BME 301

11 - Operational Amplifiers

Homework

1. What is the summing point constraint?
   - Applies to Op amps which is put into a negative feedback arrangement
   - In an Op-amp, the negative feedback returns a fraction of the output to the inverting input terminal forcing the differential input to zero.
   - Since the Op-amp is ideal and has infinite gain, the differential input will exactly be zero. This is called a virtual short circuit
   - Since the input impedance is infinite the current flowing into the input is also zero.
   - These latter two points are called the summing-point constraint.

Homework

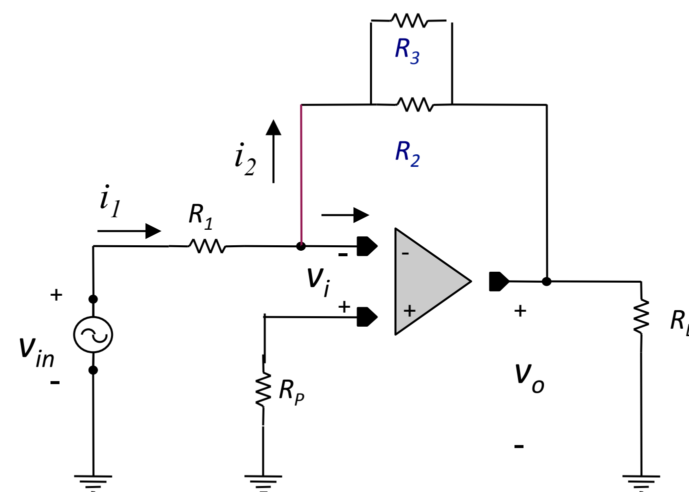
2. Calculate the gain for this amplifier (in terms of R1, R2, and R3.

(1) $v_{in} = i_1 R_1 + 0$ since v_i is zero due to the summing-point constraint

(2) $i_1 = i_2$ due to the summing-point constraint

$v_0 = -i_2 R_2 \| R_3 + 0$ since v_i is zero

(3) $v_0 = -i_2 \dfrac{1}{\dfrac{1}{R_2} + \dfrac{1}{R_3}} = -i_2 \dfrac{R_2 \times R_3}{R_2 + R_3} = -i_2 \dfrac{R_2 R_3}{R_2 + R_3}$



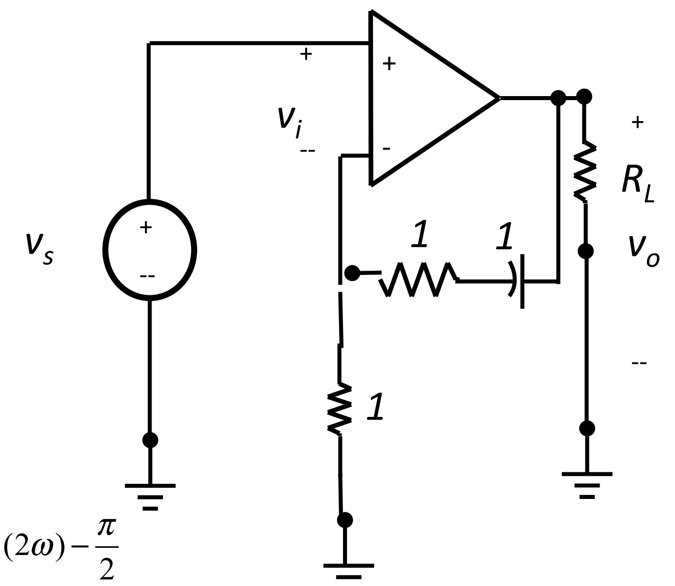
substituting for i_2 from (2) and (1) we get.

$$\frac{v_0}{v_{in}} = -\frac{\dfrac{R_2 R_3}{R_2 + R_3}}{R_1}$$ which is independent of R_L

(note that the output is opposite to the input: inverted)

