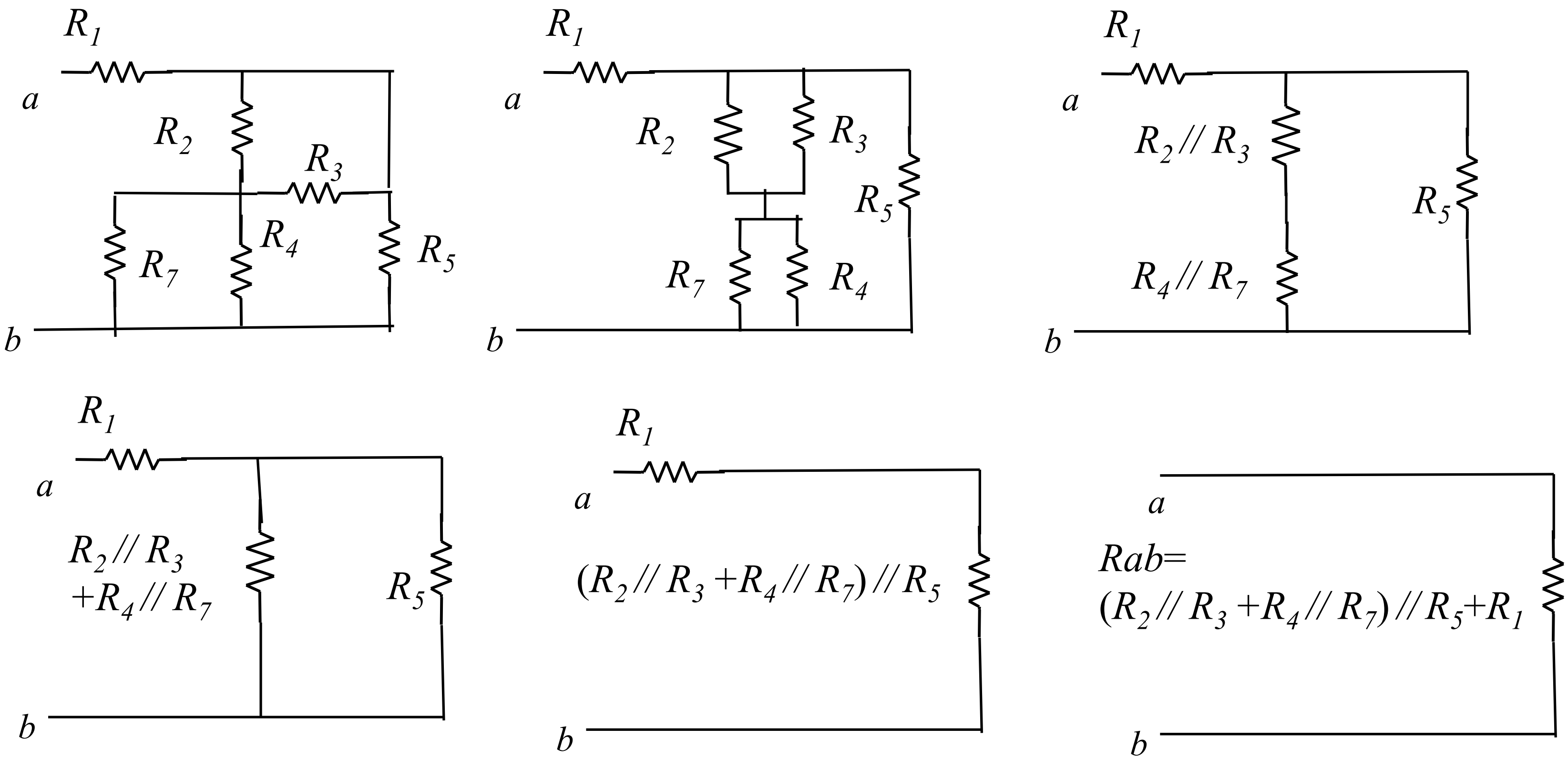
One More Example

- First let's redraw this circuit
- First we see that R2 and R3 are in parallel since the left of R3 is connected to the bottom of R2 and the top of R2 is connected to the right of R3.
- The same is true for R4 and R7 since there top terminal are connected together and there bottom terminals are connected together.
- Redrawing we see that the R2/R3 parallel combination is in series with the R4/ R7parallel combination.
- Furthermore, since the the top terminal of R5 is connected to the top terminals of R2 and R3 and the bottom terminal of R5 is connected to the bottom terminals of R4 and R7, we see that R5 is in parallel with the series combination of R2 in parallel with R3 and R7 in parallel with R4.



