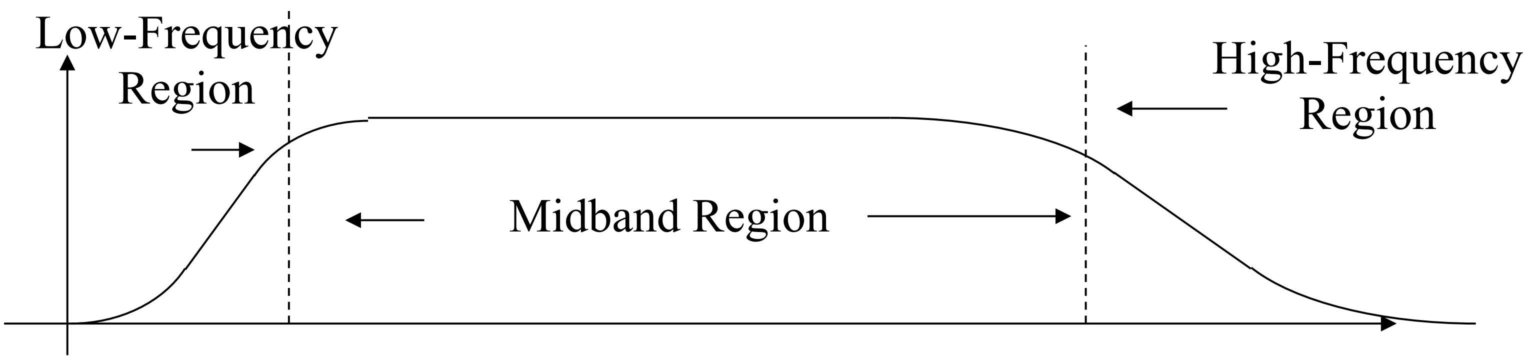
BME 301

9-Plotting Filters

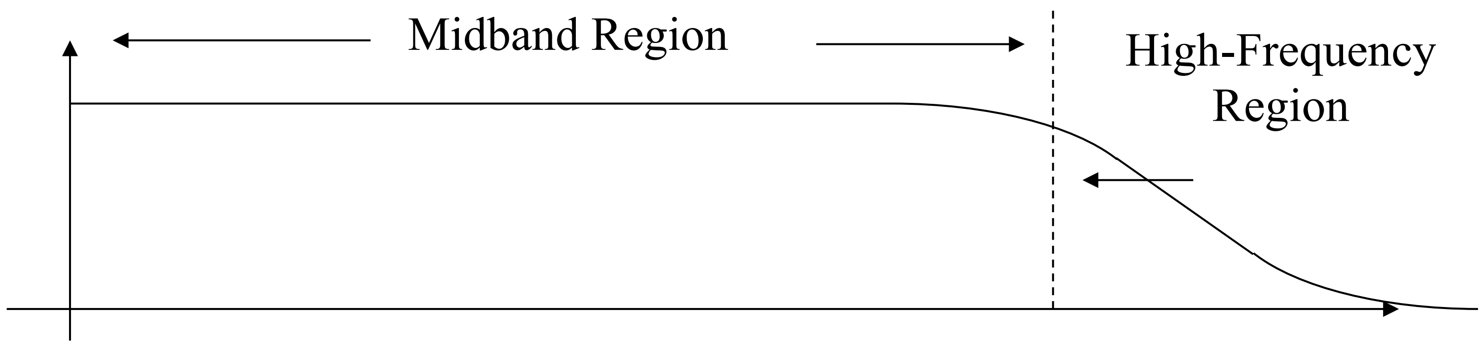
Frequency Response

- The Bode plot of the transfer function of a filter is called the Frequency Response.
- In general, a frequency response looks like the following and is made up of 3 regions or bands: Low, Mid, and High frequency bands
- Usually the filter, the stop bands are the high and low frequency bands and the pass band is the mid band region.
- The filter shown is a band pass filter.