Homework

3. Calculate and plot the voltage gain of the following circuit as function of frequency, ω.



This is an inverter and has the gain

$$\frac{v_o}{v_{in}} = 1 + \frac{Z_2}{Z_1} \text{ where } Z_2 \text{ is the branch with the series combination}$$

of a 1 ohm resistor and 1 farad capacitor

$$Z_2 = 1 + \frac{1}{j\omega} = \frac{1+j\omega}{j\omega}$$

And Z_1 is the branch with the single resistor

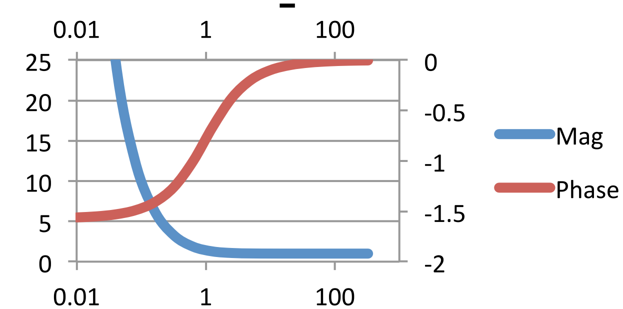
$$Z_1 = 1$$

$$\frac{v_o}{v_{in}} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{\frac{1+j\omega}{j\omega}}{1} = 1 + \frac{1+j\omega}{j\omega} = \frac{1+j2\omega}{j\omega} = \frac{\sqrt{1+(2\omega)^2}}{\omega}\angle \tan^{-1}(2\omega) - \frac{\pi}{2}$$

$$\frac{v_o}{v_{in}}\Big|_{\omega=0} = \frac{1+j2\omega}{j\omega}\Big|_{\omega=0} \to \frac{1}{j\omega} \to \infty\angle -\frac{\pi}{2}$$

$$\frac{v_o}{v_{in}}\Big|_{\omega\to\infty} = \frac{1+j2\omega}{j\omega}\Big|_{\omega\to\infty} \to \frac{j2\omega}{j\omega} = 2\angle 0$$

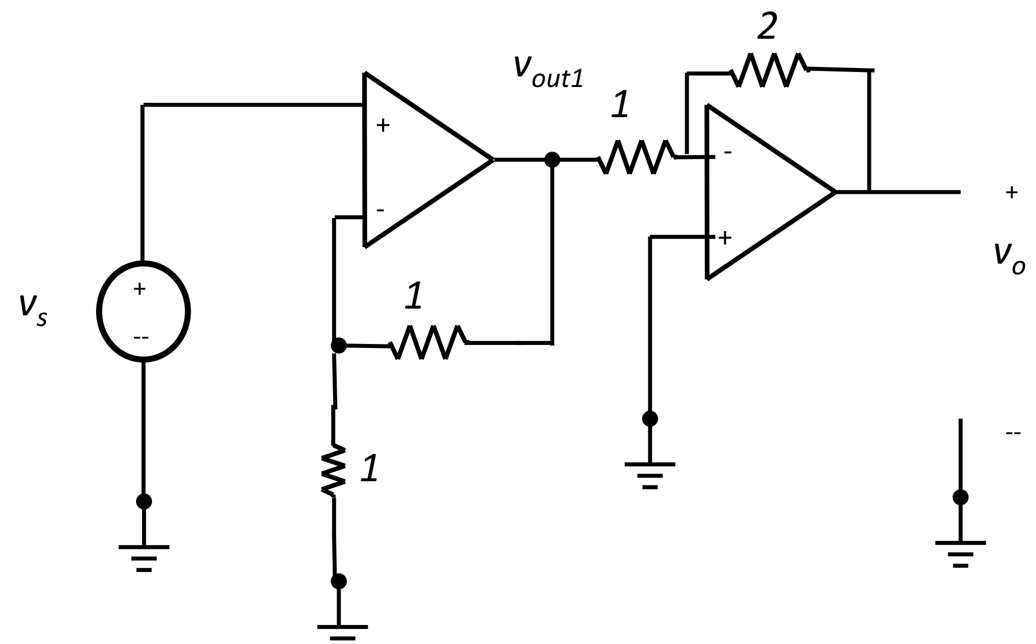
$$\frac{v_o}{v_{in}}\Big|_{\omega=.5} = \frac{1+j2\omega}{j\omega}\Big|_{\omega=.5} = \frac{1+j}{j0.5} = \frac{\sqrt{2}}{0.5}\angle\frac{\pi}{4} - \frac{\pi}{2} = 2\sqrt{2}\angle -\frac{\pi}{4}$$



Homework

4. HONORS STUDENTS ADD THE FOLLOWING

We need to calculate $v_o/v_s = v_{out1}/v_s \times v_o/v_{out1}$
Note that the output of the first stage, v_{out1} is the input to the second stage. This is called cascading.



