Transistors

Lesson #10

Chapter 4
Small Signal Equivalent Circuits and Parameters for the BJT

• If the variable portion of the input signal is small (in amplitude), it is possible to approximate the (non-linear) transistor as a linear device by representing it by an equivalent circuit

• Here’s how we do it:
  – Recall \( i_E = i_C + i_B \)

Define: \( \alpha = \frac{i_C}{i_E} \); then \( i_B = (1 - \alpha)i_E \)

and \( i_C = \frac{\alpha}{1-\alpha} i_B = \beta i_B \) where \( \beta = \frac{\alpha}{1-\alpha} \)

  – Since the base-emitter junction is a forward-biased diode and the current that crosses this junction is \( I_E \), then we can use the Shockley equation

\[
i_E = I_{ES}(e^{\frac{v_{BE}}{V_T}} - 1)\]
Small Signal Equivalent Circuits and Parameters for the BJT

• Then: \( i_B = (1 - \alpha)I_{ES} \left( e^{\frac{v_{BE}}{V_T}} - 1 \right) \)

• Before we continue let’s define the following notation:
  – A lower case signal (voltage or current) with upper case subscripts represents the total of the DC portion of the signal and the AC portion of the signal
  – An upper case signal with upper case subscripts represents the DC portion (the Q-point)
  – A lower case signal with lower case subscripts represents the AC portion

\[
i_B(t) = I_{BQ} + i_b(t) \\
v_{BE}(t) = V_{BEQ} + v_{be}(t)
\]
Small Signal Equivalent Circuits and Parameters for the BJT

\[ i_B = (1 - \alpha)I_{ES}\left(e^{\frac{v_{BE}}{V_T}} - 1\right) \]

\[ I_{BQ} + i_b(t) = (1 - \alpha)I_{ES}\left(e^{\frac{v_{BE} + v_{be}(t)}}{V_T} - 1\right) \]

Ignoring the -1 term since it is negligible compared to the exponential term

\[ I_{BQ} + i_b(t) = (1 - \alpha)I_{ES}\left(e^{\frac{v_{BE}}{V_T}} e^{\frac{v_{be}(t)}{V_T}}\right) \]

Noting that the Q-point values must also satisfy the Shockley equation,

\[ I_{BQ} = (1 - \alpha)I_{ES}\frac{v_{BE}}{V_T} \]

\[ \therefore I_{BQ} + i_b(t) = I_{BQ}e^{\frac{v_{be}(t)}{V_T}} \]

Using the first two terms of the Taylor expansion of the exponential function: \(e^x \approx 1 + x\)

\[ I_{BQ} + i_b(t) \approx I_{BQ}(1 + \frac{v_{be}(t)}{V_T}) = I_{BQ} + I_{BQ} \frac{v_{be}(t)}{V_T} \]

\[ i_b(t) = I_{BQ} \frac{v_{be}(t)}{V_T} \]
Small Signal Equivalent Circuits and Parameters for the BJT

This is a relationship of the AC portion of the base current to the base-emitter voltage:

\[ i_b(t) = I_{BQ} \frac{v_{be}(t)}{V_T} \Rightarrow v_{be}(t) = i_b(t) \frac{V_T}{I_{BQ}} = i_b(t)r_\pi \]

\[ r_\pi = \frac{V_T}{I_{BQ}} \]

This resistance is a function of the Q-point and represents the input resistance of our equivalent circuit.