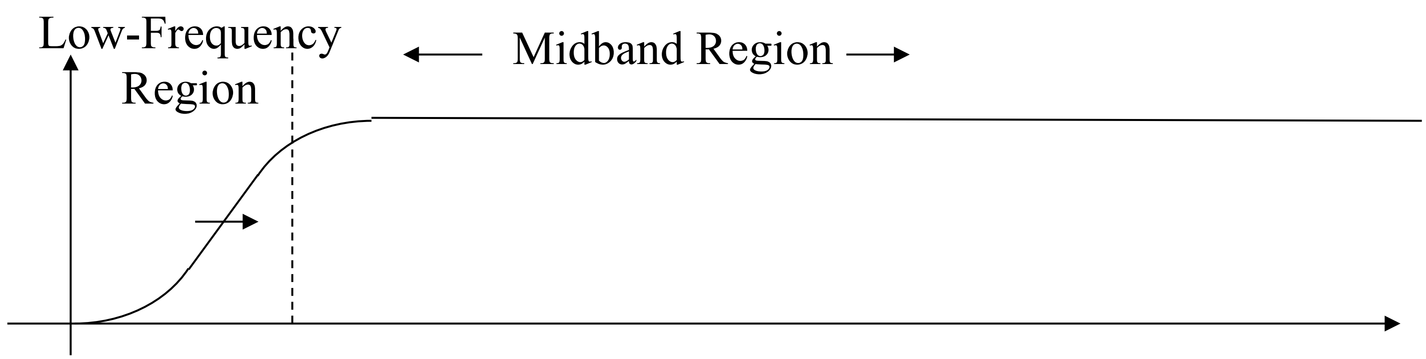


Frequency Response

- A low pass filter has no low frequency region.



- A high pass filter has no high frequency region.



Cutoff Frequencies

- The pass band is defined as those frequencies where more than ½ the maximum power passes through.
- The cutoff frequencies are the frequencies where exactly ½ the maximum power pass through.
- The range of frequencies is the pass band is called the bandwidth of the filter.

Electrical Power

Recall that the electrical power delivered by an active elements or consumed by a passive element is the product of the voltage across it and the current through it.

$P = VI$ where V is the voltage and I is the current.

For a resistor $V = IR \Rightarrow P = \frac{V^2}{R}$ or $P = I^2 R$

Let's assume that the maximum power to the output of the filter is $P_{\max} = \frac{V_{\max}^2}{R}$.

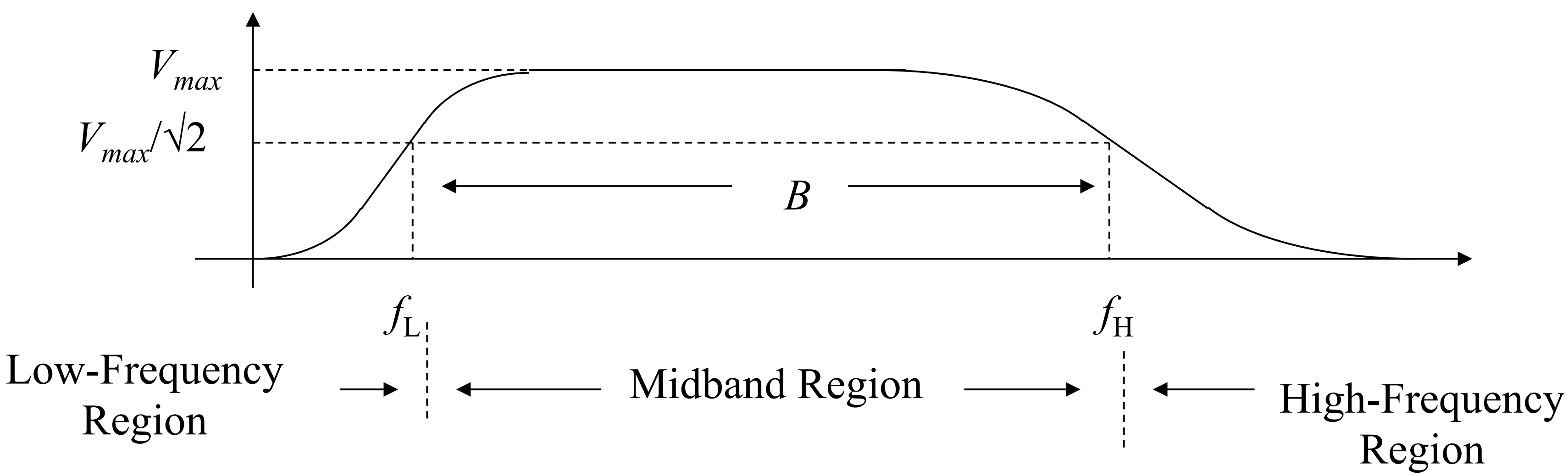
The half of the maximum power is $P_{half\ \max} = \frac{V_{\max}^2}{2R}$.

