BME 301

5-Complex Circuits

Homework

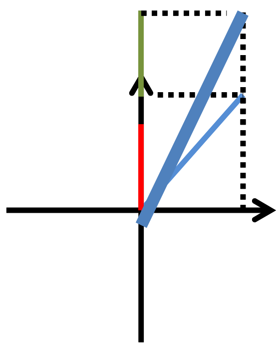
1. Simplify the following complex numbers

$$2a)\ 2e^{j\pi/4}+2e^{j\pi/2}=2[(\cos(\pi/4)+j\sin(\pi/4))+(\cos(\pi/2)+j\sin(\pi/2))]$$

$$=2[(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}})+(0+j1)]=2[\frac{1}{\sqrt{2}}+0+j(\frac{1}{\sqrt{2}}+1)]$$

$$=\sqrt{2}+j(\sqrt{2}+2)=\sqrt{2}+j3.4=\sqrt{(\sqrt{2})^2+(3.4)^2}\angle\tan^{-1}(\frac{3.4}{\sqrt{2}})$$

$$=3.7\angle 1.18$$

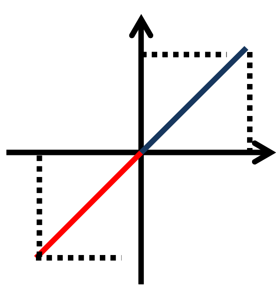


Homework

1. Simplify the following complex numbers

$$2b)\ e^{j5\pi/4} + e^{j\pi/4} = (\cos(5\pi/4) + j\sin(5\pi/4)) + (\cos(\pi/4) + j\sin(\pi/4))$$

$$= -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0\angle 0$$



Homework

1. Simplify the following complex numbers

$$2c)\ (3+4j)^5 = (\sqrt{3^2+4^2}\,e^{+j\tan^{-1}(\frac{4}{3})})^5 = (5e^{j0.92})^5$$

$$= 5^5 e^{j0.92\times 5} = 3125e^{j4.63} = 3125e^{-j1.64}$$

$$2d)\ \Re e\{\frac{\sqrt{2}e^{j\pi/2}}{-j}\} = \Re e\{\frac{\sqrt{2}e^{j\pi/2}}{e^{-j\pi/2}}\} = \Re e\{\sqrt{2}e^{j\pi/2}e^{+j\pi/2}\}$$

$$= \Re e\{\sqrt{2}e^{j\pi}\} = \Re e\{\sqrt{2}\cos\pi + j\sqrt{2}\sin\pi\} = \Re e\{-\sqrt{2}+j0\}$$