Secondly, since the collector current \( i_c(t) = \beta i_b(t) \)

\[ I_C + i_c(t) = \beta I_B + \beta i_b(t) \]

\[ \therefore i_c(t) = \beta i_b(t) \]

And we have a dependent current source in the collector to represent this relationship.
Small Signal Equivalent Circuits and Parameters for the BJT

- Here is our equivalent circuit:

- Here is a second version equivalent circuit:

\[ \beta i_b = \frac{\beta}{r_\pi} v_{be} = g_m v_{be} \]
Small Signal Equivalent Circuits

Example

- We see that we have capacitors and extra resistors.
- The extra resistors represent:
  - the load to the next stage (input impedance of the next stage)
  - the source resistance (output impedance of the previous stage)
- The extra capacitors are to:
  - protect the DC design of our amplifier
  - allow the passing the AC portion of the input signal from the source to our amplifier and on to the load (coupling capacitors)
  - eliminate the loading of the emitter resistor for the AC signal (bypass capacitors)
Small Signal Equivalent Circuits - Example
DC Analysis

- For DC the capacitors are open-circuits so for DC analysis the circuit becomes the same circuit as we have just analyzed.
- The components not affected by the DC voltage are “grayed out”.
- The next step is to perform the AC analysis.
Small Signal Equivalent Circuits-Example

AC Analysis

• The next step is to perform the AC analysis by
  – Shorting any DC voltage sources
  – Opening any DC current sources
  – Shorting the capacitors
  – Eliminating any resistors which are shorted by the capacitors (e.g., the emitter resistor
  – Connecting the source, source resistance, and load resistance
  – Replace the transistor with its small signal equivalent circuit.

• Short circuits are drawn in thick lines, the removed capacitor is “greyed-out”, and the equivalent circuit is in red.

• This circuit looks too messy, let’s redraw
Small Signal Equivalent Circuits-Example

AC Analysis

\[ R_L' = R_c \parallel R_L = \frac{R_c R_L}{R_c + R_L} = \frac{1 \times 2k}{1 + 2} = 667 \]

\[ R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10k}{5 + 10} = 3.33k \]

\[ r_\pi = \frac{V_T}{I_{BQ}} = \frac{26m}{14.1\mu} = 1844 \]

\[ v_o = -\beta i_b R_L' \]

\[ v_i = i_b r_\pi \]

\[ A_v = \frac{v_o}{v_i} = -\frac{\beta R_L'}{r_\pi} = -109 \]

\[ Z_{in} = R_B \parallel r_\pi = \frac{R_B r_\pi}{R_B + r_\pi} = 1.19k \]

\[ i_o = \frac{v_o}{R_L} = -\frac{\beta i_b R_L'}{R_L} = -\frac{\beta R_L' v_i}{R_L r_\pi} = A_v \frac{v_i}{R_L} \]

\[ i_i = \frac{v_i}{Z_{in}} \]

\[ A_i = \frac{i_o}{i_i} = A_v \frac{R_L'}{v_i} = A_v \frac{Z_{in}}{R_L} = -64.4 \]

\[ G = A_i A_v = 7000 \]
Small Signal Equivalent Circuits-Example

AC Analysis Alternative Method for $A_i$

$R_L' = R_c \parallel R_L = \frac{R_c R_L}{R_c + R_L} = \frac{1 \times 2k}{1+2} = 667$

$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10k}{5 + 10} = 3.33k$

$r_\pi = \frac{V_T}{I_{BQ}} = \frac{26m}{14.1\mu} = 1844$

By current division

$i_o = \frac{R_c}{R_c + R_L} (-\beta i_b)$

$i_b = \frac{R_B}{R_B + r_\pi} i_i$

$i_o = \frac{R_c}{R_c + R_L} (-\beta \frac{R_B}{R_B + r_\pi}) i_i$

$A_i = \frac{i_o}{i_i} = -\beta \frac{R_c}{R_c + R_L} \times \frac{R_B}{R_B + r_\pi}$

$= -300 \left( \frac{1k}{1k + 2k} \right) \left( \frac{3.33k}{3.33k + 1.844k} \right) = -64.36$

Note they are the same:

$A_i = A_v \frac{Z_{in}}{R_L} = -\beta \frac{R_L'}{r_\pi} \frac{R_B r_\pi}{R_B + r_\pi}$

