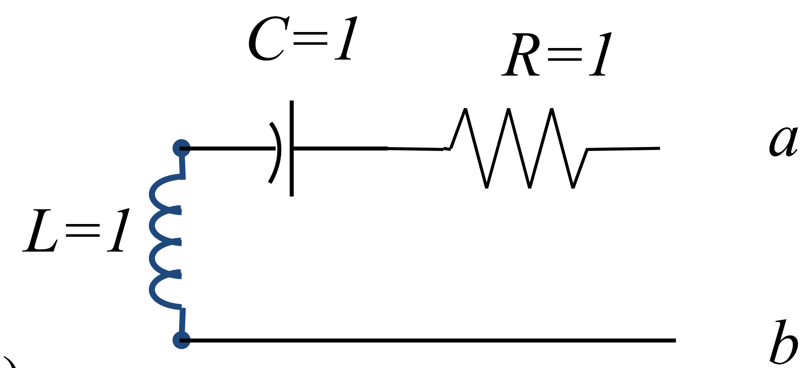
BME 301

9-Plotting Filters

Homework

1. A series RLC circuit has an impedance given as:

Sketch the impedance as a function of frequency and calculate three interesting points.



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

$$= \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle \tan^{-1}(\frac{(\omega L - \frac{1}{\omega C})}{R})$$

$$Z|_{\omega \to 0} = R + j(\omega L - \frac{1}{\omega C})|_{\omega = 0} \to \frac{1}{j\omega C} \to \infty \angle -\frac{\pi}{2}$$

$$Z|_{\omega \to \infty} = R + j(\omega L - \frac{1}{\omega C})|_{\omega \to \infty} \to j\omega L = \to \infty \angle \frac{\pi}{2}$$

$$Z|_{\omega = \frac{1}{\sqrt{LC}}} = R + j(\omega L - \frac{1}{\omega C})|_{\omega = \frac{1}{\sqrt{LC}}} = R + j(\frac{\omega^2 LC - 1}{\omega C})|_{= \frac{1}{\sqrt{LC}}} = R \angle 0$$

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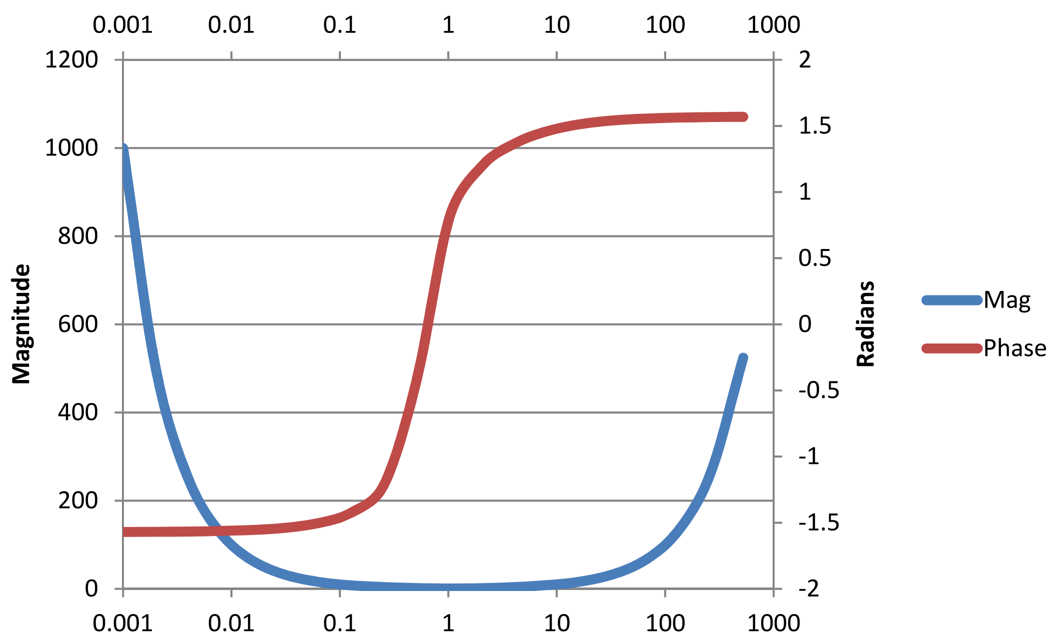
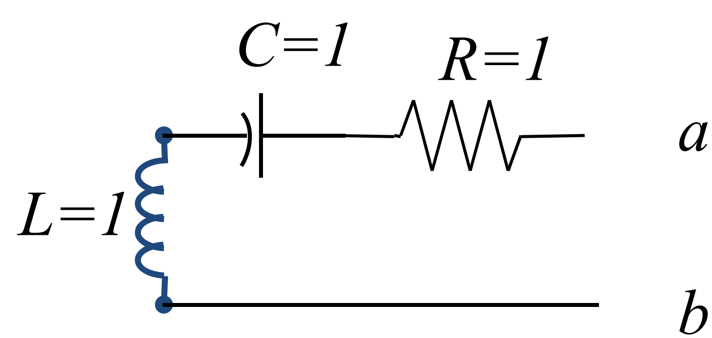