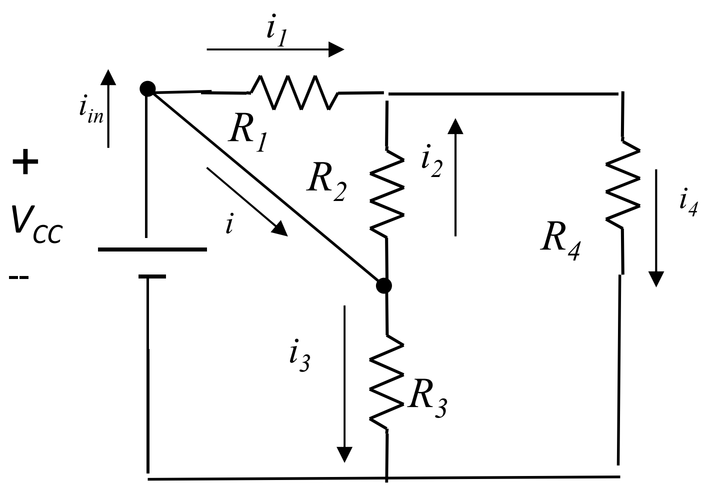
$$= -\sqrt{2}$$

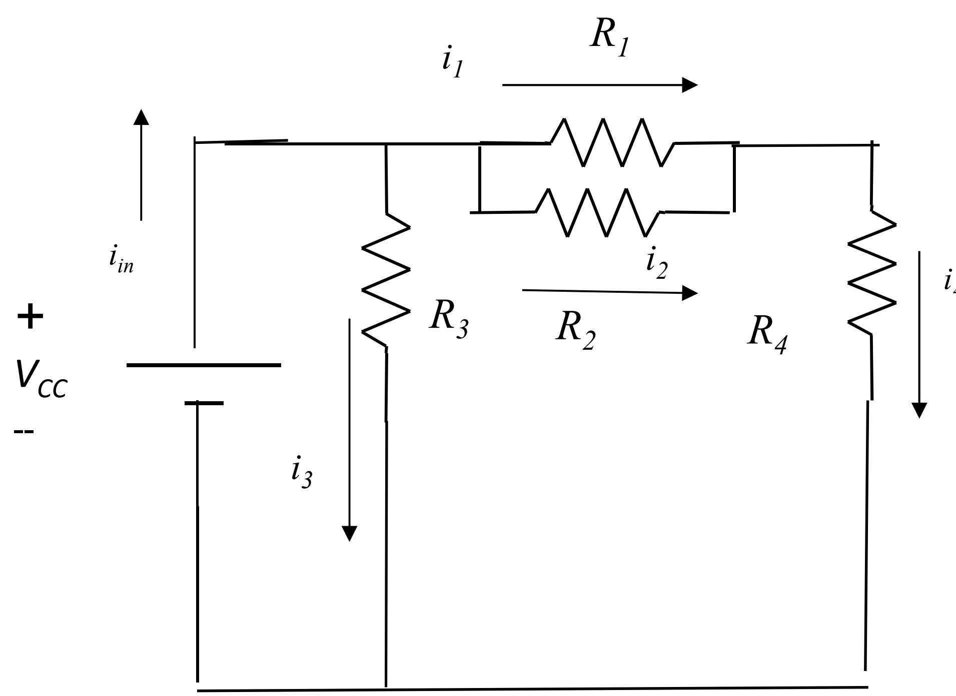
Homework

2. Calculate the current labeled, *i.*

R_1 = 5Ω, R_2 = 5Ω, R_3 = 5Ω, R_4 = 2.5Ω, V_{cc} = 5v

Note that after being redrawn i seems to disappear . But we know that $i=i_{in}-i_1$





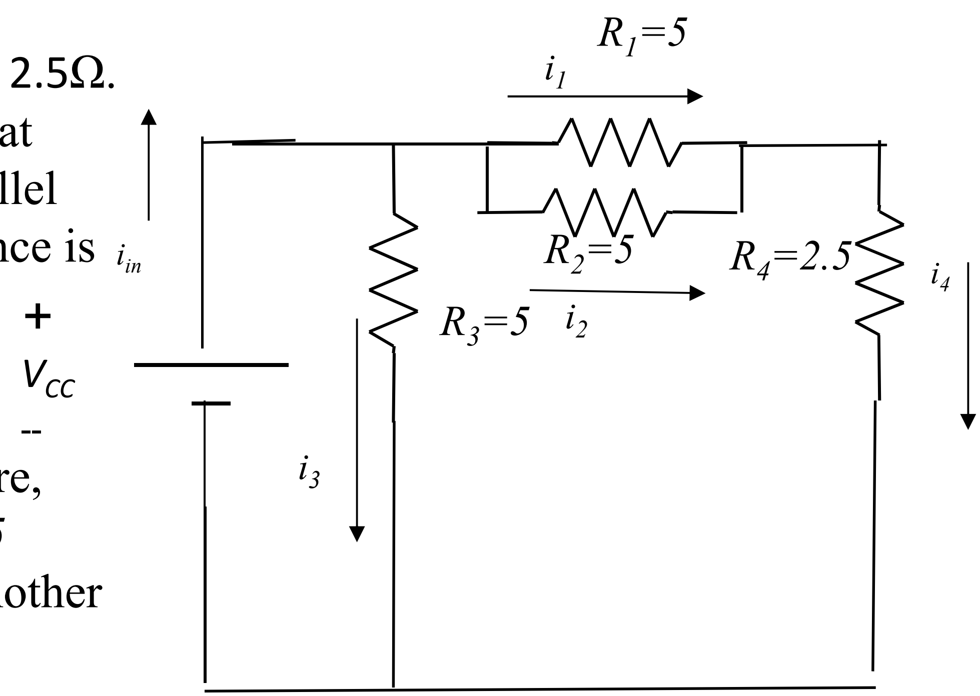
Homework

2. Calculate the current labeled, *i*.

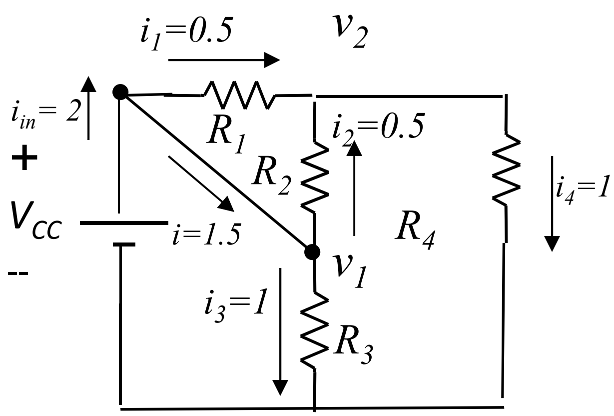
$R_1 = 5\Omega, R_2 = 5\Omega, R_3 = 5\Omega, R_4 = 2.5\Omega, V_{cc} = 5v$

Note that after being redrawn i seems to disappear . But we know that $i=i_{in}-i_1$

Since R2 and R1 are in parallel and equal 2.5Ω. This is in series with R4 which makes that combination 5Ω. However, R3 is in parallel with this 5Ω resistor. So the total resistance is 3 in parallel with 3 = 2.5Ω. Therefore, $i_{in}=5/2.5$= 2a. Using current division $i_3=1/5/(1/5+1/5)$x2=1/2*2= 1 a. This means that $i_4 = 1$ a. Furthermore, $i_1 = 0.5\ a$ and that means that $i=i_{in}-i_1=2-.5 =1.5$amp*s*. See the following page for another method.