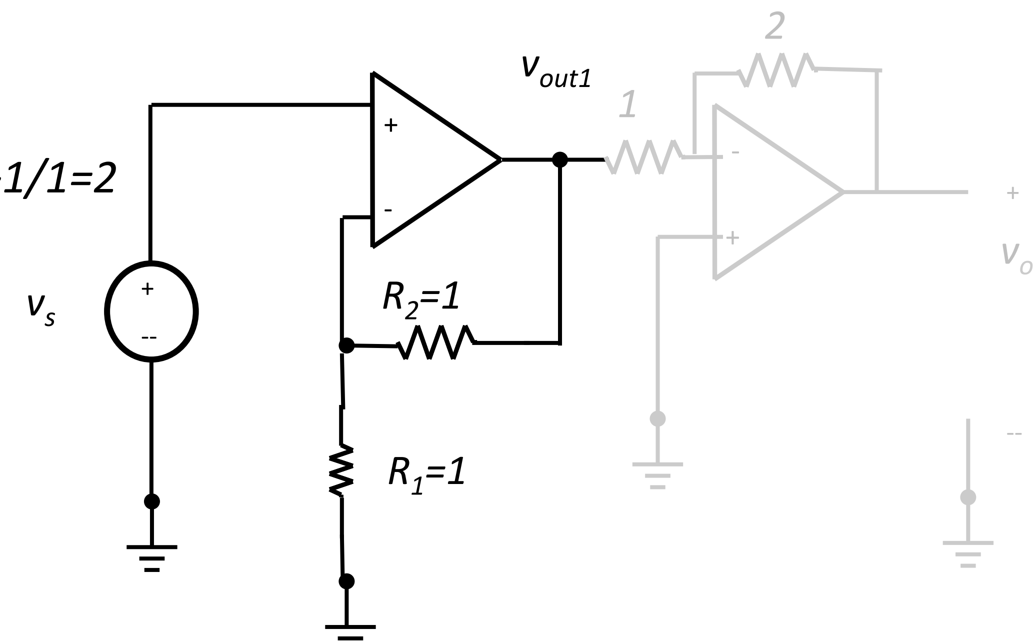
Homework

4. HONORS STUDENTS ADD THE FOLLOWING

Taking the first stage, calculating its gain.