$= -\beta \frac{R_c R_L}{R_c + R_L} \times \frac{R_B r_\pi}{R_B + r_\pi}$

$= -\beta \frac{R_c}{R_c + R_L} \times \frac{R_B}{R_B + r_\pi}$
Small Signal Equivalent Circuits-Example

AC Analysis

\[ v_{in} = 0.001 \sin \omega t \]

\[ A_v = \frac{v_o}{v_i} = -\frac{\beta R_L'}{r_\pi} = -109 \]

\[ Z_{in} = R_o \parallel r_\pi = \frac{R_or_\pi}{R_o + r_\pi} = 1.19k \]

\[ A_{vs} = \frac{v_o}{v_{in}} = \frac{v_o}{v_i} \frac{v_i}{v_{in}} \]

\[ \frac{v_i}{Z_{in}} = \frac{v_{in}}{Z_{in} + R_s} \]

\[ \frac{v_i}{v_{in}} = \frac{Z_{in}}{Z_{in} + R_s} = .7 \]

\[ A_{vs} = \frac{v_o}{v_{in}} = \frac{v_o}{v_i} \frac{v_i}{v_{in}} = A_v \frac{v_i}{v_{in}} = A_v \frac{Z_{in}}{Z_{in} + R_s} = -76.4 \]

\[ v_o = -76.4v_i = -76.4 \sin \omega t \text{ mV} \]
Small Signal Equivalent Circuits-Example

AC Analysis

\[ Z_o = \frac{v_x}{i_x} \]
\[ i_x = \frac{v_x}{R_c} + \beta i_b \]
\[ i_b = 0; \quad i_x = \frac{v_x}{R_c} \]
\[ Z_o = \frac{v_x}{i_x} = R_c = 1k \]
Small Signal Equivalent Circuits
Emitter Follower Example

- We see that the load is across the emitter resistor
- The extra resistors and capacitors are represent
  - the load to the next stage (input impedance of the next stage)
  - the source resistance (output impedance of the previous stage)
  - Coupling capacitors
- Our analysis plan
  - First, DC analysis to determine Q-point and equivalent circuit parameters
  - AC analysis to calculate the gains

\[
V_{BEQ} = 0.7 \text{ V} \\
\beta = 200 \\
V_{CC} = 20\text{V}
\]
Small Signal Equivalent Circuits

Emitter Follower Example DC Analysis

- Remove the circuit elements which are not affected by the DC voltages
- Opening the coupling capacitors
- Redraw the circuit and replace the base circuit with its Thevenin’s equivalent

\[ R_{th} = R_1 \parallel R_2 = 50k \]

\[ V_{th} = \frac{R_2}{R_2 + R_1} V_{cc} = 10V \]

Next, use the DC equivalent circuit for the active region and then write KVL for the base circuit

\[ V_{th} = I_{bq}R_{th} + V_{beq} + I_{eq}R_E = I_{bq}R_{th} + V_{beq} + (1+\beta)I_{bq}R_E \]

\[ I_{bq} = \frac{V_{th} - V_{beq}}{R_T + (1+\beta)R_E} = \frac{10 - 0.7}{50k + (1+200)2k} = 20.6 \mu A \]

\[ I_{cq} = \beta I_{bq} = 4.12 mA; I_{eq} = (1+\beta)I_{bq} = 4.14 mA \]

\[ V_{ceq} = V_{cc} - I_{eq}R_E = 11.7V; r_\pi = \frac{V_T}{I_{beq}} = \frac{26m}{20.6 \mu} = 1260 \]
The next step is to perform the AC analysis by

- Shorting any DC voltage sources
- Opening any DC current sources
- Shorting the capacitors
- Connecting the source, source resistance, and load resistance
- Replace the transistor with its small signal equivalent circuit.