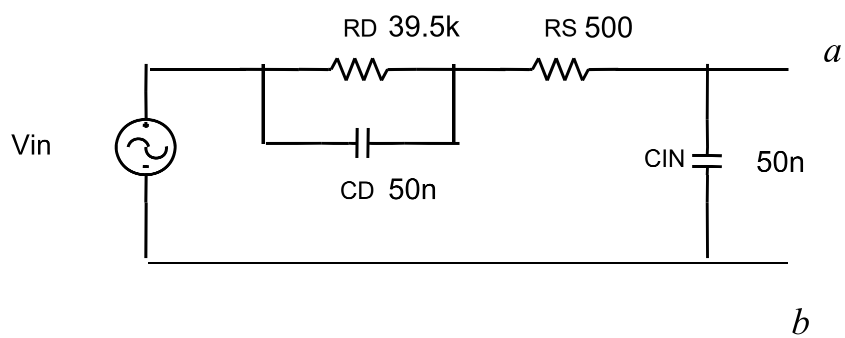


Homework

2. Plot using Matlab the transfer function of the electrode connected to an oscilloscope you calculated in Lecture 8. And calculate three interesting points

Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of *ω.*



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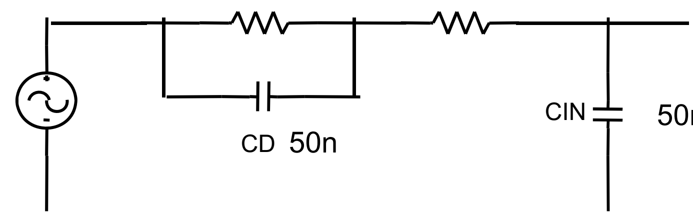


Using voltage division:

$$\frac{Vab}{Vin} = \frac{Zout}{Zout + Zseries}$$

where $Zout = \dfrac{1}{j\omega C_{IN}}$

and $Zseries = R_S + R_D // C_D = R_S + \dfrac{1}{\dfrac{1}{R_D} + \dfrac{1}{\dfrac{1}{j\omega C_D}}} = R_S + \dfrac{1}{\dfrac{1}{R_D} + j\omega C_D}$

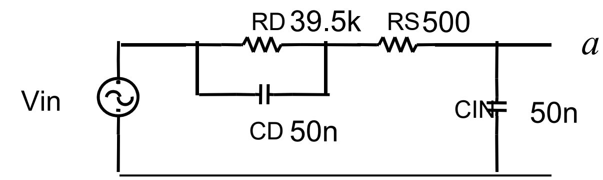
$$= R_S + \frac{R_D}{1 + j\omega R_D C_D} = \frac{R_S(1 + j\omega R_D C_D) + R_D}{1 + j\omega R_D C_D} = \frac{R_S + R_D + j\omega R_D R_S C_D}{1 + j\omega R_D C_D}$$

$$Zout + Zseries = \frac{1}{j\omega C_{IN}} + \frac{R_S + R_D + j\omega R_D R_S C_D}{1 + j\omega R_D C_D} = \frac{1 + j\omega R_D C_D + j\omega C_{IN}(R_S + R_D + j\omega R_D R_S C_D)}{j\omega C_{IN}(1 + j\omega R_D C_D)} = \frac{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}{j\omega C_{IN}(1 + j\omega R_D C_D)}$$

$$\frac{Vab}{Vin} = \frac{Zout}{Zout + Zseries} = \frac{\dfrac{1}{j\omega C_{IN}}}{\dfrac{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}{j\omega C_{IN}(1 + j\omega R_D C_D)}} = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S + R_D) + R_D C_D]}$$

Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of *ω.*



$$\frac{Vab}{Vin}=\frac{1+j\omega R_D C_D}{1-\omega^2 R_D C_{IN} R_S C_D + j\omega[C_{IN}(R_S+R_D)+R_D C_D]}=\frac{\sqrt{1+(\omega R_D C_D)^2}\angle\tan^{-1}(\omega R_D C_D)}{\sqrt{(1-\omega^2 R_D C_{IN} R_S C_D)^2+(\omega[C_{IN}(R_S+R_D)+R_D C_D])^2}\angle\tan^{-1}(\frac{\omega[C_{IN}(R_S+R_D)+R_D C_D]}{1-\omega^2 R_D C_{IN} R_S C_D})}$$

$$=\frac{\sqrt{1+(\omega R_D C_D)^2}}{\sqrt{(1-\omega^2 R_D C_{IN} R_S C_D)^2+(\omega[C_{IN}(R_S+R_D)+R_D C_D])^2}}\angle[\tan^{-1}(\omega R_D C_D)-\tan^{-1}(\frac{\omega[C_{IN}(R_S+R_D)+R_D C_D]}{1-\omega^2 R_D C_{IN} R_S C_D})]$$

$$\frac{Vab}{Vin}\Big|_{\omega=0}=\frac{1}{1}=1\angle 0$$

$$\frac{Vab}{Vin}\Big|_{\omega\to\infty}\to\frac{j\omega R_D C_D}{-\omega^2 R_D C_{IN} R_S C_D}=\frac{j}{-\omega C_{IN} R_S}=0\angle-\frac{\pi}{2}$$

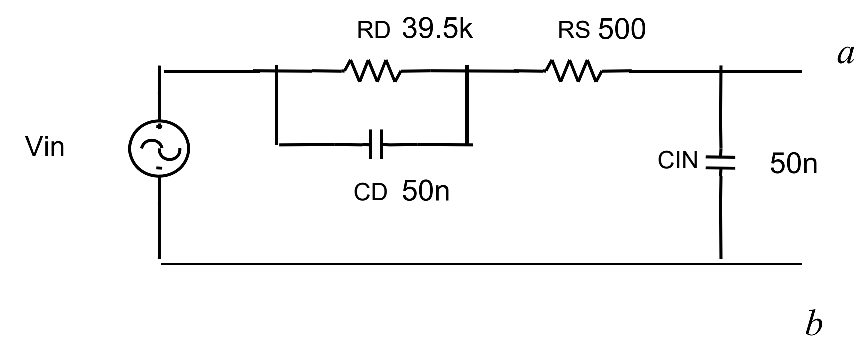
$$\omega=\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}=1.69\times 10^4$$

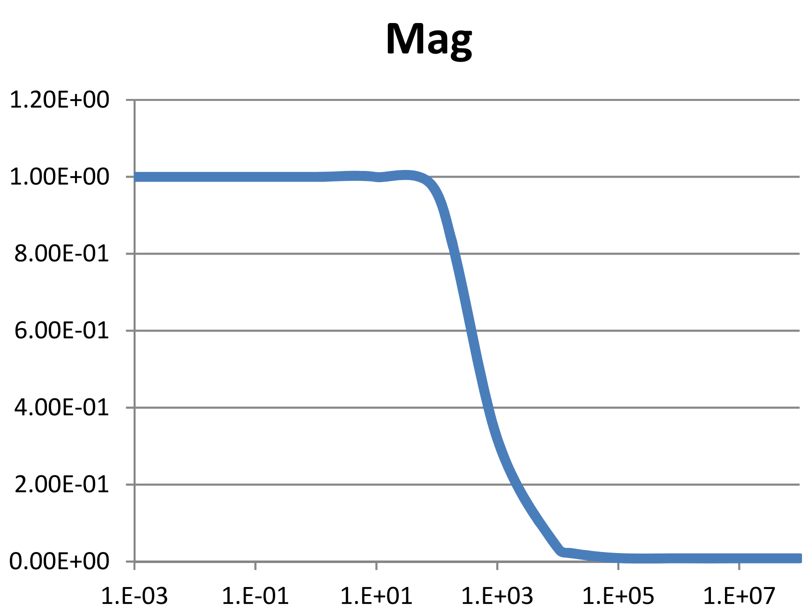
$$\frac{Vab}{Vin}\Big|_{\omega=\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}}=\frac{1+j\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D}{j\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}[C_{IN}(R_S+R_D)+R_D C_D]}=\frac{\sqrt{1+(\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D)^2}}{\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}[C_{IN}(R_S+R_D)+R_D C_D]}\angle[\tan^{-1}(\frac{1}{\sqrt{R_D C_{IN} R_S C_D}}R_D C_D)-\frac{\pi}{2}]$$