First

$v_{out1}/v_s = 1 + R_2/R_1 = 1 + 1/1 = 2$

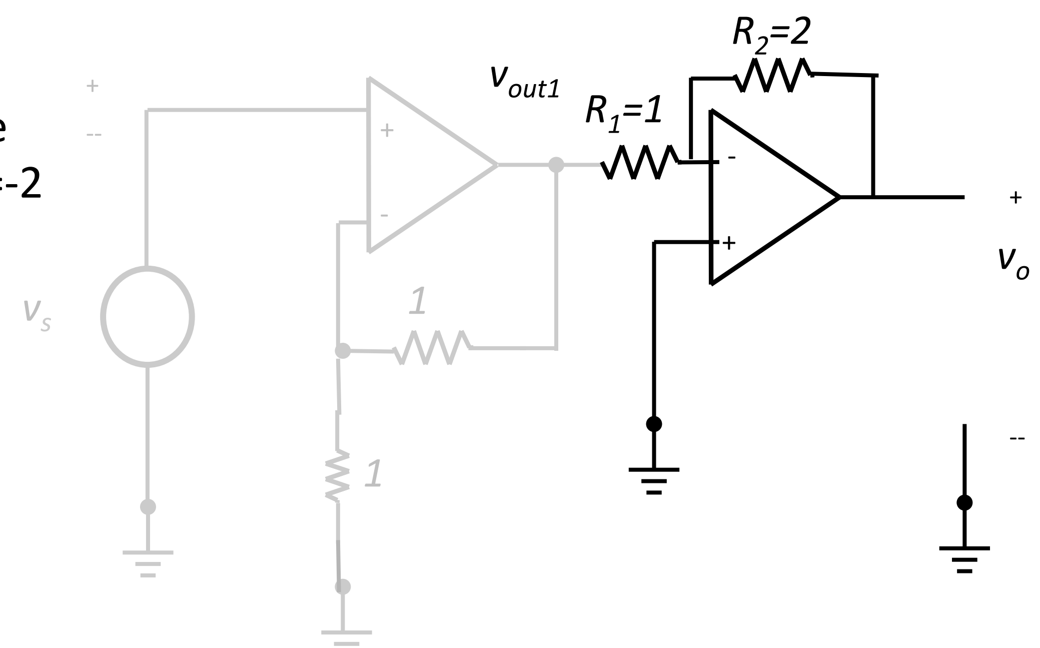


Homework

4. HONORS STUDENTS ADD THE FOLLOWING

   Next stage 2

We need to calculate
v_o/v_{out1}=-R_2/R_1=-2/1=-2

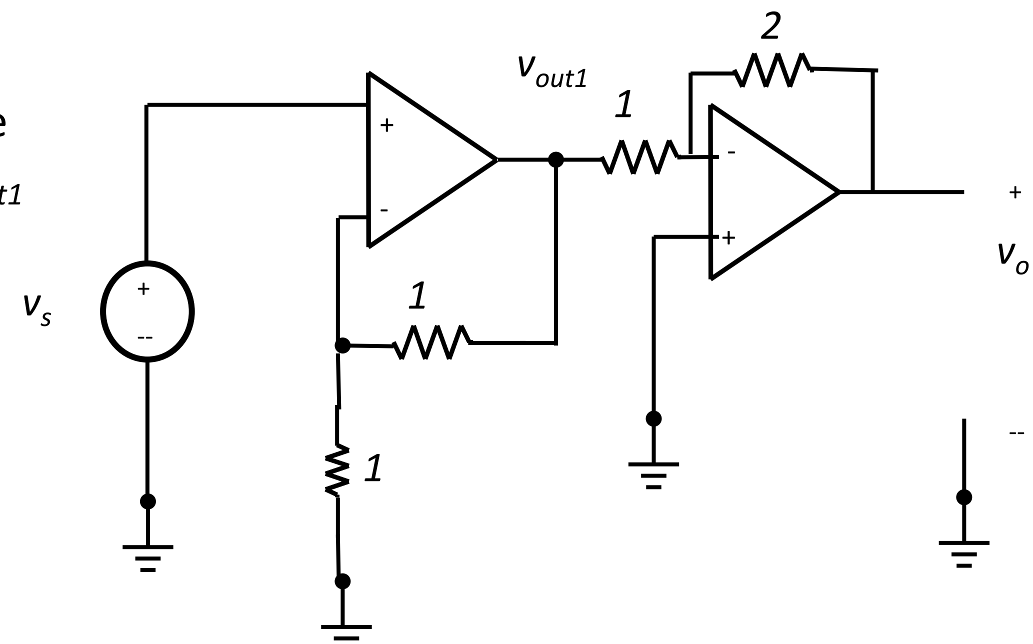


Homework

4. HONORS STUDENTS ADD THE FOLLOWING

Putting it all together.

We need to calculate
$v_o/v_s = v_{out1}/v_s \times v_o/v_{out1}$
$v_o/v_s = 2 \times -2 = -4$



Homework

5. HONORS STUDENTS ADD THE FOLLOWING

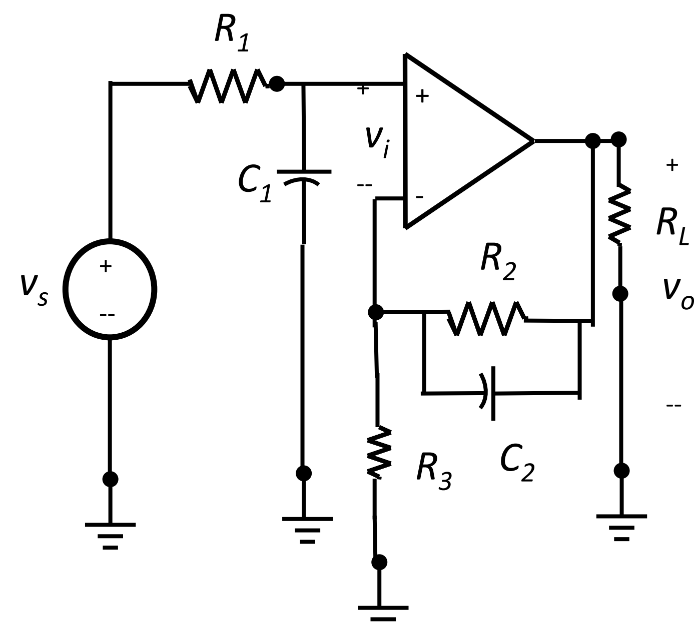
   The criteria for a proper negative feedback opamp circuit is the summing point constraint. What would it be for a proper positive feedback circuit?

