BME 301

8-Filters

What is a filter

- A filter is a system
- A filter can separate unwanted information from a signal
- Like any system a filter has at least one output and at least one input
- A filter manipulates the signal
- Some filters operate on the frequencies of the input signal.

Filter Basics

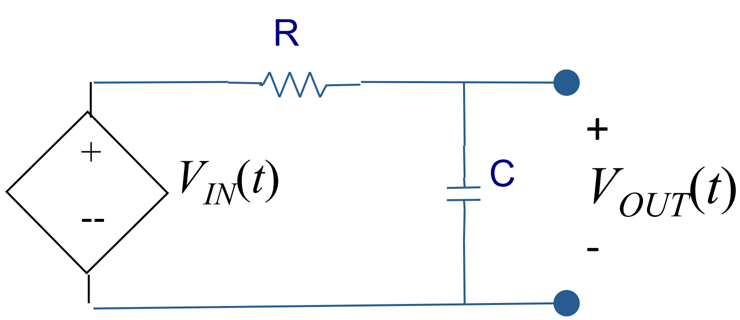
- Allows certain frequencies of a signal to pass through the filter; this is called the pass band
- Other frequencies do not get through the filters; this is called the stop band
- Some filters have multiple pass and stop bands
- These bands are defined by certain frequencies; these frequencies are called break frequencies or cutoff frequencies.

Examples of Filters

- Low Pass Filter (LP) : allows signal frequencies below a defined cutoff frequency to pass through the filter
- High Pass Filter (HP): allows signal frequencies above a defined cutoff frequency to pass through the filter
- Band Pass Filter (BP): allows signal frequencies between a range of a defined cutoff frequencies to pass through the filter

Typical Filter

- This is a typical electric circuit filter
- It has an input and an output (across the capacitor)
- The relationship of the output to the input as a function of frequency, called the transfer function, $V_{out}(t)/V_{in}(t)$, is calculated
- Once the transfer function is calculation, a plot of the magnitude and phase as a function of frequency is made. This is called Bode plot.
- This transfer function and it's Bode will demonstrate the operation of the filter; e.g., HP or LP.

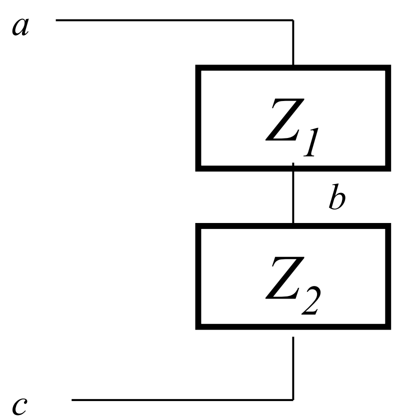


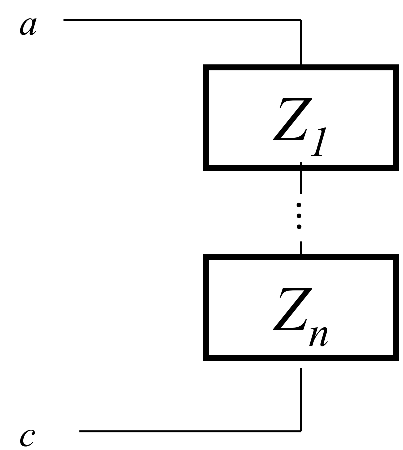
Transfer function Calculation

- In order to determine the Transfer Function of this circuit, we need to learn some circuit analysis techniques.
- Examples of circuit analysis techniques:
  - KVL & KCL
  - Circuit Reduction
  - Voltage Division
  - Current Division
  - Nodal and Mesh Analysis
  - Superposition

Voltage Division

- The voltage across impedances in series divides in proportion to the impedances.





$$\mathbf{V}_{ac} = \mathbf{V}_{ab} + \mathbf{V}_{bc} = \mathbf{I}(Z_1 + Z_2); \text{KVL} + \text{Ohm's Law}$$

