BME 301

11 - Operational Amplifiers

Homework

1. What is the summing point constraint?
   - Applies to Op amps which is put into a negative feedback arrangement
   - In an Op-amp, the negative feedback returns a fraction of the output to the inverting input terminal forcing the differential input to zero.
   - Since the Op-amp is ideal and has infinite gain, the differential input will exactly be zero. This is called a virtual short circuit
   - Since the input impedance is infinite the current flowing into the input is also zero.
   - These latter two points are called the summing-point constraint.

Homework

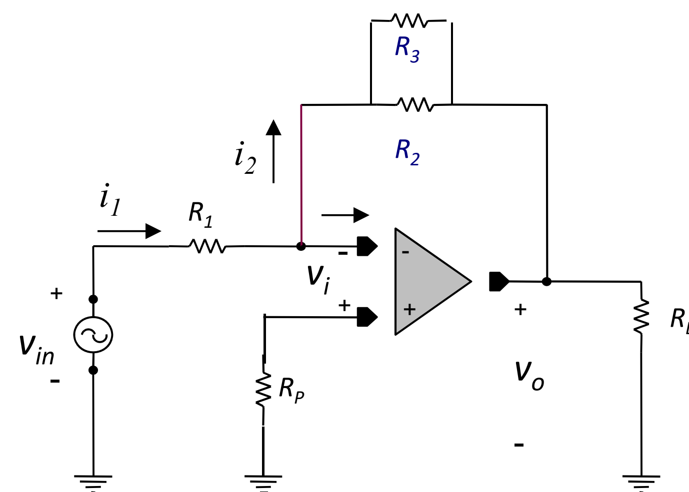
2. Calculate the gain for this amplifier (in terms of R1, R2, and R3.

(1) $v_{in} = i_1 R_1 + 0$ since v_i is zero due to the summing-point constraint

(2) $i_1 = i_2$ due to the summing-point constraint

$v_0 = -i_2 R_2 \parallel R_3 + 0$ since v_i is zero

(3) $v_0 = -i_2 \dfrac{1}{\dfrac{1}{R_2} + \dfrac{1}{R_3}} = -i_2 \dfrac{R_2 \times R_3}{R_2 + R_3} = -i_2 \dfrac{R_2 R_3}{R_2 + R_3}$



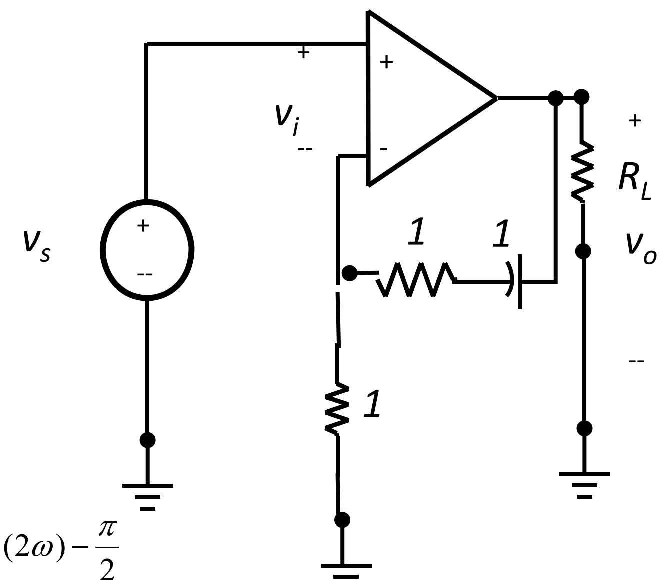
substituting for i_2 from (2) and (1) we get.

$$\frac{v_0}{v_{in}} = -\frac{\dfrac{R_2 R_3}{R_2 + R_3}}{R_1}$$ which is independent of R_L

(note that the output is opposite to the input: inverted)

Homework

3. Calculate and plot the voltage gain of the following circuit as function of frequency, *ω*.



This is an inverter and has the gain

$\frac{v_o}{v_{in}} = 1 + \frac{Z_2}{Z_1}$ where Z_2 is the branch with the series combination of a 1 ohm resistor and 1 farad capacitor

