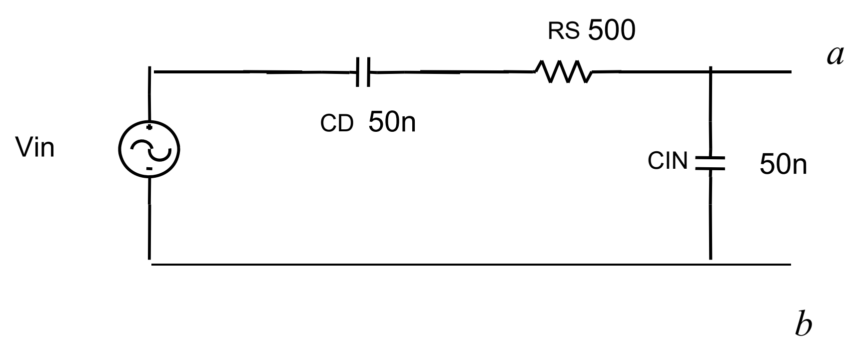
BME 301

8-Filters

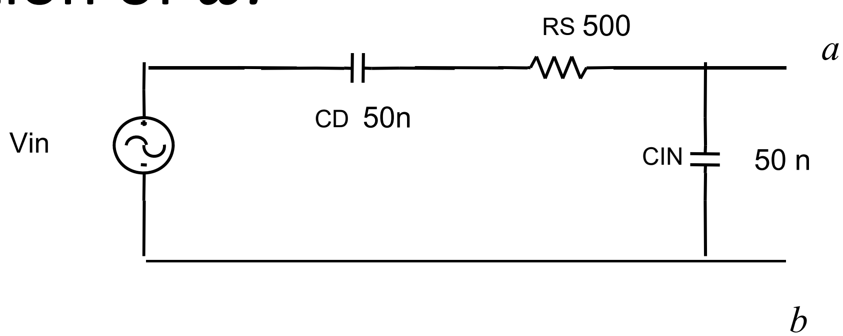
Homework

1. If the circuit in problem 1 was too challenging, try this circuit, first. Find the transfer function of this circuit as function of ω.



Homework

1. If the circuit in problem 1 was too challenging, try this circuit. Find the transfer function of this circuit as function of *ω.*



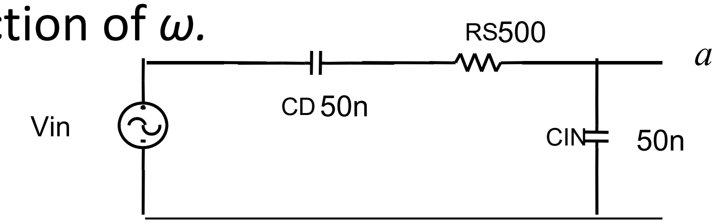
$$\frac{Vab}{Vin} = \frac{\dfrac{1}{j\omega C_{IN}}}{\dfrac{1}{j\omega C_{IN}} + R_S + \dfrac{1}{j\omega C_D}} = \frac{j\omega C_D}{j\omega C_D + j\omega C_{IN} j\omega C_D R_S + j\omega C_{IN}} = \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S + j\omega(C_{IN} + C_D)}$$

$$= \frac{\omega C_D \angle \dfrac{\pi}{2}}{\sqrt{(\omega^2 C_{IN} C_D R_S)^2 + (\omega(C_{IN} + C_D))^2} \angle \tan^{-1}(\dfrac{\omega(C_{IN} + C_D)}{-\omega^2 C_{IN} C_D R_S}}$$

$$= \frac{\omega C_D}{\sqrt{(\omega^2 C_{IN} C_D R_S)^2 + (\omega(C_{IN} + C_D))^2}} \angle [\frac{\pi}{2} - \tan^{-1}(\frac{\omega(C_{IN} + C_D)}{-\omega^2 C_{IN} C_D R_S}]$$

Homework (Extra Credit if done)