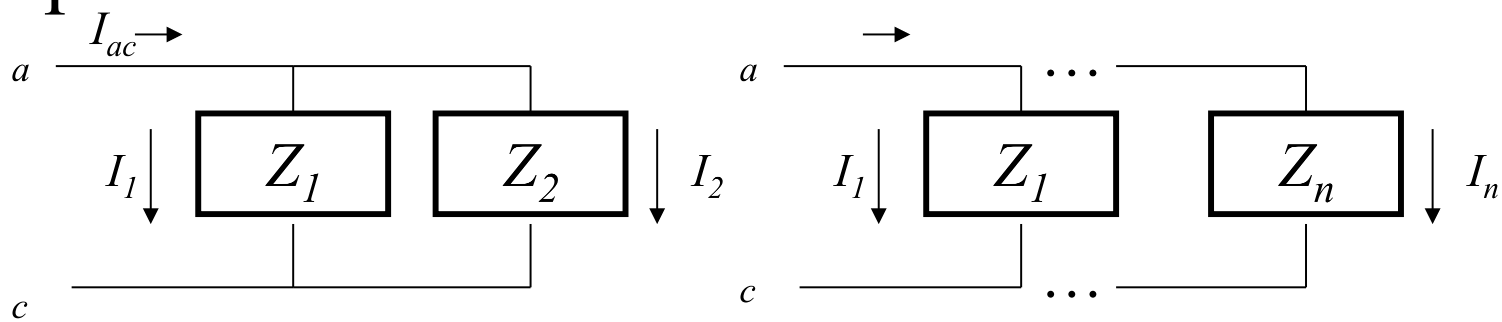
$$\mathbf{V}_{bc} = \mathbf{I}Z_2$$

$$\frac{\mathbf{V}_{bc}}{\mathbf{V}_{ac}} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbf{V}_i}{\mathbf{V}_{ac}} = \frac{Z_i}{Z_1 + Z_2 + \cdots Z_n}$$

Current Division

- The current into impedances in parallel divides in proportion to the inverse of the impedances.



$$\mathbf{I}_{ac} = \mathbf{I}_1 + \mathbf{I}_2 = \mathbf{V}(\frac{1}{Z_1} + \frac{1}{Z_2}); \text{KCL} + \text{Ohm's Law}$$

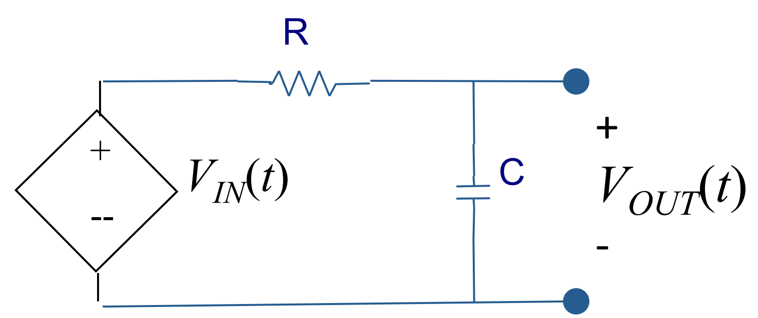
$$\mathbf{I}_1 = \frac{\mathbf{V}}{Z_1}$$

$$\frac{\mathbf{I}_1}{\mathbf{I}_{ac}} = \frac{1/Z_1}{(1/Z_1)+(1/Z_2)} = \frac{Z_2}{Z_1+Z_2}$$

$$\frac{\mathbf{I}_i}{\mathbf{I}_{ac}} = \frac{(1/Z_i)}{(1/Z_1)+(1/Z_2)+\cdots+(1/Z_n)}$$

Let’s try Voltage division on this circuit

- Since the capacitor and the resistor are in series and since the voltage across the capacitor is the output, we can use voltage division.
- An easy way to plot the transfer function using Matlab or Excel or one can sketch it by finding 3 points and generate a sketch. Two of the points are when the frequency is zero and when the frequency approaches infinity. The third can be any convenient point in between.



Let’s try Voltage division on this circuit

Using Voltage division

$$\frac{Vout}{Vin} = \frac{Z_2}{Z_1 + Z_2}$$; where Z_1 is the resistor which is R and Z_2 is the capacitor which is $\frac{1}{j\omega C}$

$$\frac{Vout}{Vin} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle - \tan^{-1}(\omega RC)$$

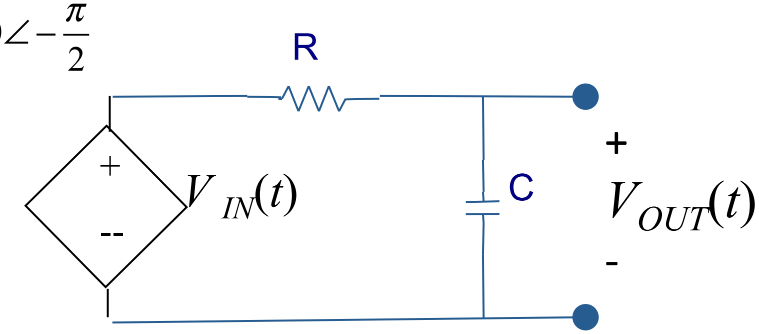
An easy way to plot the transfer function is determine 3 or more points and sketch its shape.
Two of these points are usually taken at ω=0 and $\omega \to \infty$. The third can be somewhere in between at any easy point to calculate.