$$\frac{\text{Half power}}{\text{Max power}} = \frac{\frac{V_{\max}^2}{2R}}{\frac{V_{\max}^2}{R}} = \frac{1}{2}$$

Then the voltage at half power $= \frac{1}{\sqrt{2}} = 0.707$

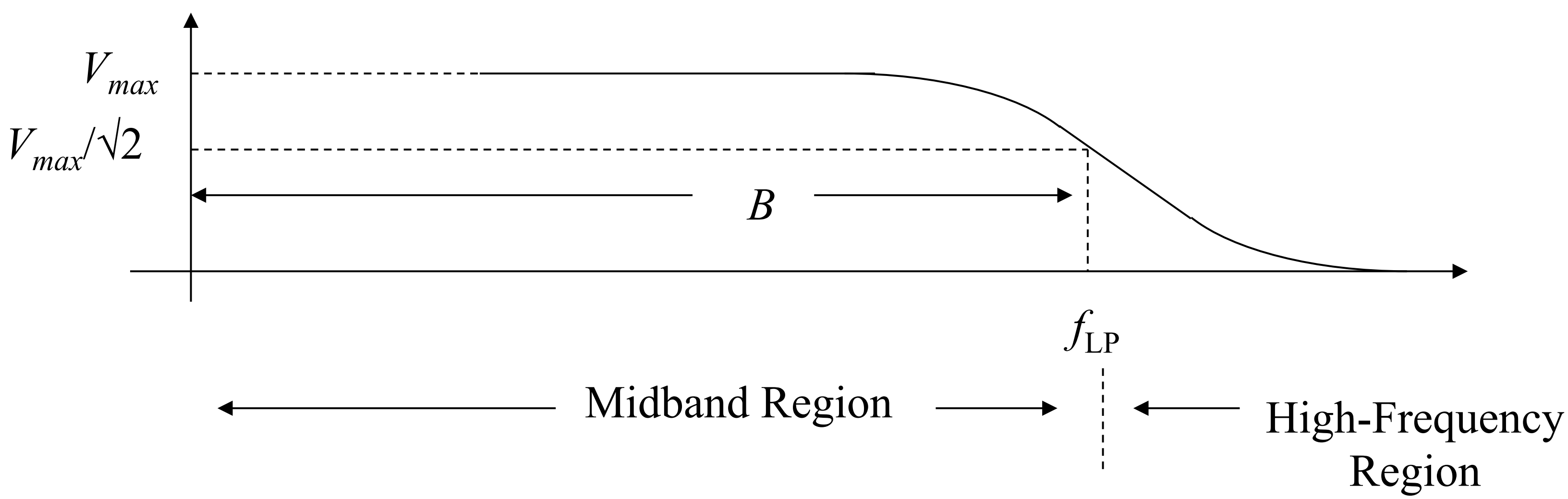
Cutoff Frequencies

- So to determine the cutoff frequencies, a line is drawn across the pass band at 0.707 of the maximum and the intersection of this line and the frequency response determines the cutoff Frequencies.
- For the band pass filter there are 2 cutoff frequencies: one for the low band f_L and one for the high band, f_H.
- And the bandwidth, $B = f_H - f_L$