$$Z_2 = 1 + \frac{1}{j\omega} = \frac{1+j\omega}{j\omega}$$

And Z_1 is the branch with the single resistor

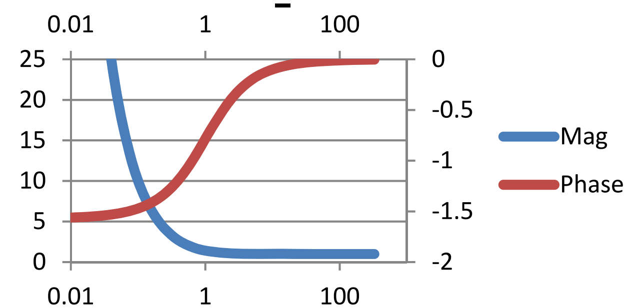
$$Z_1 = 1$$

$$\frac{v_o}{v_{in}} = 1 + \frac{Z_2}{Z_1} = 1 + \frac{\frac{1+j\omega}{j\omega}}{1} = 1 + \frac{1+j\omega}{j\omega} = \frac{1+j2\omega}{j\omega} = \frac{\sqrt{1+(2\omega)^2}}{\omega} \angle \tan^{-1}(2\omega) - \frac{\pi}{2}$$

$$\frac{v_o}{v_{in}}\Big|_{\omega=0} = \frac{1+j2\omega}{j\omega}\Big|_{\omega=0} \rightarrow \frac{1}{j\omega} \rightarrow \infty\angle -\frac{\pi}{2}$$

$$\frac{v_o}{v_{in}}\Big|_{\omega\to\infty} = \frac{1+j2\omega}{j\omega}\Big|_{\omega\to\infty} \rightarrow \frac{j2\omega}{j\omega} = 2\angle 0$$

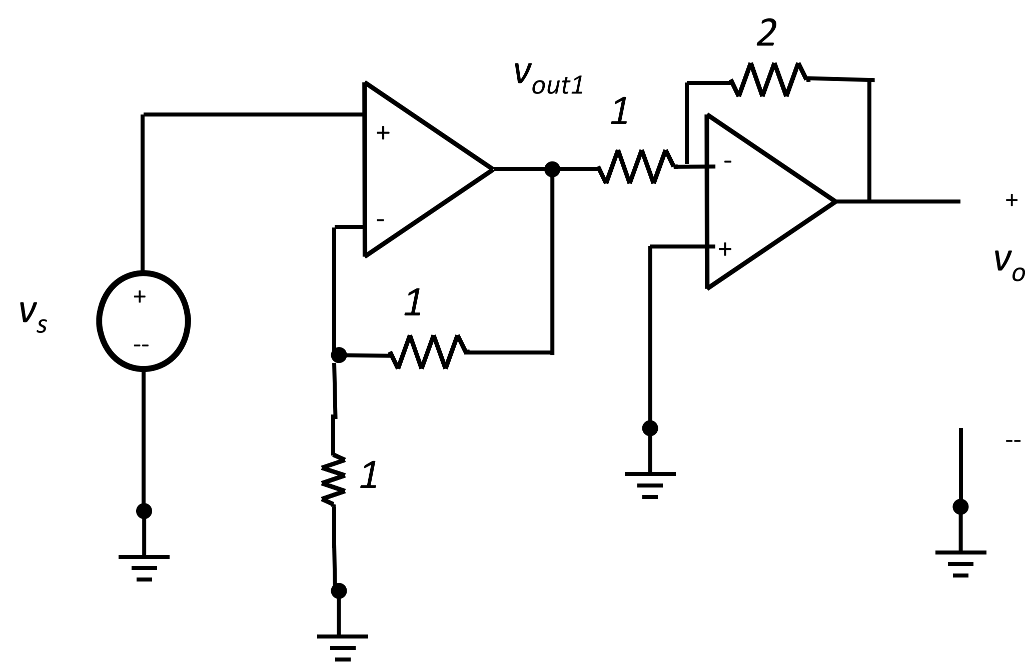
$$\frac{v_o}{v_{in}}\Big|_{\omega=.5} = \frac{1+j2\omega}{j\omega}\Big|_{\omega=.5} = \frac{1+j}{j0.5} = \frac{\sqrt{2}}{0.5}\angle\frac{\pi}{4} - \frac{\pi}{2} = 2\sqrt{2}\angle -\frac{\pi}{4}$$



Homework

4. HONORS STUDENTS ADD THE FOLLOWING

We need to calculate $v_o/v_s = v_{out1}/v_s \times v_o/v_{out1}$
Note that the output of the first stage, v_{out1} is the input to the second stage. This is called cascading.