1. If the circuit in problem 1 was too challenging, try this circuit. Find the transfer function of this circuit as function of *ω.*



$$\frac{Vab}{Vin} = \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S + j\omega(C_{IN}+C_D)} = \frac{\omega C_D}{\sqrt{(\omega^2 C_{IN} C_D R_S)^2 + (\omega(C_{IN}+C_D))^2}} \angle[\frac{\pi}{2} - \tan^{-1}(\frac{\omega(C_{IN}+C_D)}{-\omega^2 C_{IN} C_D R_S}]$$

$$\frac{Vab}{Vin}|_{\omega\to 0} = \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S + j\omega(C_{IN}+C_D)}|_{\omega\to 0} \to \frac{j\omega C_D}{j\omega(C_{IN}+C_D)} = \frac{C_D}{(C_{IN}+C_D)}\angle 0$$

$$= \frac{1/C_{IN}}{(1/C_{IN} + 1/C_D)}\angle 0 = 0.5\angle 0 \text{ voltage divides according to the capacitor impedances.}$$

$$\frac{Vab}{Vin}|_{\omega\to\infty} = \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S + j\omega(C_{IN}+C_D)}|_{\omega\to\infty} \to \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S} = \frac{j}{-\omega C_{IN} R_S} = \frac{-j}{\omega C_{IN} R_S}|_{\omega\to\infty}$$

$$\to 0\angle -\frac{\pi}{2} \text{ output capacitor is a short circuit.}$$

$$\omega = \frac{1}{C_{IN} R_S} = 4\times 10^4$$

$$\frac{Vab}{Vin}|_{\omega=\frac{1}{C_{IN}R_S}} = \frac{j\omega C_D}{-\omega^2 C_{IN} C_D R_S + j\omega(C_{IN}+C_D)}|_{\omega=\frac{1}{C_{IN}R_S}} = \frac{j}{-\omega C_{IN} R_S + j(\frac{C_{IN}+C_D}{C_D})}|_{\omega=\frac{1}{C_{IN}R_S}} = \frac{j}{-1 + j(\frac{C_{IN}+C_D}{C_D})}$$

$$= \frac{1}{\sqrt{1+(\frac{C_{IN}+C_D}{C_D})^2}}\angle\frac{\pi}{2} - \tan^{-1}(\frac{C_{IN}+C_D}{-C_D}) = 0.45\angle -0.46$$

Homework (Extra Credit if done)