$$\frac{Vout}{Vin}\Big|_{\omega=0} = \frac{1}{1 + j\omega RC}\Big|_{\omega=0} = 1 = 1\angle 0$$

$$\frac{Vout}{Vin}\Big|_{\omega\to\infty} = \frac{1}{1 + j\omega RC}\Big|_{\omega\to\infty} \to \frac{1}{j\omega RC}\Big|_{\omega\to\infty} = \frac{1}{\omega RC}\angle - \frac{\pi}{2}\Big|_{\omega\to\infty} \to 0\angle - \frac{\pi}{2}$$

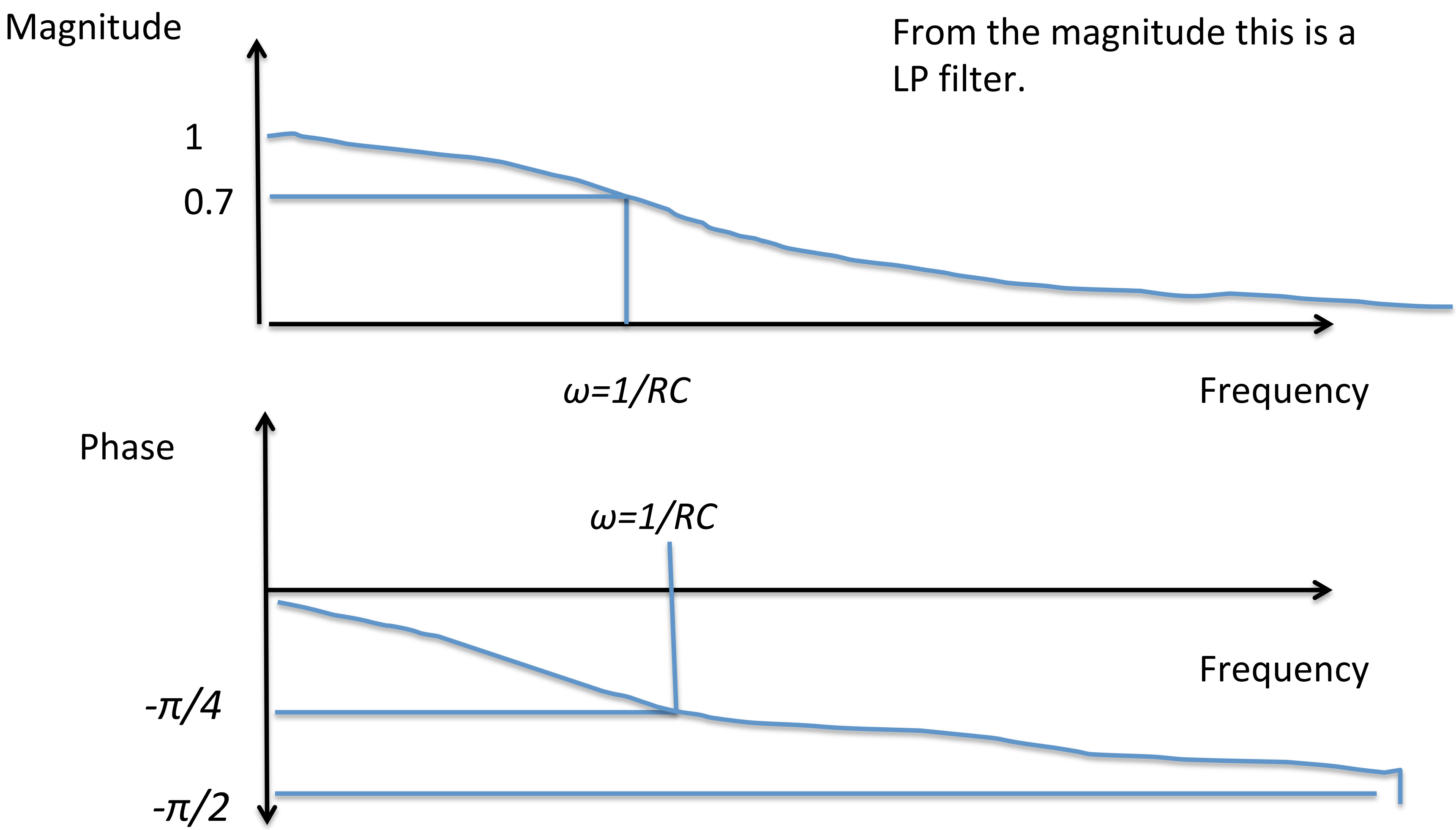
A convenient midway point is a $\omega = \frac{1}{RC}$