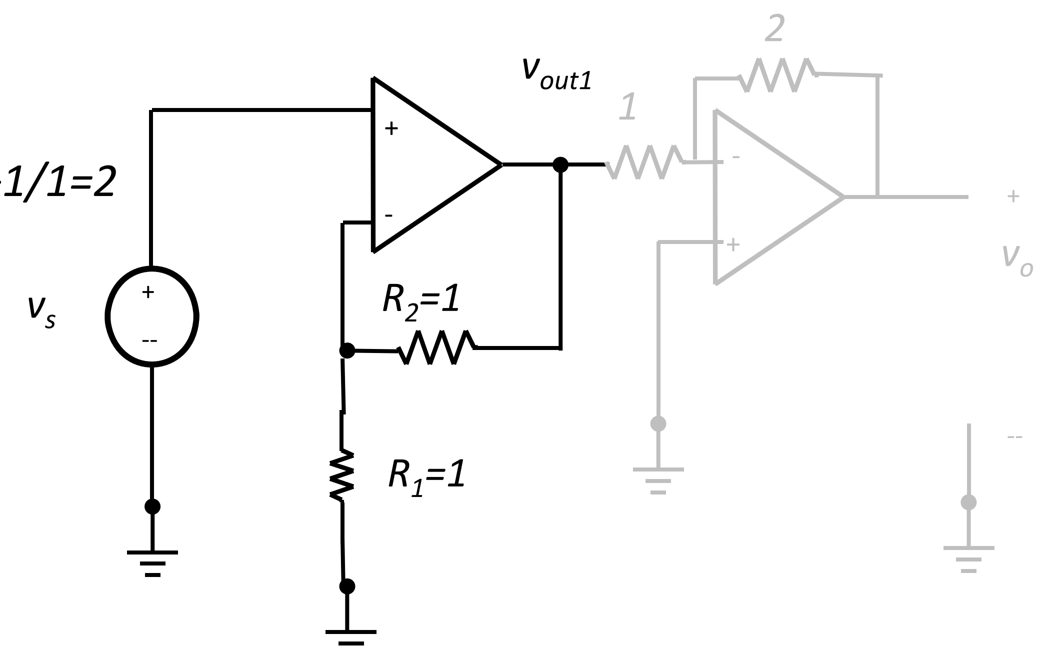
Homework

4. HONORS STUDENTS ADD THE FOLLOWING

Taking the first stage, calculating its gain.