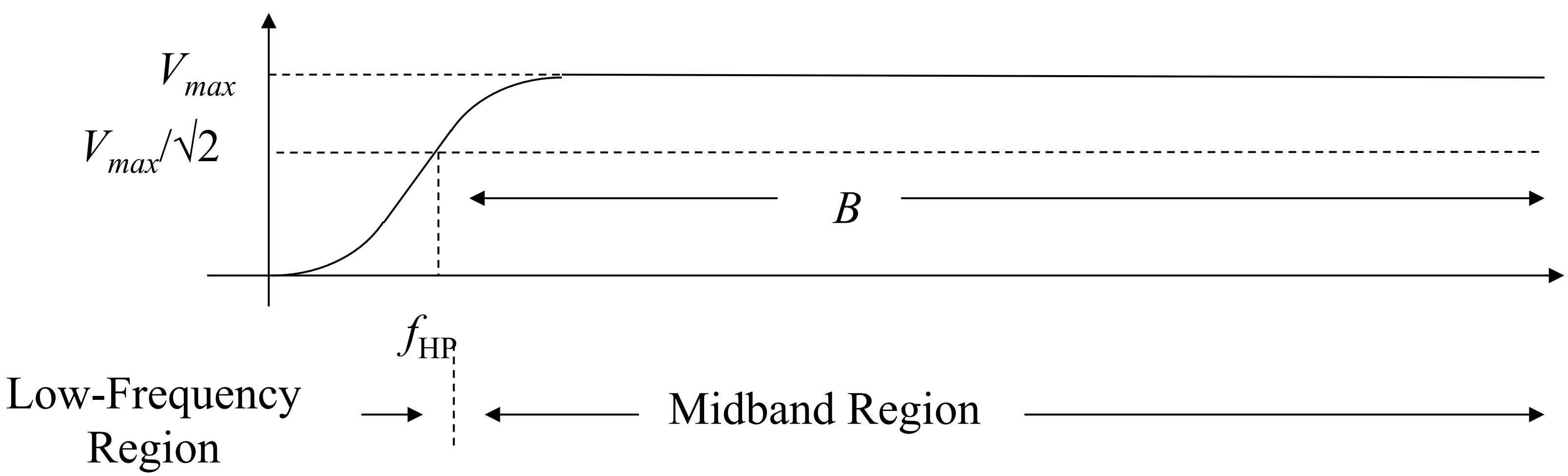
Cutoff Frequencies

- For the low pass filter there is one cutoff frequencies: f_{LP}.
- And the bandwidth, $B = f_{LP}$



Cutoff Frequencies

- For the hig pass filter there is one cutoff frequencies: f_{HP}.
- And the bandwidth is infinite but starts at $f_{HP.}$



The Transfer Function of a LP

Recall that the transfer function for the low pass filter was

$$\frac{Vout}{Vin} = \frac{1}{1 + j\omega RC}$$

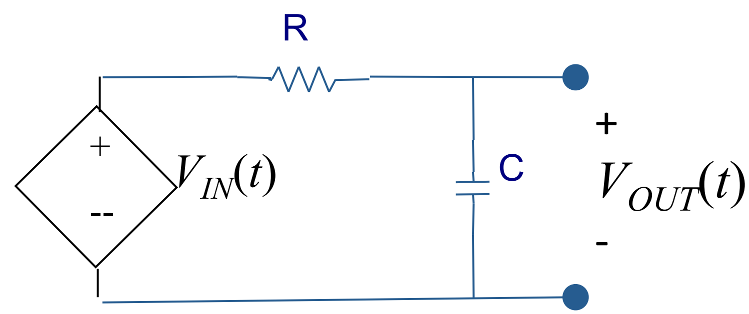
Since we normally speak of frequency, f, is units of Hertz (Hz)

let's rewrite the tranfer in term of f.