$$\frac{Vout}{Vin}\Big|_{\omega=\frac{1}{RC}} = \frac{1}{1 + j\omega RC}\Big|_{\omega=\frac{1}{RC}} = \frac{1}{1 + j} = \frac{1}{\sqrt{2}}\angle - \frac{\pi}{4}$$

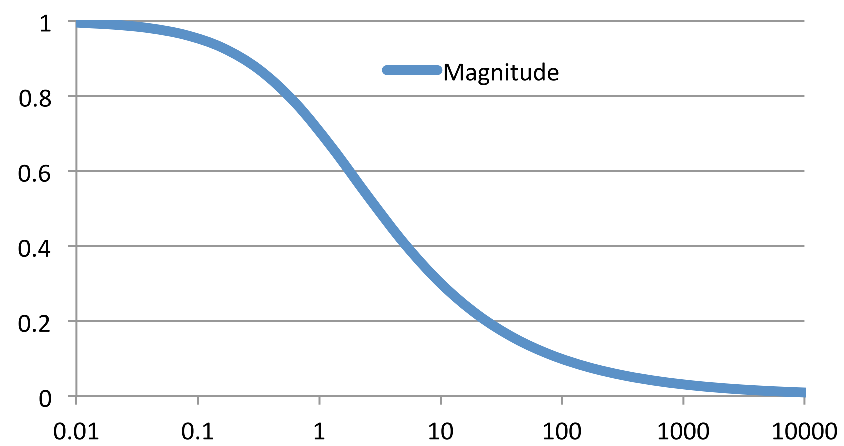


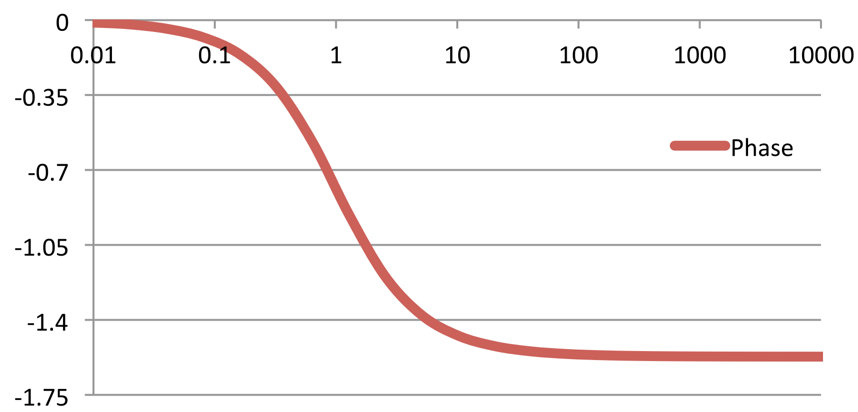
Sketching the Bode plot of the transfer function

From the magnitude this is a LP filter.



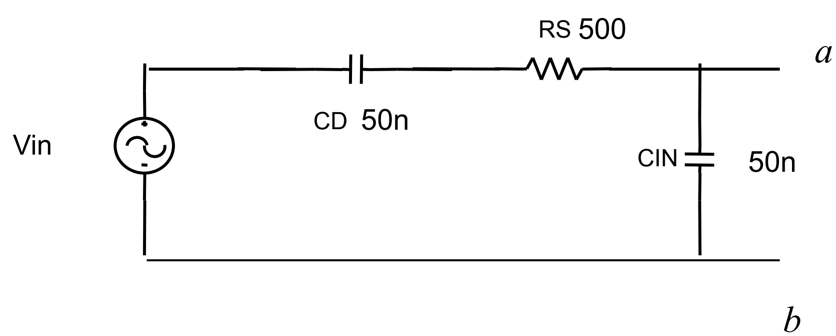
Excel Bode Plot





Homework

1. Find the transfer function of this circuit as function of ω. Plot using Matlab the magnitude and phase as a function of ω.



Homework

2. HONORS STUDENTS ADD THE FOLLOWING

An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find and plot using Matlab the transfer function of this circuit as function of ω.