And Redraw to simplify
**Small Signal Equivalent Circuits**

**Emitter Follower Example AC Analysis**

\[ R_B = R_1 \parallel R_2 = 50k \]
\[ R_L' = R_E \parallel R_L = 667 \]
\[ v_o = (1 + \beta) i_b R_L' \]
\[ v_i = i_b r_\pi + v_o = \frac{v_o}{(1 + \beta) R_L'} r_\pi + v_o \]
\[ A_v = \frac{v_o}{v_i} = \frac{(1 + \beta) R_L'}{(1 + \beta) R_L' + r_\pi} \]
\[ A_v = \frac{(1 + 200) \times 667}{(1 + 200) \times 667 + 1260} = .991 \]
\[ Z_{ii} = \frac{v_i}{i_b} = r_\pi + (1 + \beta) R_L' = 134k \]
\[ Z_{in} = R_B \parallel Z_{ii} = 36.5k \]
\[ i_o = \frac{v_o}{R_L} = \frac{A_v v_i}{R_L} ; \quad i_i = \frac{v_i}{Z_{in}} \]
\[ A_i = \frac{i_o}{i_i} = \frac{A_v Z_{in}}{R_L} = 36.2 \]
\[ G = A_v A_i = 35.8 \]
Small Signal Equivalent Circuits
Emitter Follower Example AC Analysis

\[ i_e = (1 + \beta) i_b \]

\[ A_v = \frac{v_o}{v_i} = \frac{(1 + 200) \times 667}{(1 + 200) \times 667 + 1260} = 0.991 \]

\[ A_{vs} = \frac{v_o}{v_s} = \frac{v_o}{v_i} \frac{v_i}{v_s} = 0.991 \frac{v_i}{v_s} \]

\[ v_i = \frac{Z_{in}}{Z_{in} + R_S} = \frac{36.5}{36.5 + 10} = 0.78 \]

\[ A_{vs} = \frac{v_o}{v_s} = \frac{v_o}{v_i} \frac{v_i}{v_s} = 0.991 \times 0.78 = 0.78 \]
Small Signal Equivalent Circuits

Emitter Follower Example AC Analysis

\[ Z_o = \frac{1}{1/R_E + (1 + \beta)/(R_s^' + r_\pi)} \Rightarrow R_E \parallel \frac{R_s^' + r_\pi}{1 + \beta} = 46.6 \]

\[ i_x = \frac{v_x}{R_E} \]

\[ i_x = \frac{v_x}{R_s^'} - \frac{v_x}{R_B} = (1 + \beta) i_b \]

\[ R_s^' = R_s \parallel R_B = 8.33k \]

\[ v_x = -i_b (R_s^' + r_\pi) \]

\[ v_x = i_x \left( \frac{1}{1/R_E + (1 + \beta)/(R_s^' + r_\pi)} \right) \]
In Saturation, if

\[
V_o = V_{CE} = V_{CC} - R_C i_C = V_{CC} - R_C \beta \quad i_B = V_{CC} - R_C \beta \quad \frac{V_{in} - V_{BEQ}}{R_B} < .2V
\]

Then, \( V_o = V_{CE} = .2 \)

- Operating between cutoff and saturation (i.e., bypassing the active region), the BJT acts like an inverter.
- From this behavior, logic circuits such as NOR gates can be developed.

BME 372 Electronics I – J.Schesser
The BJT as a Digital Switch

In Saturation, if

\[ V_o = V_{cc} - R_c \beta \frac{V_{in} - V_{BEQ}}{R_B} < .2V \]

\[ V_o = 3 - 2 \times \beta \frac{V_{in} - .7}{5} = .2 \]

\[ V_{in} = 5 \times \frac{3 - .2}{2 \beta} + .7 = \frac{7}{\beta} + .7 \]

In Cutoff, if \( V_{in} < .7V \)

Then, \( V_o = V_{cc} \)
Switching and Timing

$t_r = \text{rise time}$

$t_f = \text{fall time}$

$t_d = \text{delay time} (\text{propagation delay from High to Low})$

$t_s = \text{storage time} (\text{propagation delay from Low to High})$
Homework

• Probs. 4.40, 4.42, 4.43, 4.45, 4.46, 4.51, 4.53, 4.54, 4.56,