Recall $\omega = 2\pi f$ where f is the frequency in Hz

(where ω is the radian frequency in radians/sec).

$$\frac{Vout}{Vin} = \frac{1}{1 + j2\pi fRC} = \frac{1}{1 + jf\,2\pi RC} = \frac{1}{1 + j\frac{f}{f_o}}$$

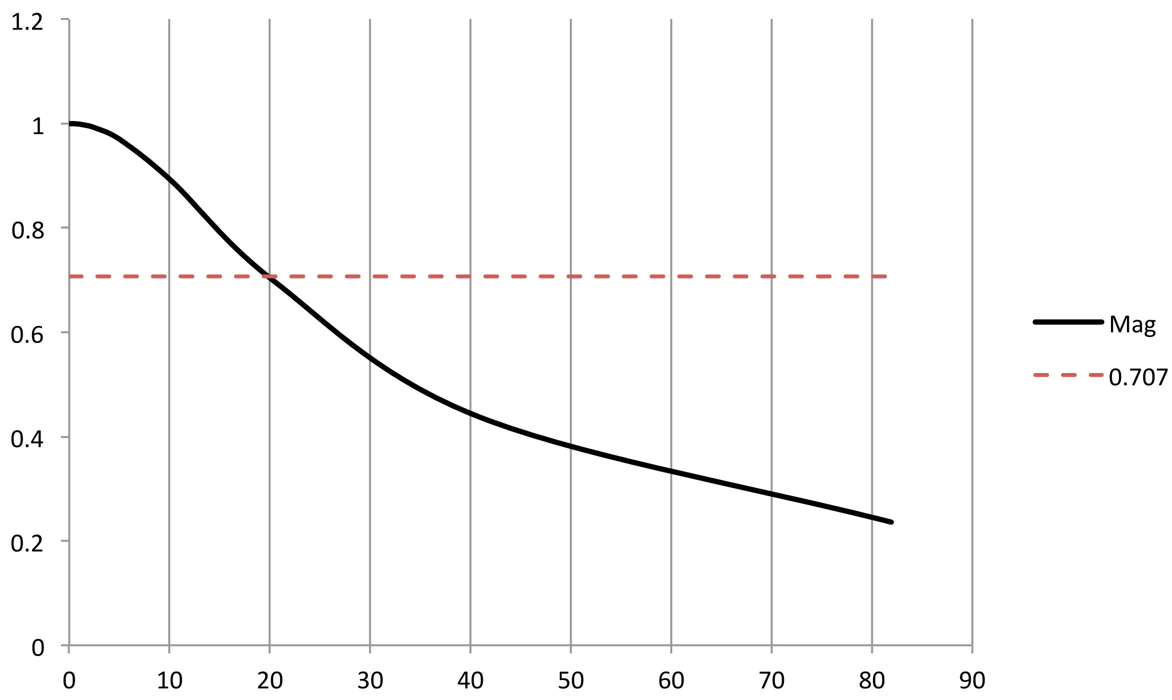


where $f_o = \frac{1}{2\pi RC}$

$$\frac{Vout}{Vin} = \frac{1}{1 + j\frac{f}{f_o}} = \frac{1}{\sqrt{1 + (\frac{f}{f_o})^2}} \angle -\tan^{-1}(\frac{f}{f_o})$$

The Transfer Function of a LP

- Here RC was set at 0.008 and f_o=19.9, we see that the cutoff frequency is about 20 Hz.
- Why?



Why?

The magnitude of transfer function should be equal to $\frac{1}{\sqrt{2}}$ at the cutoff frequency.

$$\left|\frac{Vout}{Vin}\right|_{\text{at cutoff}} = \frac{1}{\sqrt{1+(\frac{f}{f_o})^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{1+(\frac{f}{f_o})^2} = \frac{1}{2}$$

$$2 = 1+(\frac{f}{f_o})^2$$

$$(\frac{f}{f_o})^2 = 1$$

$$f = f_o$$

Let's try a high pass filter

Now the output is across the resistor

$$\frac{Vout}{Vin} = \frac{Z_2}{Z_1 + Z_2}\text{; where } Z_1 \text{ is the capacitor which is } \frac{1}{j\omega C} \text{ and } Z_2 \text{ is the resistor which is } R$$

$$\frac{Vout}{Vin} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} \angle \frac{\pi}{2} - \tan^{-1}(\omega RC)$$