First

$v_{out1}/v_s = 1 + R_2/R_1 = 1 + 1/1 = 2$

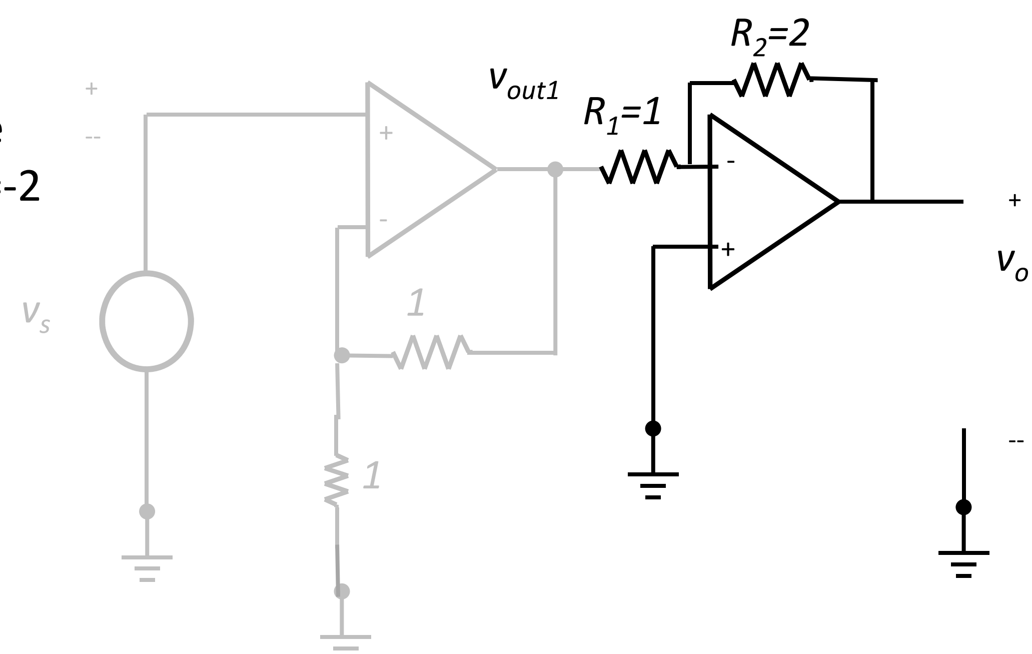


Homework

4. HONORS STUDENTS ADD THE FOLLOWING

Next stage 2

We need to calculate
v_o/v_{out1}=-R_2/R_1=-2/1=-2

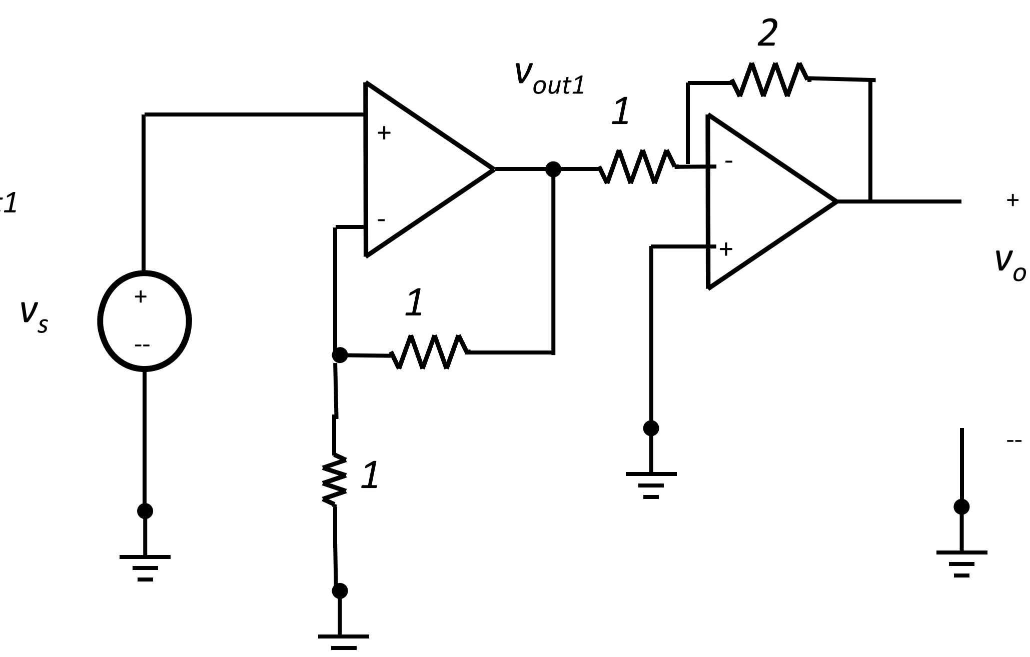


Homework

4. HONORS STUDENTS ADD THE FOLLOWING

Putting it all together.

We need to calculate
$v_o/v_s = v_{out1}/v_s \times v_o/v_{out1}$
$v_o/v_s = 2 \times -2 = -4$



Homework

5. HONORS STUDENTS ADD THE FOLLOWING

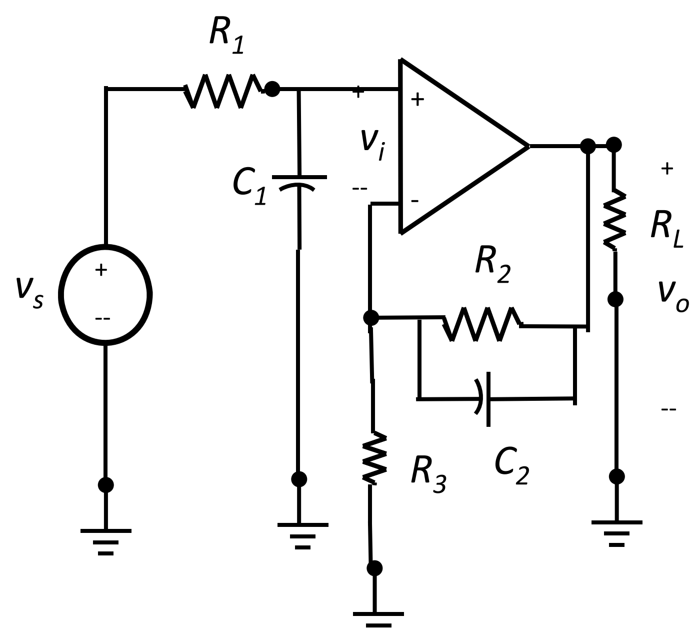
   The criteria for a proper negative feedback opamp circuit is the summing point constraint. What would it be for a proper positive feedback circuit?