Homework



Calculate the current labeled, *i*.

$R_1 = 5\Omega,\ R_2 = 5\Omega,\ R_3 = 5\Omega,\ R_4 = 2.5\Omega,\ V_{cc} = 5v$

$$R'_1 = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = 5 \| 5 = 2.5$$

$$R'_2 = R_4 + R'_1 = R_4 + \frac{R_1 R_2}{R_1 + R_2} = 2.5 + 2.5 = 5$$

$$R'_3 = R'_2 \| R_3 = 5 \| 5 = 2.5$$

$$i_{in} = \frac{Vcc}{R'_3} = \frac{5}{2.5} = 2;$$

$$i_3 = i_{in}\frac{R'_2}{R'_2 + R_3} = 2 \times \frac{5}{5+5} = 1; v_1 = 5v$$

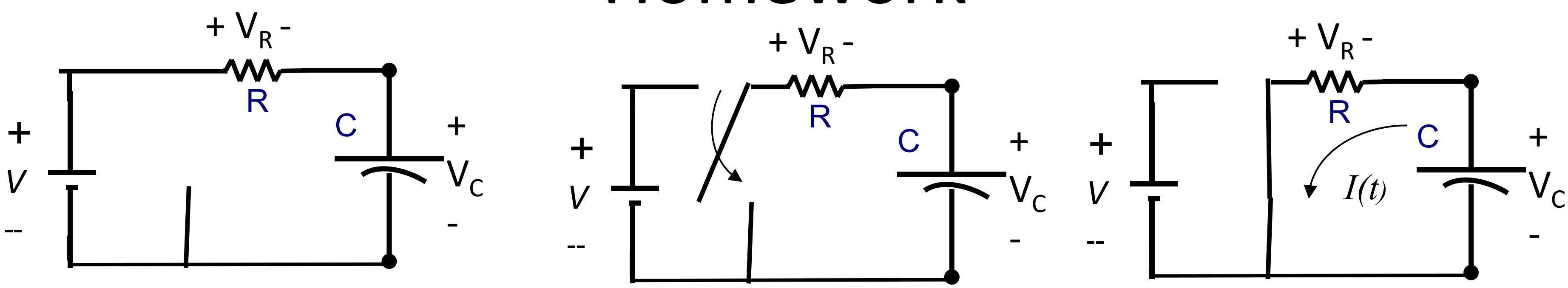
$$i_4 = i_{in}\frac{R_3}{R'_2 + R_3} = 2 \times \frac{1}{2} = 1$$

$$i_1 = i_4\frac{R_2}{R_2 + R_1} = 1 \times \frac{1}{2} = \frac{1}{2}$$

$$i_2 = i_4\frac{R_1}{R_2 + R_1} = 1 \times \frac{1}{2} = \frac{1}{2}$$

$$i = i_{in} - i_1 = 2 - \frac{1}{2} = 1.5$$

Homework



3. a. Given that R=1kOhm, and C=50µF calculate the time constant.

Since the Resistor and Capacitor are in series, then the time constant, τ, equals $RC = 1\text{k} \times 50\mu$

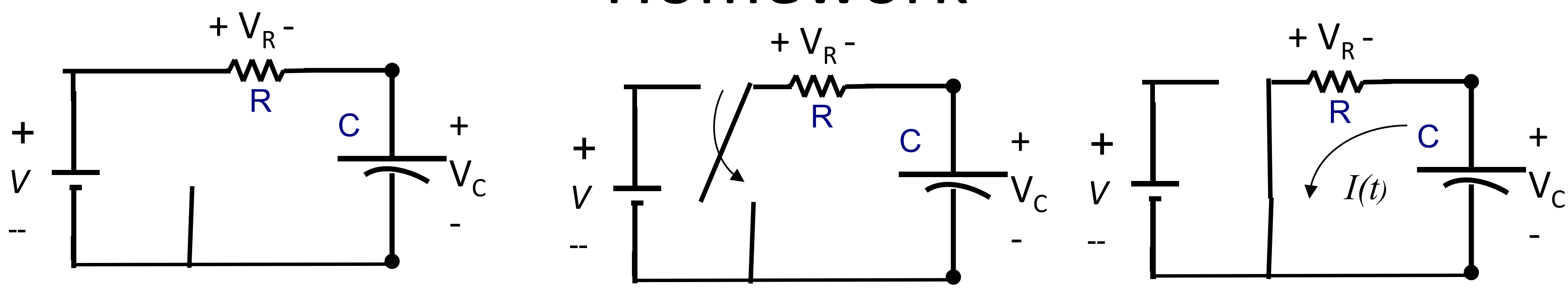
$= 10^3 \times 50 \times 10^{-6} = 0.05$ seconds = 50 milliseconds = 50 ms.

b. How would you increase the time constant of the circuit by a factor of five?

You can

1. Increase the resistance by a factor of 5.
2. Or increase the capacitance by a factor of 5.
3. Or increase both the resistance and capacitance such that RC is increased by a factor of 5.