   – Infinite input impedance, R_i is infinite
   – Zero output impedance, R_o is zero
   – Infinite gain for the differential signal, A_d is infinite
   – Zero gain for the common-mode signal
   – Infinite Bandwidth

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

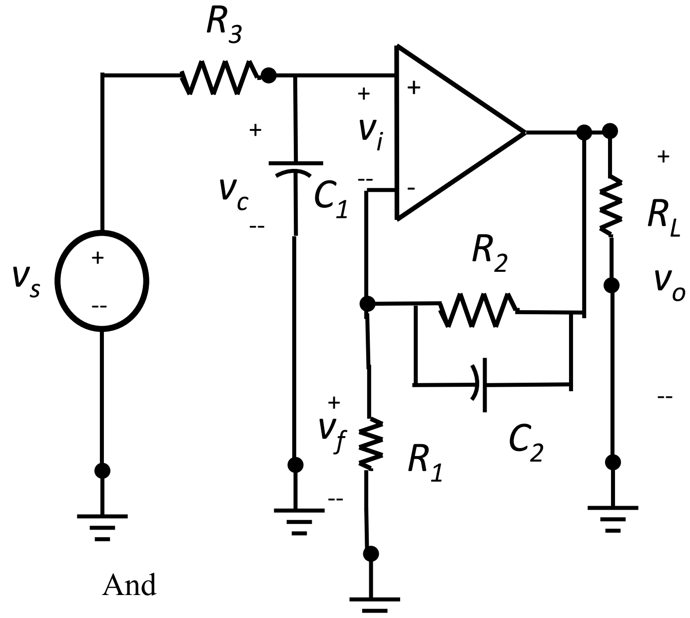
Calculate and plot the gain of this circuit. What type of filter is this?



Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Calculate and plot the gain of this circuit. What type of filter is this?



Note that

$$v_c = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R_3} v_s = \frac{1}{1 + j\omega C_1 R_3} v_s$$

$$v_f = \frac{R_1}{R_1 + R_2 \parallel \frac{1}{j\omega C_2}} v_o = \frac{R_1}{R_1 + \frac{R_2}{1 + j\omega C_2 R_2}} v_o$$

$$R_2 \parallel \frac{1}{j\omega C_2} = \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$v_f = \frac{R_1(1 + j\omega C_2 R_2)}{R_1(1 + j\omega C_2 R_2) + R_2} v_o = \frac{R_1(1 + j\omega C_2 R_2)}{R_1 + R_2 + j\omega C_2 R_2 R_1} v_o$$

And

$$v_c = v_i + v_f$$

Due to negative feedback and the summing point constraint

$$v_c = v_f$$

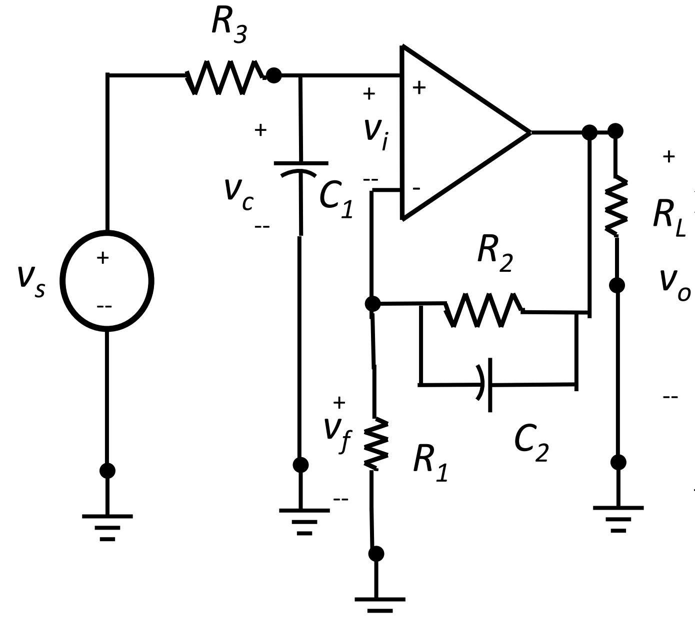
Therefore,

$$\frac{1}{1 + j\omega C_1 R_3} v_s = \frac{R_1(1 + j\omega C_2 R_2)}{R_1 + R_2 + j\omega C_2 R_2 R_1} v_o$$

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Calculate and plot the gain of this circuit. What type of filter is this?