   – Infinite input impedance, R_i is infinite
   – Zero output impedance, R_o is zero
   – Infinite gain for the differential signal, A_d is infinite
   – Zero gain for the common-mode signal
   – Infinite Bandwidth

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Calculate and plot the gain of this circuit. What type of filter is this?



Homework

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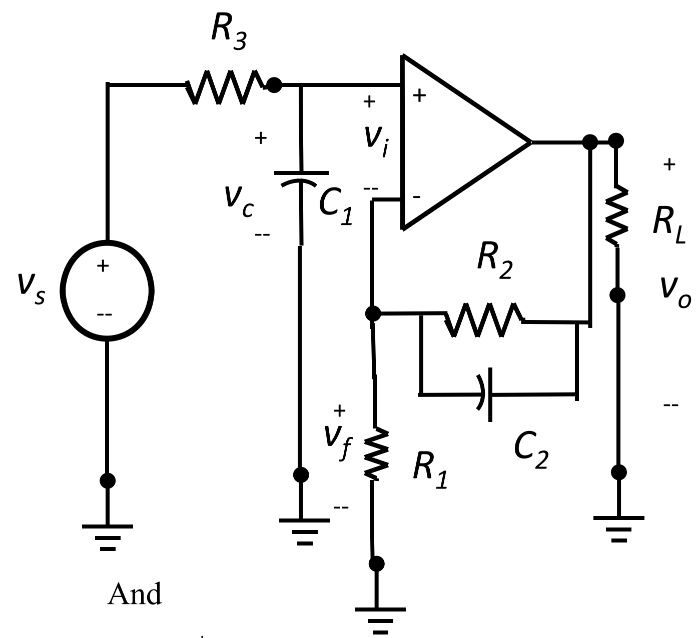

Note that

$$v_c = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + R_3} v_s = \frac{1}{1 + j\omega C_1 R_3} v_s$$

$$v_f = \frac{R_1}{R_1 + R_2 \| \frac{1}{j\omega C_2}} v_o = \frac{R_1}{R_1 + \frac{R_2}{1 + j\omega C_2 R_2}} v_o$$

$$R_2 \| \frac{1}{j\omega C_2} = \frac{R_2 \times \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega C_2 R_2}$$

$$v_f = \frac{R_1(1 + j\omega C_2 R_2)}{R_1(1 + j\omega C_2 R_2) + R_2} v_o = \frac{R_1(1 + j\omega C_2 R_2)}{R_1 + R_2 + j\omega C_2 R_2 R_1} v_o$$

And

$$v_c = v_i + v_f$$

Due to negative feedback and the summing point constraint

$$v_c = v_f$$

Therefore,

$$\frac{1}{1 + j\omega C_1 R_3} v_s = \frac{R_1(1 + j\omega C_2 R_2)}{R_1 + R_2 + j\omega C_2 R_2 R_1} v_o$$

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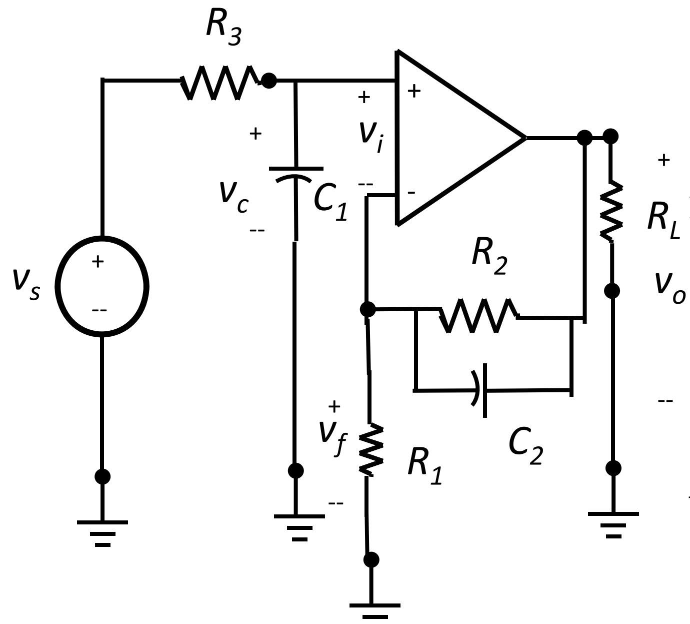