Homework



3.

c. Given that the voltage V=10V, calculate the voltage of the capacitor at 1,2,3,4, and 5 time constants.

$$V_C(t) = V(1-e^{-\frac{t}{RC}}) = 10(1-e^{-\frac{t}{0.05}})$$

One Time Constant $\Rightarrow V_C(t=0.05) = 10(1-e^{-\frac{1\times 0.05}{0.05}}) = 10(1-e^{-\frac{0.5}{0.5}}) = 10(1-e^{-1}) = 10(1-0.368) = 6.32 volts$

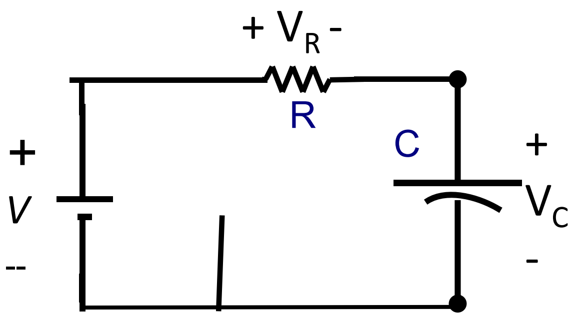
Two Time Constants $\Rightarrow V_C(t=0.1) = 10(1-e^{-\frac{2\times 0.05}{0.05}}) = 10(1-e^{-2}) = 10(1-0.135) = 8.65 volts$

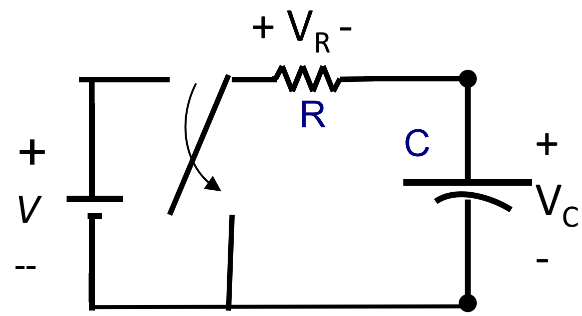
Three Time Constants $\Rightarrow V_C(t=0.15) = 10(1-e^{-\frac{3\times 0.05}{0.05}}) = 10(1-e^{-3}) = 10(1-0.05) = 9.5 volts$

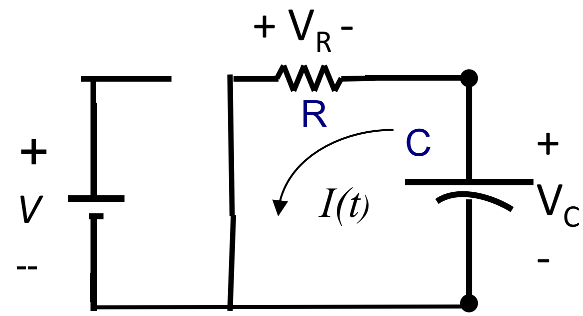
Four Time Constants $\Rightarrow V_C(t=0.2) = 10(1-e^{-\frac{4\times 0.05}{0.05}}) = 10(1-e^{-4}) = 10(1-0.018) = 9.82 volts$

Five Time Constants $\Rightarrow V_C(t=0.25) = 10(1-e^{-\frac{5\times 0.05}{0.05}}) = 10(1-e^{-5}) = 10(1-0.007) = 9.93 volts$

Homework







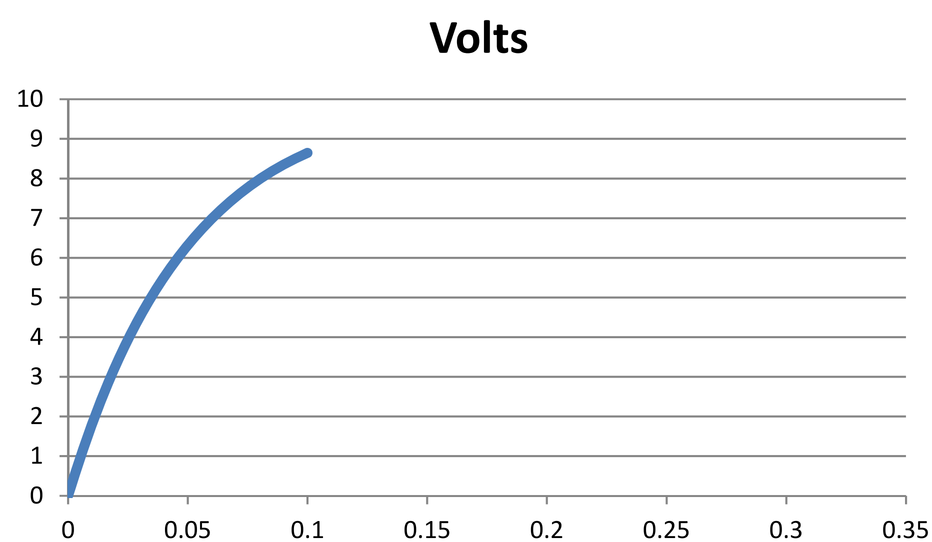
3. d. At time = 0s, the switch closes and stays closed for 2 time constants (see figure on the left).