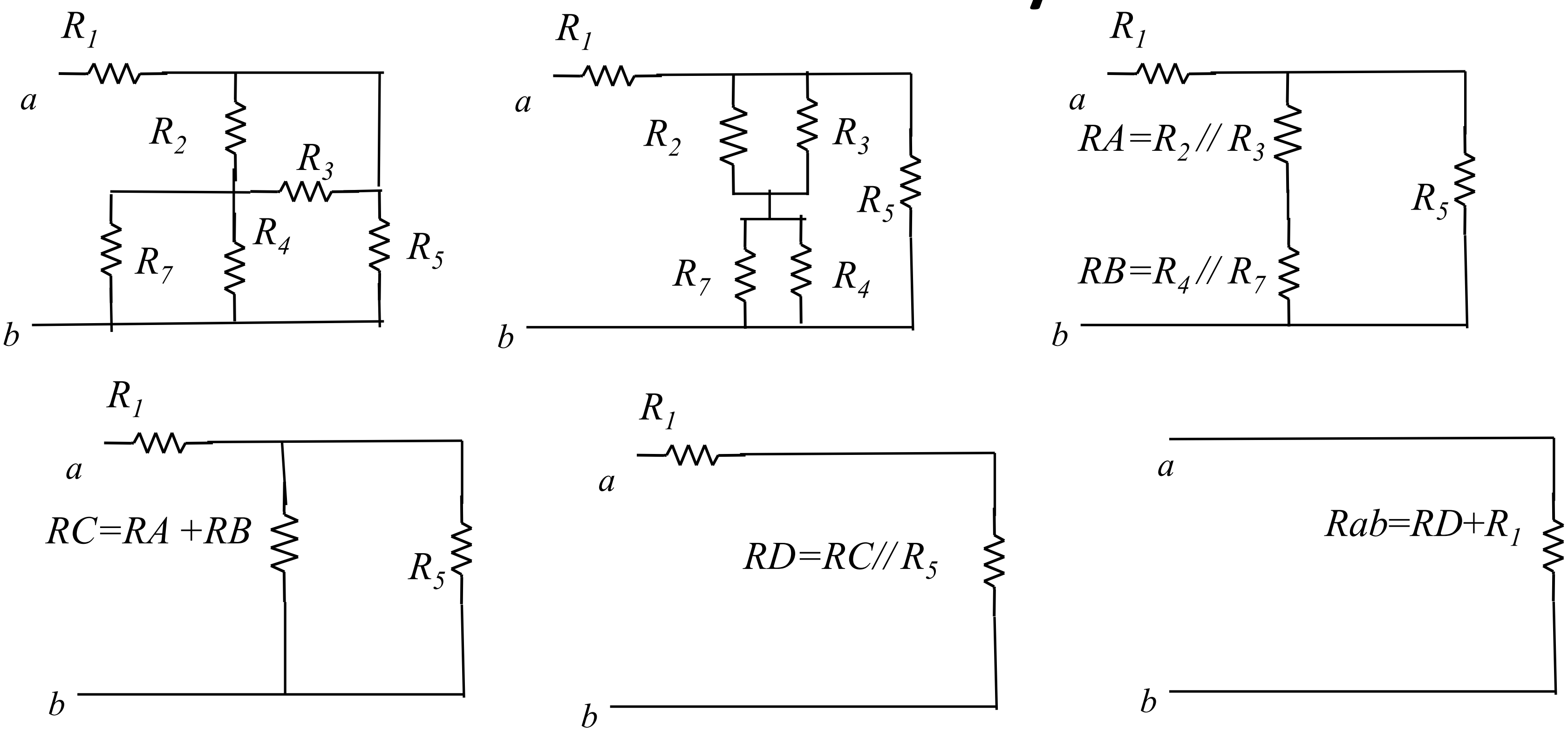


One More Example



1. R2 and R3 are in parallel
2. R7 and R4 are in parallel
3. These two parallel combinations are in series
4. This series combination is in parallel with R5
5. And R1 is in series with this parallel combination; this is Rab.

Another way



1. R2 and R3 are in parallel; call this RA
2. R7 and R4 are in parallel; call this RB
3. RA and Rb are in series; call this RC
4. RC is in parallel with R5; call this RD
5. And R1 is in series with RD; this is Rab.

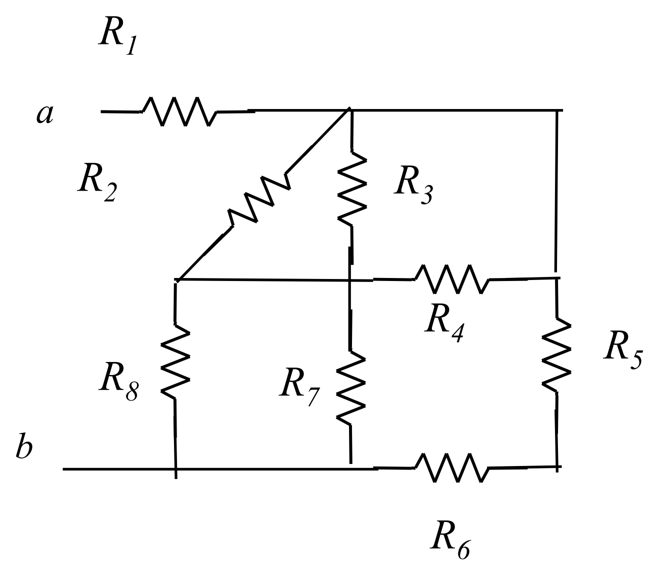