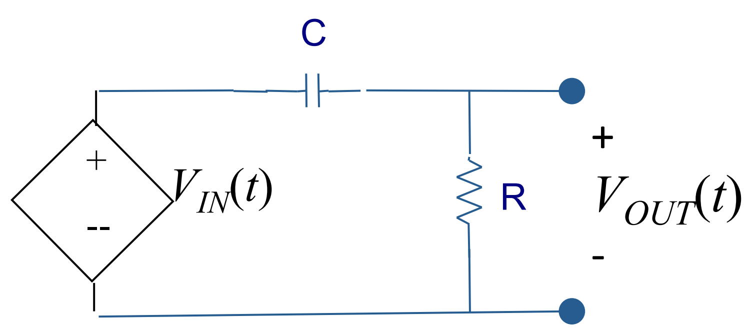
An easy way to plot the transfer function is determine 3 or more points and sketch its shape.
Two of these points are usually taken at $\omega=0$ and $\omega \to \infty$. The third can be somewhere in between at any easy point to calculate.

$$\frac{Vout}{Vin}\Big|_{\omega=0} = \frac{j\omega RC}{1 + j\omega RC}\Big|_{\omega=0} = \frac{j0}{1} = 0\angle\frac{\pi}{2}$$

$$\frac{Vout}{Vin}\Big|_{\omega\to\infty} = \frac{j\omega RC}{1 + j\omega RC}\Big|_{\omega\to\infty} \to \frac{j\omega RC}{j\omega RC}\Big|_{\omega\to\infty} = 1\angle 0$$

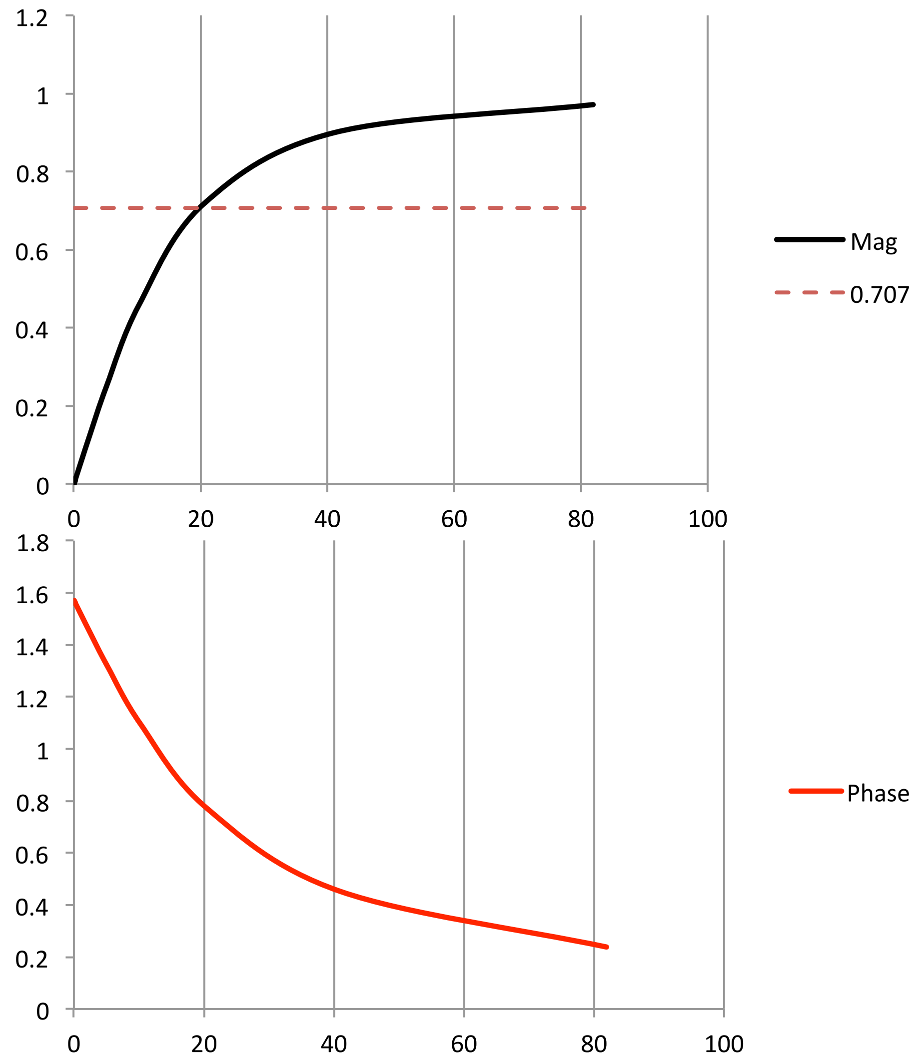
A convenient midway point is a $\omega = \frac{1}{RC}$