At t=0 the switch is closed the capacitor begins to charge and it stays closed for 2 time constants.

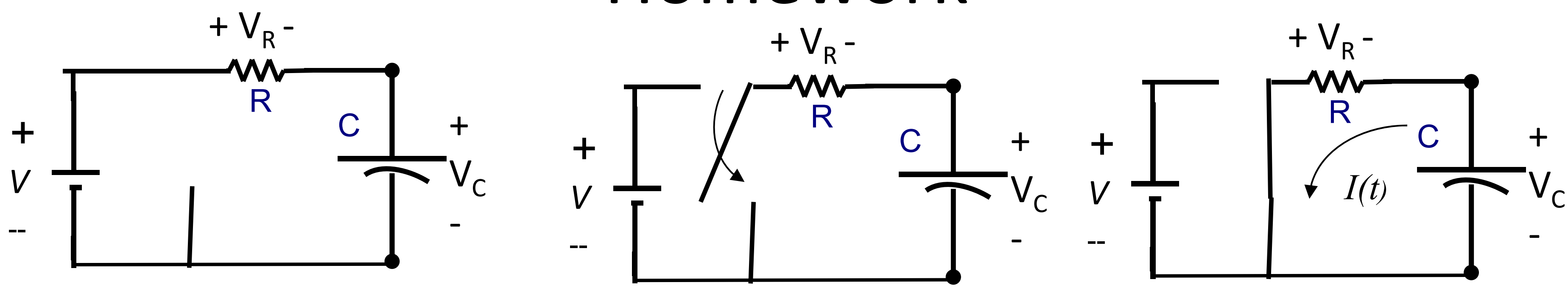
Therefore the voltage across the capacitor is $V_C(t) = 10(1 - e^{-\frac{t}{0.05}})$

Therefore at the end of 2 time constant the voltage across the capacitor reaches 8.65 volts.

See the plot following for t=0 to t=2 time constants or 0.1 second.



Homework

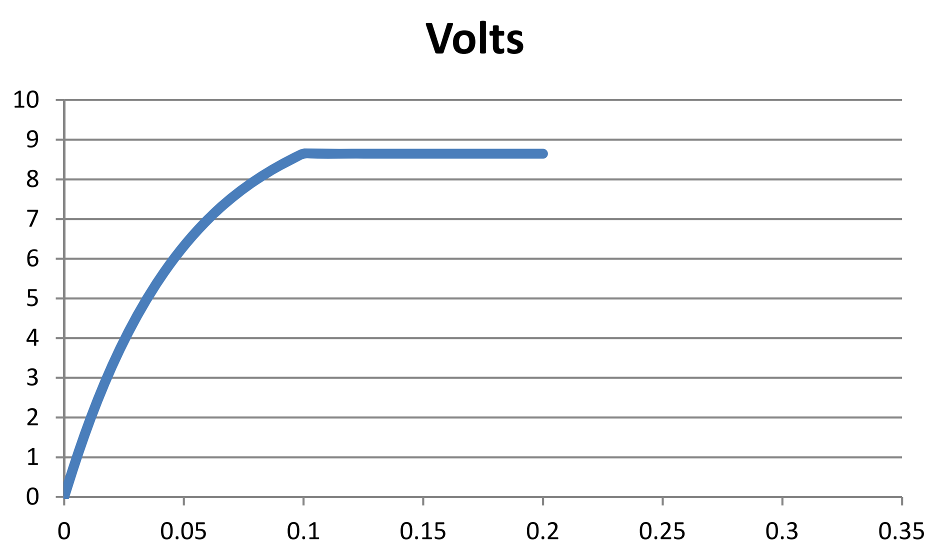


3. d. At that point the switch is opened for 2 time constants (see center figure).

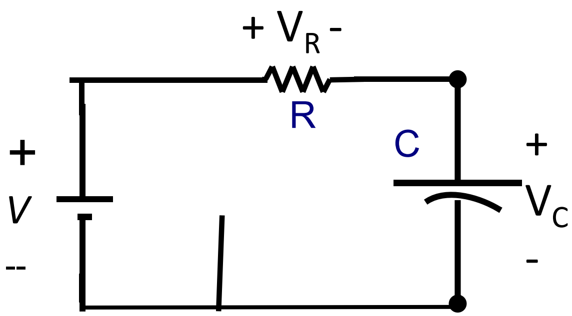
The circuit opens (no path for current to flow)

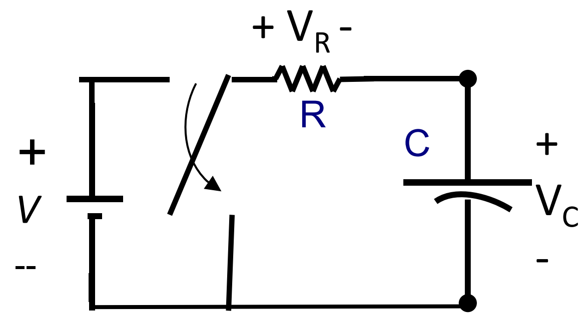
and the capacitor remains charged for the next 2 time constants.