Due to negative Solving for the transfer function $\frac{v_o}{v_s}$

$$\frac{1}{1+j\omega C_1R_3}v_s = \frac{R_1(1+j\omega C_2R_2)}{R_1+R_2+j\omega C_2R_2R_1}v_o \Rightarrow \frac{v_o}{v_s} = \frac{\frac{1}{1+j\omega C_1R_3}}{\frac{R_1(1+j\omega C_2R_2)}{R_1+R_2+j\omega C_2R_2R_1}}$$

$$\frac{v_o}{v_s} = \frac{R_1+R_2+j\omega C_2R_2R_1}{[R_1(1+j\omega C_2R_2)](1+j\omega C_1R_3)}$$

$$\frac{v_o}{v_s} = \frac{R_1+R_2+j\omega C_2R_2R_1}{[R_1(1+j\omega C_2R_2)](1+j\omega C_1R_3)} = \frac{R_1+R_2(1+j\omega\frac{C_2R_2R_1}{R_1+R_2})}{R_1[1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)]}$$

$$= \frac{R_1+R_2}{R_1}\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)} = \frac{R_1+R_2}{R_1}\times\frac{\sqrt{1+(\omega\frac{C_2R_2R_1}{R_1+R_2})^2}}{\sqrt{(1-\omega^2C_2R_2C_1R_3)^2+(\omega(C_1R_3+C_2R_2))^2}}\angle[\tan^{-1}(\omega\frac{C_2R_2R_1}{R_1+R_2})-\tan^{-1}(\frac{\omega(C_1R_3+C_2R_2)}{1-\omega^2C_2R_2C_1R})]$$

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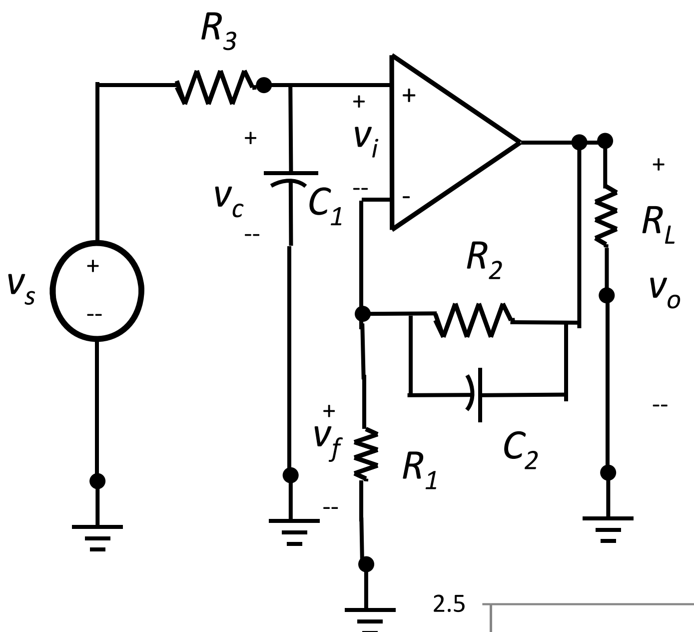

$$\frac{v_o}{v_s} = \frac{R_1+R_2}{R_1}\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{1-\omega^2C_2R_2C_1R_3+j\omega(C_1R_3+C_2R_2)}$$

$$\frac{v_o}{v_s}\Big|_{\omega=0} = \frac{R_1+R_2}{R_1}\frac{1+j0\frac{C_2R_2R_1}{R_1+R_2}}{1-0^2C_2R_2C_1R_3+j0(C_1R_3+C_2R_2)} = \frac{R_1+R_2}{R_1}\angle 0$$

$$\frac{v_o}{v_s}\Big|_{\omega\to\infty} \to \frac{R_1+R_2}{R_1}\frac{j\omega\frac{C_2R_2R_1}{R_1+R_2}}{-\omega^2C_2R_2C_1R_3} = \frac{R_1+R_2}{R_1}\frac{j\frac{1}{R_1+R_2}}{-\omega C_1} = \frac{j}{-\omega R_1C_1} = 0\angle\frac{\pi}{2}$$

$$\frac{v_o}{v_s}\Big|_{\omega=\frac{1}{\sqrt{C_2R_2C_1R_3}}} = \frac{R_1+R_2}{R_1}\frac{1+j\omega\frac{C_2R_2R_1}{R_1+R_2}}{j\omega(C_1R_3+C_2R_2)} = \frac{R_1+R_2}{R_1}\frac{\sqrt{1+(\omega\frac{C_2R_2R_1}{R_1+R_2})^2}}{\omega(C_1R_3+C_2R_2)}\angle[\tan^{-1}(\omega\frac{C_2R_2R_1}{R_1+R_2})-\frac{\pi}{2}]$$