$$\frac{Vout}{Vin}\Big|_{\omega=\frac{1}{RC}} = \frac{j\omega RC}{1 + j\omega RC}\Big|_{\omega=\frac{1}{RC}} = \frac{j}{1+j} = \frac{1}{\sqrt{2}}\angle\frac{\pi}{2} - \frac{\pi}{4} = \frac{1}{\sqrt{2}}\angle\frac{\pi}{4}$$



The Bode Plot of the HP Filter

- Again RC was set at 0.008 and f_o=19.9, we see that the cutoff frequency is about 20 Hz.
- Why?



Why?

The magnitude of transfer function should be equal to $\frac{1}{\sqrt{2}}$ at the cutoff frequency.

$$\left|\frac{Vout}{Vin}\right|_{\text{at cutoff}} = \frac{\frac{f}{f_o}}{\sqrt{1+(\frac{f}{f_o})^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{(\frac{f}{f_o})^2}{1+(\frac{f}{f_o})^2} = \frac{1}{2}$$

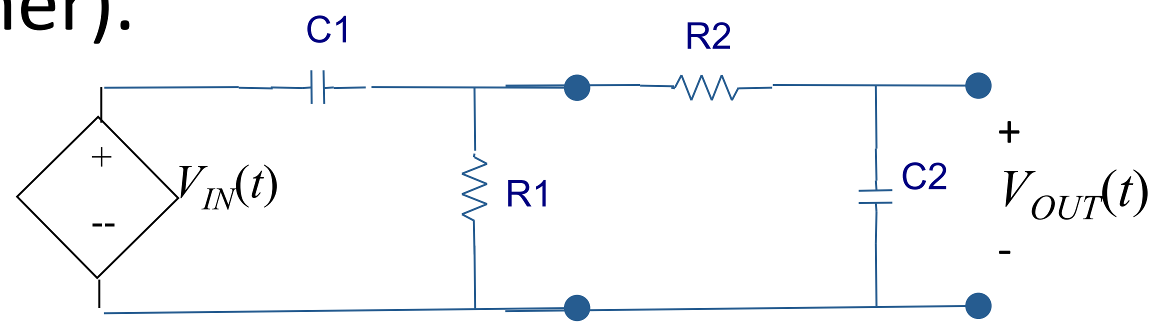
$$2(\frac{f}{f_o})^2 = 1+(\frac{f}{f_o})^2$$