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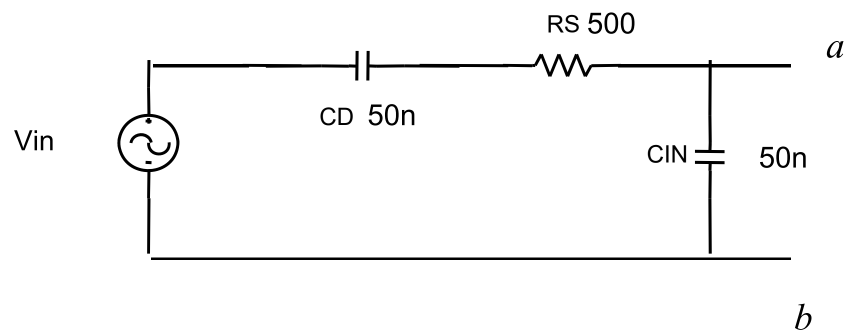
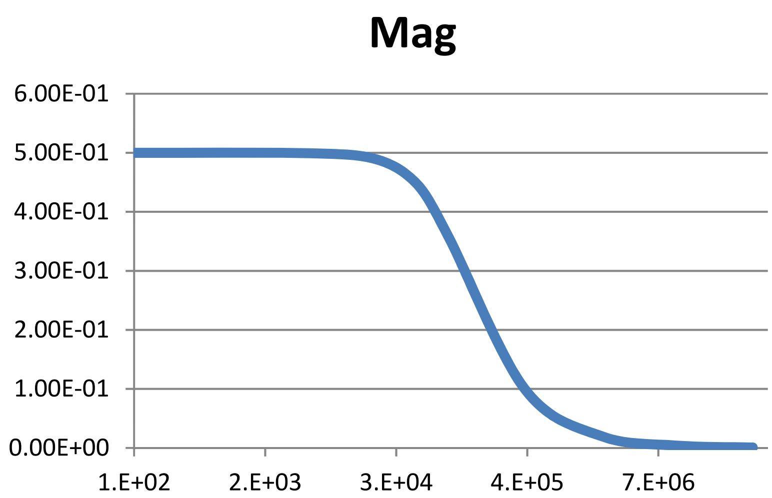
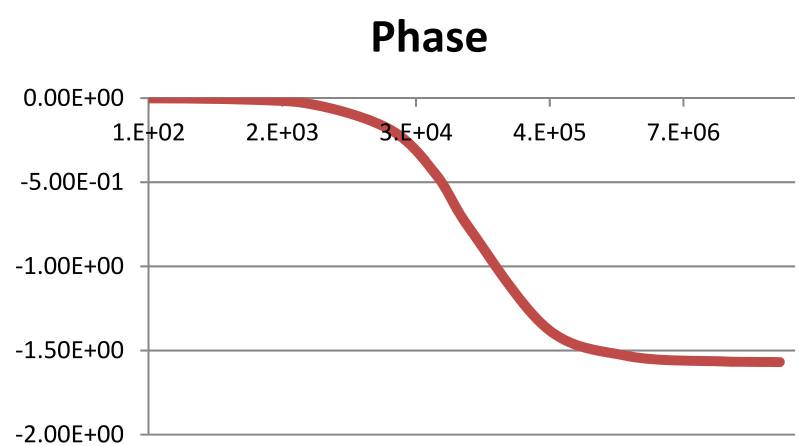


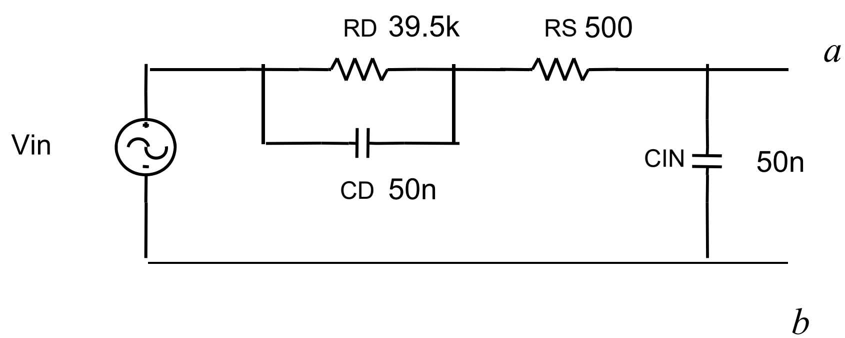




Homework

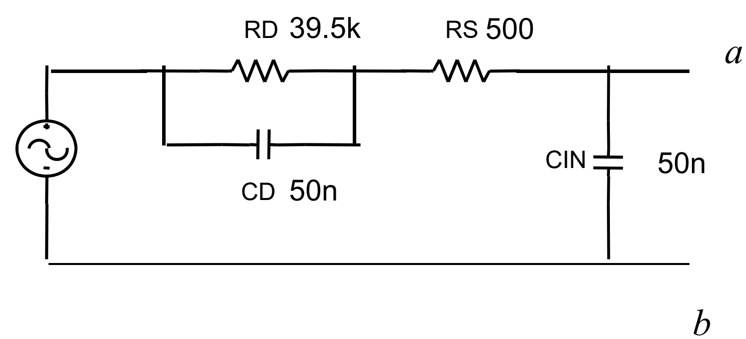
2. HONORS STUDENTS ADD THE FOLLOWING

An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of ω.



Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of *ω.*



Using voltage division:

$$\frac{Vab}{Vin} = \frac{Zout}{Zout + Zseries}$$

where $Zout = \dfrac{1}{j\omega C_{IN}}$

and $Zseries = R_S + R_D // C_D = R_S + \dfrac{1}{\dfrac{1}{R_D} + \dfrac{1}{\dfrac{1}{j\omega C_D}}} = R_S + \dfrac{1}{\dfrac{1}{R_D} + j\omega C_D}$

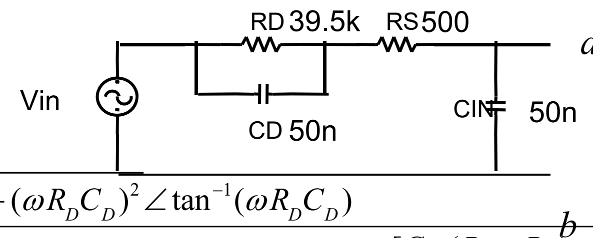
$$= R_S + \frac{R_D}{1 + j\omega R_D C_D} = \frac{R_S(1 + j\omega R_D C_D) + R_D}{1 + j\omega R_D C_D} = \frac{R_S + R_D + j\omega R_D R_S C_D}{1 + j\omega R_D C_D}$$

$$Zout + Zseries = \frac{1}{j\omega C_{IN}} + \frac{R_S + R_D + j\omega R_D R_S C_D}{1 + j\omega R_D C_D} = \frac{1 + j\omega R_D C_D + j\omega C_{IN}(R_S + R_D + j\omega R_D R_S C_D)}{j\omega C_{IN}(1 + j\omega R_D C_D)} = \frac{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}{j\omega C_{IN}(1 + j\omega R_D C_D)}$$

$$\frac{Vab}{Vin} = \frac{Zout}{Zout + Zseries} = \frac{\dfrac{1}{j\omega C_{IN}}}{\dfrac{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}{j\omega C_{IN}(1 + j\omega R_D C_D)}} = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}$$

Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of *ω.*



$$\frac{Vab}{Vin}=\frac{1+j\omega R_D C_D}{1-\omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S+R_D)+R_D C_D]}=\frac{\sqrt{1+(\omega R_D C_D)^2}\angle\tan^{-1}(\omega R_D C_D)}{\sqrt{(1-\omega^2 R_D C_{IN} R_S C_D)^2+(\omega[C_{IN}(R_S+R_D)+R_D C_D])^2}\angle\tan^{-1}(\frac{\omega[C_{IN}(R_S+R_D)+R_D C_D]}{1-\omega^2 R_D C_{IN} R_S C_D})}$$

$$=\frac{\sqrt{1+(\omega R_D C_D)^2}}{\sqrt{(1-\omega^2 R_D C_{IN} R_S C_D)^2+(\omega[C_{IN}(R_S+R_D)+R_D C_D])^2}}\angle[\tan^{-1}(\omega R_D C_D)-\tan^{-1}(\frac{\omega[C_{IN}(R_S+R_D)+R_D C_D]}{1-\omega^2 R_D C_{IN} R_S C_D})]$$

$$\frac{Vab}{Vin}|_{\omega=0}=\frac{1}{1}=1\angle 0$$

$$\frac{Vab}{Vin}|_{\omega\to\infty}\to\frac{j\omega R_D C_D}{-\omega^2 R_D C_{IN} R_S C_D}=\frac{j}{-\omega C_{IN} R_S}=0\angle-\frac{\pi}{2}$$

$$\omega=\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}=1.69\times 10^4$$

$$\frac{Vab}{Vin}|_{\omega=\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}}=\frac{1+j\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D}{j\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}[C_{IN}(R_S+R_D)+R_D C_D]}=\frac{\sqrt{1+(\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D)^2}}{\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}[C_{IN}(R_S+R_D)+R_D C_D]}\angle[\tan^{-1}(\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D)-\frac{\pi}{2}]$$

$$=2.2\times 10^{-2}\angle-1.18$$

Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of ω.