Due to negative Solving for the transfer function $\frac{v_o}{v_s}$

$$\frac{1}{1+j\omega C_1R_3}v_s=\frac{R_1(1+j\omega C_2R_2)}{R_1+R_2+j\omega C_2R_2R_1}v_o\Rightarrow\frac{v_o}{v_s}=\frac{\frac{1}{1+j\omega C_1R_3}}{\frac{R_1(1+j\omega C_2R_2)}{R_1+R_2+j\omega C_2R_2R_1}}$$

$$\frac{v_o}{v_s}=\frac{R_1+R_2+j\omega C_2R_2R_1}{[R_1(1+j\omega C_2R_2)](1+j\omega C_1R_3)}$$

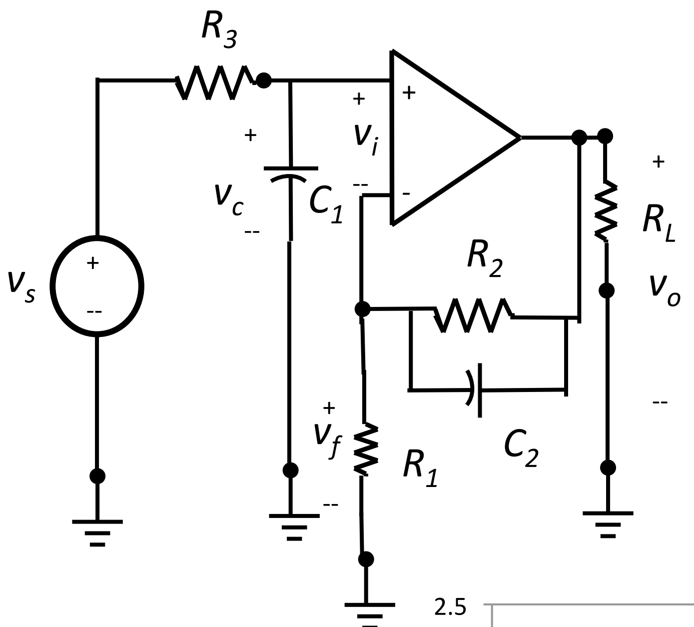
$$\frac{v_o}{v_s}=\frac{R_1+R_2+j\omega C_2R_2R_1}{[R_1(1+j\omega C_2R_2)](1+j\omega C_1R_3)}=\frac{R_1+R_2(1+j\omega\frac{C_2R_2R_1}{R_1+R_2})}{R_1[1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)]}$$

$$=\frac{R_1+R_2}{R_1}\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)}=\frac{R_1+R_2}{R_1}\times\frac{\sqrt{1+(\omega\frac{C_2R_2R_1}{R_1+R_2})^2}}{\sqrt{(1-\omega^2C_2R_2C_1R_3)^2+(\omega(C_1R_3+C_2R_2))^2}}\angle[\tan^{-1}(\omega\frac{C_2R_2R_1}{R_1+R_2})-\tan^{-1}(\frac{\omega(C_1R_3+C_2R_2)}{1-\omega^2C_2R_2C_1R})]$$

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Calculate and plot the gain of this circuit. What type of filter is this?



$$\frac{v_o}{v_s} = \frac{R_1+R_2}{R_1}\,\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)}$$

$$\frac{v_o}{v_s}\Big|_{\omega=0} = \frac{R_1+R_2}{R_1}\,\frac{1+j0\frac{C_2R_2R_1}{R_1+R_2}}{1-0^2C_2R_2C_1R_3+j0(C_1R_3+C_2R_2)} = \frac{R_1+R_2}{R_1}\angle 0$$

$$\frac{v_o}{v_s}\Big|_{\omega\to\infty} \to \frac{R_1+R_2}{R_1}\,\frac{j\omega\frac{C_2R_2R_1}{R_1+R_2}}{-\omega^2C_2R_2C_1R_3} = \frac{R_1+R_2}{R_1}\,\frac{j\frac{1}{R_1+R_2}}{-\omega C_1} = \frac{j}{-\omega R_1C_1} = 0\angle\frac{\pi}{2}$$

$$\frac{v_o}{v_s}\Big|_{\omega=\frac{1}{\sqrt{C_2R_2C_1R_3}}} = \frac{R_1+R_2}{R_1}\,\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{j\omega(C_1R_3+C_2R_2)} = \frac{R_1+R_2}{R_1}\,\frac{\sqrt{1+(\omega\frac{C_2R_2R_1}{R_1+R_2})^2}}{\omega(C_1R_3+C_2R_2)}\angle[\tan^{-1}(\omega\frac{C_2R_2R_1}{R_1+R_2})-\frac{\pi}{2}]$$