$$(\frac{f}{f_o})^2 = 1$$

$$f = f_o$$

Band Pass Filter

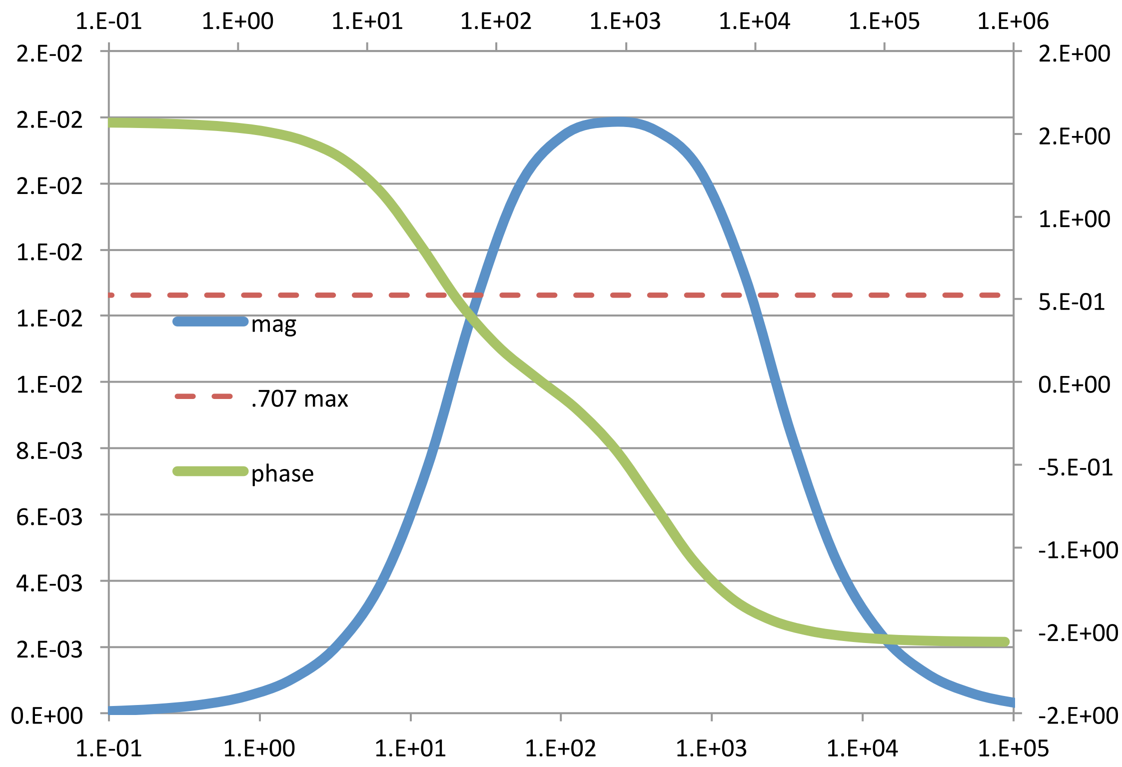
- Here we see a LP and a HP connected in cascade (the output of one is the input to the other).



- What do you expect the transfer function to be?

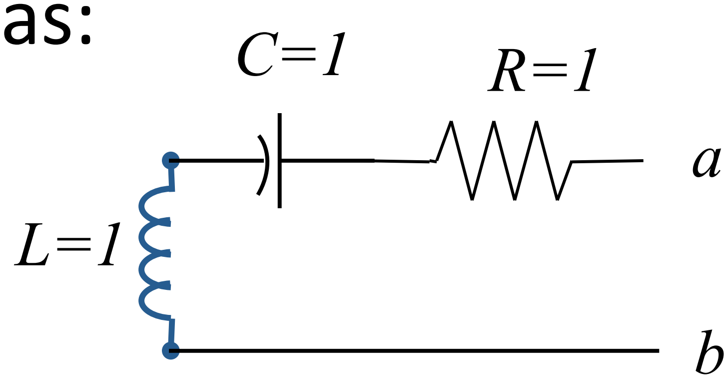
Band pass

R1=100 C1=1microf R2=1k C2=5microf



Homework

1. A series RLC circuit has an impedance given as:



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

Plot using Matlab the impedance as a function of frequency and calculate three interesting points.

Homework

2. Sketch using Matlab the transfer function of the electrode connected to an oscilloscope you calculated in Lecture 8. And calculate three interesting points.
3. HONORS STUDENTS ADD THE FOLLOWING

For the following circuit, calculate the transfer function and plot its magnitude using Matlab. Graphically determine the upper and lower cutoff frequencies.