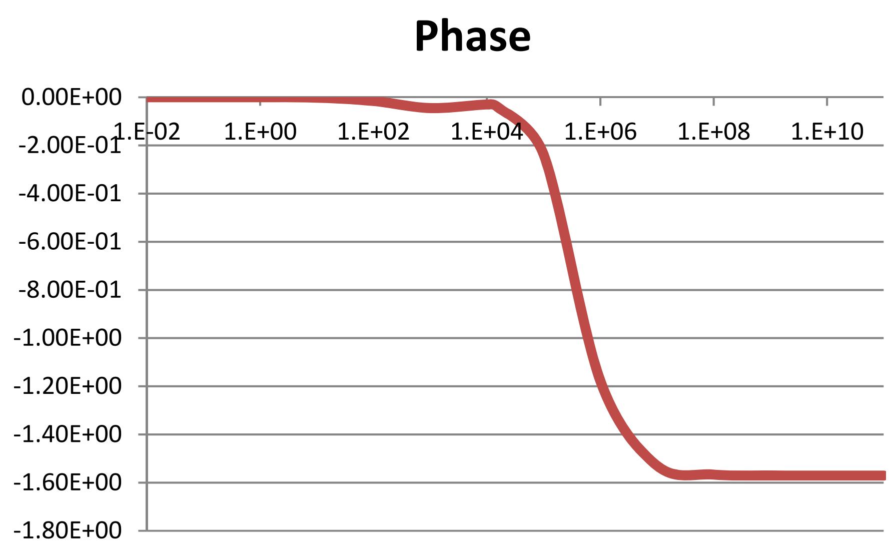
$$=2.2\times 10^{-2}\angle-1.18$$

Homework

2. An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find the transfer function of this circuit as function of ω.



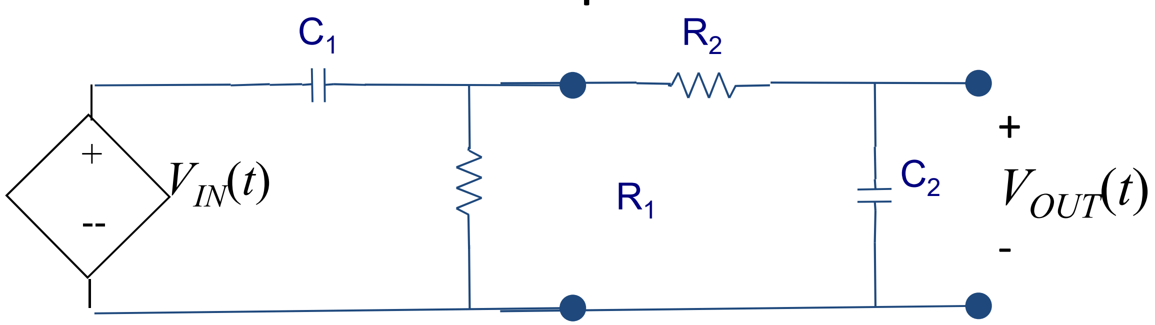




Homework

3. HONORS STUDENTS ADD THE FOLLOWING

For the following circuit, calculate the transfer function and plot its magnitude using Matlab. Graphically determine the upper and lower cutoff frequencies.

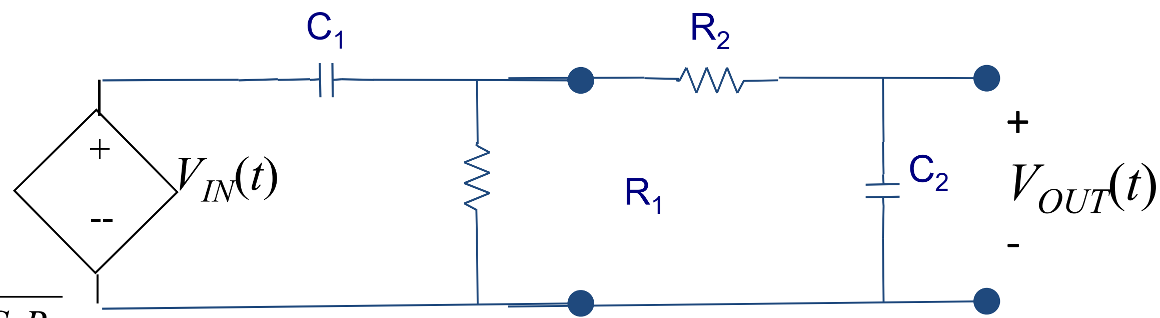


Homework

Note:

$$\frac{Vout}{VIN}=\frac{Vout}{VMID}\times\frac{VMID}{VIN}$$

$$\frac{VMID}{VIN}=\frac{\frac{1}{j\omega C_2}}{R_2+\frac{1}{j\omega C_2}}=\frac{1}{1+j\omega C_2R_2}$$