Homework

1. Find the total resistance R_{ab} where

$R_1 = 10\Omega$, $R_2 = 15\Omega$, $R_3 = 15\Omega$,

$R_4 = 15\Omega$, $R_5 = 10\Omega$, $R_6 = 10\Omega$,

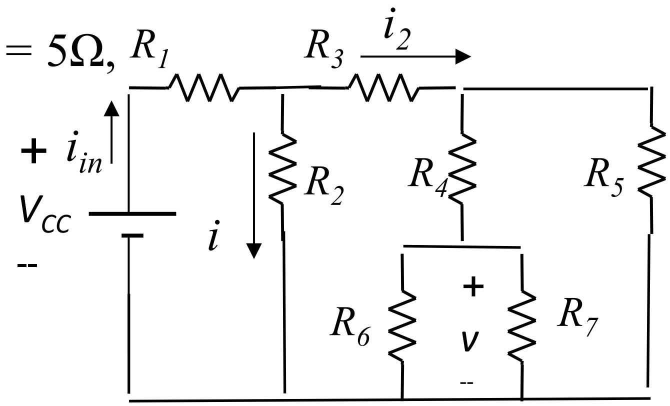
$R_7 = 10\Omega$, $R_8 = 10\Omega$



Homework

2. Calculate the current labeled, *I,* and voltage labeled *v.*

$R_1 = 2.5\Omega$, $R_2 = 5\Omega$, $R_3 = 2.5\Omega$, $R_4 = 2.5\Omega$, $R_5 = 5\Omega$, $R_6 = 5\Omega$, $R_7 = 5\Omega$, $V_{cc} = 5$



3. HONORS STUDENTS ADD THE FOLLOWING

Find the total resistance R_{ab} for this infinite resistive network.