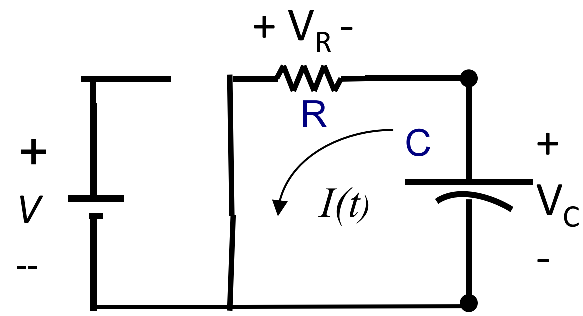
See the plot below for the voltage across the capacitor extended to 4 time constants or 0.2 seconds.



Homework







3. d. The switch is then switched as shown in the right figure for a total of2 time constants before it is opened again. Plot the voltage across the capacitor, the charge on the capacitor as a function of time from time = 0 until 2 time constants after the circuits opened for the final time.

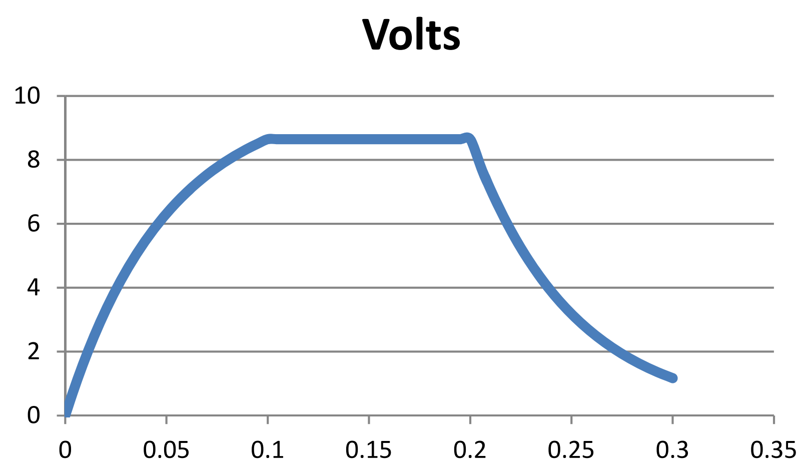
At this point the circuit is closed and the capacitor will be able to discharge through the resistor for a time period of another 2 time constants. See the plot following extended to 6 times constants or to 0.3 seconds. Note that when the capacitor starts to discharge the equation becomes

$V_C(t) = 8.65e^{-\frac{t}{0.05}}$ and not $V_C(t) = 10e^{-\frac{t}{0.05}}$.

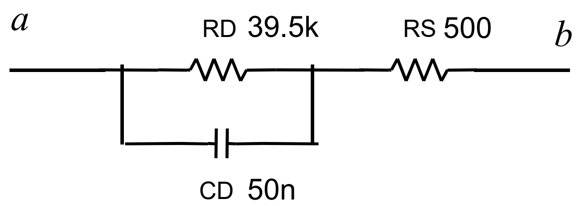
Therefore the voltage at the end of 0.3 seconds is 1.17 volts and not 0 volts.

Note the the plot is a little bump at t=0.2 seconds. This bump is a artifact due to Excel's plotting function and this bump is an error.

The curve should be drop exponetially after t=0.2 and not rise.



Homework



4. The circuit shown is an equivalent circuit of an electrode where RD and CD are the resistance and capacitance associated with the interface of the electrode and the body and RS is the resistance of the device itself. Find the impedance $Z_{ab}(j\omega)$ as function of ω.

$$R_S + R_D // C_D$$

$$R_D // C_D = \frac{1}{\frac{1}{R_D} + \frac{1}{\frac{1}{j\omega C_D}}} = \frac{1}{\frac{1}{R_D} + j\omega C_D}$$

$$= \frac{R_D}{1 + j\omega R_D C_D}$$

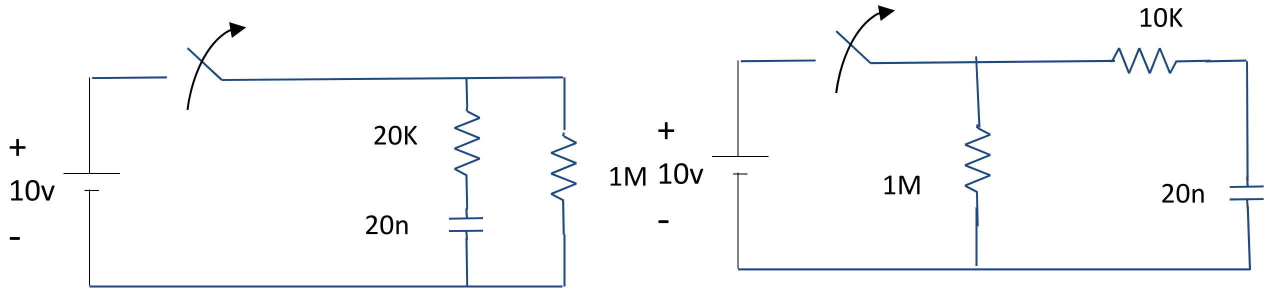
$$R_S + R_D // C_D = R_S + \frac{R_D}{1 + j\omega R_D C_D} = \frac{R_S + j\omega R_S R_D C_D + R_D}{1 + j\omega R_D C_D}$$