Note that the impedance at $VMID$, $ZMID$, is C_1 in parallel with the series comnination of R_2 and C_2:

$$\text{ZMID}=R_1\,||\,(R_2+\frac{1}{j\omega C_2})=R_1\,||\,(\frac{1+j\omega C_2R_2}{j\omega C_2})$$

$$=\frac{R_1\times(\frac{1+j\omega C_2R_2}{j\omega C_2})}{R_1+(\frac{1+j\omega C_2R_2}{j\omega C_2})}=\frac{R_1\frac{(1+j\omega C_2R_2)}{j\omega C_2}}{\frac{j\omega C_2R_1+(1+j\omega C_2R_2)}{j\omega C_2}}=\frac{R_1(1+j\omega C_2R_2)}{1+j\omega(C_2R_2+C_2R_1)}$$

$$\frac{Vout}{VMID}=\frac{\text{ZMID}}{\text{ZMID}+\frac{1}{j\omega C_1}}=\frac{\frac{R_1(1+j\omega C_2R_2)}{1+j\omega(C_2R_2+C_2R_1)}}{\frac{R_1(1+j\omega C_2R_2)}{1+j\omega(C_2R_2+C_2R_1)}+\frac{1}{j\omega C_1}}=\frac{\frac{R_1(1+j\omega C_2R_2)}{1+j\omega(C_2R_2+C_2R_1)}}{\frac{j\omega C_1R_1(1+j\omega C_2R_2)+1+j\omega(C_2R_2+C_2R_1)}{j\omega C_1(1+j\omega(C_2R_2+C_2R_1))}}$$

$$=\frac{j\omega C_1R_1(1+j\omega C_2R_2)}{1-\omega^2C_2R_2C_1R_1+j\omega(C_2R_2+C_2R_1+C_1R_1)}$$

Homework

$$\frac{Vout}{VIN}=\frac{Vout}{VMID}\times\frac{VMID}{VIN}=\frac{j\omega C_1R_1(1+j\omega C_2R_2)}{1-\omega^2C_2R_2C_1R_1+j\omega(C_2R_2+C_2R_1+C_1R_1)}\times\frac{1}{1+j\omega C_2R_2}$$

$$=\frac{j\omega C_1R_1}{1-\omega^2R_2R_1C_2C_1+j\omega(C_2R_1+R_2C_2+R_1C_1)}=\frac{\omega C_1R_1\angle\frac{\pi}{2}}{\sqrt{(1-\omega^2R_2R_1C_2C_1)^2+(\omega(C_2R_1+R_2C_2+R_1C_1))^2}\angle\tan^{-1}(\frac{\omega(C_2R_1+R_2C_2+R_1C_1)}{1-\omega^2R_2R_1C_2C_1})}$$

$$\frac{Vout}{VIN}=\frac{j\omega C_1R_1}{1-\omega^2R_2R_1C_2C_1+j\omega(C_2R_1+R_2C_2+R_1C_1)}=\frac{\omega C_1R_1}{\sqrt{(1-\omega^2R_2R_1C_2C_1)^2+(\omega(C_2R_1+R_2C_2+R_1C_1))^2}}\angle\frac{\pi}{2}-\angle\tan^{-1}(\frac{\omega(C_2R_1+R_2C_2+R_1C_1)}{1-\omega^2R_2R_1C_2C_1})$$

$$\frac{Vout}{VIN}\Big|_{\omega=0}=\frac{j0}{1-0}=0\angle\frac{\pi}{2}$$

$$\frac{Vout}{VIN}\Big|_{\omega\to\infty}\to\frac{j\omega C_1R_1}{-\omega^2R_2R_1C_2C_1)}\to\frac{j}{-\omega R_2C_2}=0\angle-\frac{\pi}{2}$$

$$\frac{Vout}{VIN}\Big|_{\omega=\frac{1}{\sqrt{R_2R_1C_2C_1}}}=\frac{j\omega C_1R_1}{j\omega(C_2R_1+R_2C_2+R_1C_1)}=\frac{C_1R_1}{C_2R_1+R_2C_2+R_1C_1}\angle 0=\frac{1}{3}\angle 0$$

From curve the lower and upper cutoff frequencies are 30 rad/sec (4.8 Hz) and 325 rad/sec (51.7 Hz), respectively.