$$= \frac{R_S + R_D + j\omega R_S R_D C_D}{1 + j\omega R_D C_D} = \frac{\sqrt{(R_S + R_D)^2 + (\omega R_S R_D C_D)^2} \angle \tan^{-1}(\frac{\omega R_S R_D C_D}{R_S + R_D})}{\sqrt{1 + (\omega R_D C_D)^2} \angle \tan^{-1}(\omega R_D C_D)}$$

$$= \frac{\sqrt{(R_S + R_D)^2 + (\omega R_S R_D C_D)^2}}{\sqrt{1 + (\omega R_D C_D)^2}} \angle \{\tan^{-1}(\frac{\omega R_S R_D C_D}{R_S + R_D}) - \tan^{-1}(\omega R_D C_D)\}$$

$$= \frac{40 \times 10^3 + j\omega 9.9 \times 10^{-1}}{1 + j\omega 1.98 \times 10^{-3}} = \frac{\sqrt{(40 \times 10^3)^2 + (\omega 9.9 \times 10^{-1})^2}}{\sqrt{1 + (\omega 1.99 \times 10^{-3})^2}} \angle \{\tan^{-1}(\frac{\omega 9.9 \times 10^{-1}}{40 \times 10^3}) - \tan^{-1}(\omega 1.99 \times 10^{-3})\}$$

Homework

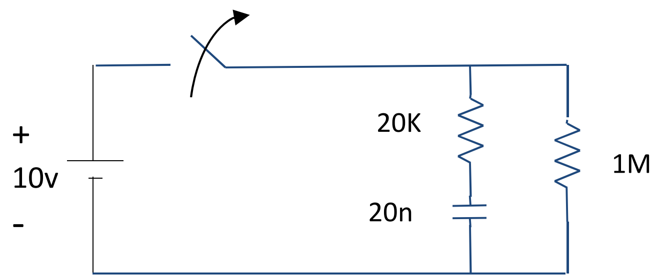


5. HONORS STUDENTS ADD THE FOLLOWING

Assume the switch has been closed for a very long time. Once the switch is opened, how long will it take for the capacitor voltage to discharge to 5volts. What will the voltage be after the switch is closed again 2 msec later?

After a long time the capacitor will charge to 10 with time constant (10k)(20n)=$(2x10^{4})(2x10^{-8})=4x10^{-4}$sec or .4msec. This is because all of the current flowing through the capacitor only flows through the 20k resistor. However, it will discharge through both the 20k and 1M resistors and so the time constant for discharge is $(1.02x10^{6})\ (2x10^{-8})=2.04x10^{-2}$sec or 20.4msec.

Homework



5. HONORS STUDENTS ADD THE FOLLOWING

Assume the switch has been closed for a very long time. Once the switch is opened, how long will it take for the capacitor voltage to discharge to 5volts. What will the voltage be after the switch is closed again 2 msec later?

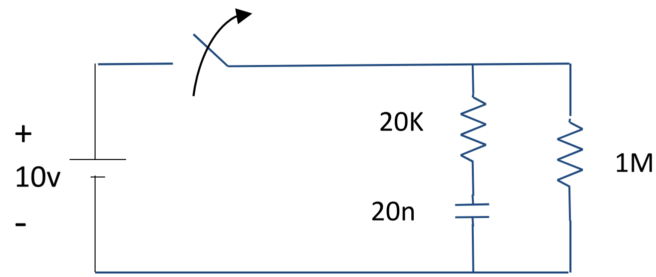
$$Vc(t) = 10e^{-\frac{t}{2.04\times10^{-2}}}$$

Time to discharge to 5 volts:

$$5 = 10e^{-\frac{t_5}{2.04\times10^{-2}}} \Rightarrow 0.5 = e^{-\frac{t_5}{2.04\times10^{-2}}} \Rightarrow 2 = e^{\frac{t_5}{2.04\times10^{-2}}}$$

$$t_5 = 2.04\times10^{-2}\ln(2) = 2.04\times10^{-2}\times0.69 = 1.4\times10^{-2} = 14\text{msec}$$

Homework



5. HONORS STUDENTS ADD THE FOLLOWING

Assume the switch has been closed for a very long time. Once the switch is opened, how long will it take for the capacitor voltage to discharge to 5volts. What will the voltage be after the switch is closed again 2 msec later?

When the switch is closed the capacitor will charge back to 10 with this equation:

$$Vc(t) = K_1 + K_2 e^{-\frac{t}{RC}}$$

In this case the initial condition of the voltage is 5 volts and the final condition is 10 volts:

$$Vc(0) = K_1 + K_2 = 5$$

$$Vc(\infty) = K_1 = 10 \Rightarrow 10 + K_2 = 5 \Rightarrow K_2 = -5$$

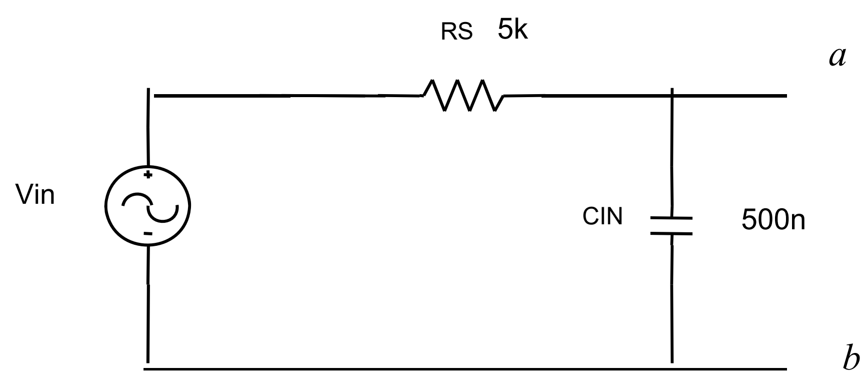
$$Vc(t) = 10 - 5e^{-\frac{t}{4\times10^{-4}}}$$

$$Vc(2\times10^{-3}) = 10 - 5e^{-\frac{2\times10^{-3}}{4\times10^{-4}}} = 10 - 5e^{-5} = 10 - 0.034 = 9.97$$

Homework

6. HONORS STUDENTS ADD THE FOLLOWING

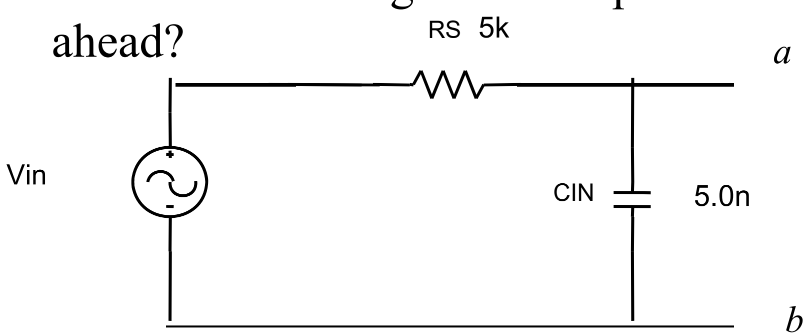
Plot the voltage phasors for an R-C series circuit where R=5k ohms, C=500 nF, and the source voltage Vin=10V is a sinusoidal signal at f=100 k Hz. What is the phase angle between the voltage of the capacitance V_C and V_R of the circuit? Which one is ahead?

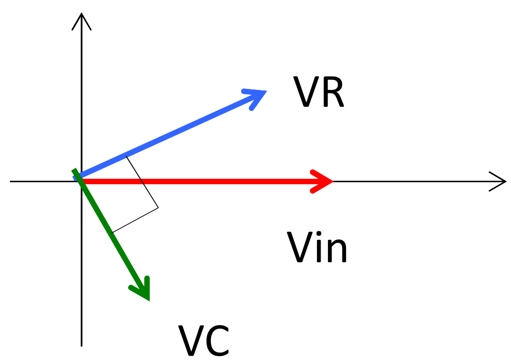


Homework

6. HONORS STUDENTS ADD THE FOLLOWING

Plot the voltage phasors for an R-C series circuit where R=5k ohms, C=500 nF, and the source voltage Vin=10V is a sinusoidal signal at f=100 k Hz. What is the phase angle between the voltage of the capacitance (Vc) and VR of the circuit? Which one is ahead?





Using KVL, $V_{in}(t) = I(t)R + \frac{1}{C}\int I(t)dt$

Converting to impedances,

$$Vin = V\angle 0 = IR + I\frac{1}{j\omega C} = (R + \frac{1}{j\omega C})I = (R + \frac{1}{j2\pi fC})I = Z(f)I$$

where $Z(f) = R + \frac{1}{j2\pi fC}$

$$Z(100) = 5000 + \frac{1}{j(2\times 3.14\times 100\times 500\times 10^{-9})} = 5000 + \frac{1}{j(3.14\times 10^{-4})}$$

$$= 5000 - j3183 = 5927\angle -0.57$$

$$I = \frac{Vin}{Z(f)} = \frac{10\angle 0}{5927\angle -0.57} = 1.6\times 10^{-3}\angle 0.57$$

$$V_R = IR = 1.6\times 10^{-3}\angle 0.57\times 5000\angle 0 = 8.4\angle 0.57$$

$$V_C = I\frac{1}{j\omega C} = 1.6\times 10^{-3}\angle 0.57\times 3183\angle -\frac{\pi}{2} = 5.4\angle -1.0$$