Electronics II

- Goal of the course is to continue to become familiar with electronic circuitry
  - Transistors
  - FETs
  - Logic Circuitry
  - Frequency Response
  - Feedback and Oscillators
  - Waveshaping
- To apply this knowledge to Biomedical applications
- But First Some Review
Review of Electronics I

Lesson #1
Circuit Analysis & Amplifiers
Chapters 1 & 2
Circuit Analysis

• Circuit Elements
  – Passive Devices
  – Active Devices

• Circuit Analysis Tools
  – Ohms Law
  – Kirchhoff’s Law
  – Impedances
  – Mesh and Nodal Analysis
  – Superposition

• Examples
Circuit Elements - Passive Devices

- **Passive**: dissipates or stores energy
- **Linear**: supports a linear relationship between the voltage across the device and the current through it.
  - **Resistor**: supports a voltage and current which are proportional, device dissipates heat, and is governed by Ohm’s Law, units: resistance or ohms $\Omega$
    \[ V_R(t) = I_R(t)R \text{ where } R \text{ is the value of the resistance associated with the resistor} \]
  - **Capacitor**: supports a current which is proportional to its changing voltage, device stores an electric field between its plates, and is governed by Gauss’ Law, units: capacitance or farads
    \[ I_C(t) = C \frac{dV_C(t)}{dt} \text{ where } C \text{ is the value of the capacitance associated with the capacitor} \]
  - **Inductor**: supports a voltage which is proportional to its changing current, device stores a magnetic field through its coils and is governed by Faraday’s Law, units: inductance or henries
    \[ V_L(t) = L \frac{dI_L(t)}{dt} \text{ where } L \text{ is the value of the inductance associated with the inductor} \]
Circuit Elements - Passive Devices Continued

• **Non-linear**: supports a non-linear relationship among the currents and voltages associated with it
  – **Diodes**: supports current flowing through it in only one direction
Circuit Elements - Active Devices

- **Active**: Provider of energy or supports power gain
- **Linear**
  - **Voltage Source**: a device which supplies a voltage as a function of time at its terminals which is independent of the current flowing through it, units: Volts
    - DC, AC, Pulse Trains, Square Waves, Triangular Waves
  - **Current Source**: a device which supplies a current as a function of time out of its terminals which is independent of the voltage across it, units: Amperes
    - DC, AC, Pulse Trains, Square Waves, Triangular Waves
Circuit Elements - Active Devices
Continued

– Ideal Sources vs Practical Sources

• An ideal source is one which only depends on the type of source (i.e., current or voltage)

• A practical source is one where other circuit elements are associated with it (e.g., resistance, inductance, etc.)
  – A practical voltage source consists of an ideal voltage source connected in series with passive circuit elements such as a resistor
  – A practical current source consists of an ideal current source connected in parallel with passive circuit elements such as a resistor
Independent vs Dependent Sources

- An independent source is one where the output voltage or current is not dependent on other voltages or currents in the device.
- A dependent source is one where the output voltage or current is a function of another voltage or current in the device (e.g., a BJT transistor may be viewed as having an output current source which is dependent on the input current).
Circuit Elements - Active Devices

Continued

• Non-Linear
  – Transistors: three or more terminal devices where its output voltage and current characteristics are a function on its input voltage and/or current characteristics, several types BJT, FETs, etc.
Circuits

- A circuit is a grouping of passive and active elements
- Elements may be connecting is series, parallel or combinations of both
**Circuits Continued**

- **Series Connection**: Same current through the devices
  - The resultant resistance of two or more Resistors connected in series is the sum of the resistance
  - The resultant inductance of two or more Inductors connected in series is the sum of the inductances
  - The resultant capacitance of two or more Capacitors connected in series is the inverse of the sum of the inverse capacitances
  - The resultant voltage of two or more Ideal Voltage Sources connected in series is the sum of the voltages
  - Two of more Ideal Current sources can not be connected in series
Series Circuits

- Resistors

\[ R_T = R_1 + R_2 \]

\[ V_{ac} = V_{ab} + V_{bc} = IR_1 + IR_2 = I(R_1 + R_2) = IR_T \]

- Inductors

\[ L_T = L_1 + L_2 \]

\[ V_{ac} = V_{ab} + V_{bc} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} = (L_1 + L_2) \frac{dI}{dt} = L_T \frac{dI}{dt} \]

- Capacitors

\[ C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1C_2}{C_1 + C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)C_T = \frac{1}{C_T} \int Idt \]

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Circuits Continued

- Parallel Connection: Same Voltage across the devices
  - The resultant resistance of two or more Resistors connected in parallel is the inverse of the sum of the inverse resistances
  - The resultant inductance of two or more Inductors connected in parallel is the inverse of the sum of the inverse inductances
  - The resultant capacitance of two or more Capacitors connected in parallel is the sum of the capacitances
  - The resultant current of two or more Ideal Current Sources connected in parallel is the sum of the currents
  - Two of more Ideal Voltage sources can not be connected in parallel
Parallel Circuits

• Resistors

\[ R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \]

\[ I_{ab} = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)V = \frac{V}{R_T} \]

• Inductors

\[ L_T = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2} \]

\[ I_{ab} = I_1 + I_2 = \frac{1}{L_1} \int V dt + \frac{1}{L_2} \int V dt = \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \int V dt = \frac{1}{L_T} \int V dt \]

• Capacitors

\[ C_T = C_1 + C_2 \]

\[ I_{ab} = I_1 + I_2 = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} = (C_1 + C_2) \frac{dV}{dt} = C_T \frac{dV}{dt} \]
Combining Circuit Elements

Kirchhoff’s Laws

• Kirchhoff Voltage Law: The sum of the voltages around a loop must equal zero
• Kirchhoff Current Law: The sum of the currents leaving (entering) a node must equal zero
Combining Rs, Ls and Cs

• We can use KVL or KCL to write and solve a differential equation associated with the circuit.
  – Example: a series RLC circuit

\[
V(t) = I(t)R_1 + L_1 \frac{dI(t)}{dt} + \frac{1}{C_1} \int I(t)dt
\]

• Or to simplify this analysis, we can concentrate on special cases
Impedances

• Our special case, signals of the form: \( V(t) \) or \( I(t) = Ae^{st} \)
where \( s \) can be a real or complex number

• For example: \( \cos(\omega t + \theta) \); Recall Euler’s formula \( e^{j\theta} = \cos \theta + j \sin \theta \) where \( j \) is the imaginary number = \( \sqrt{-1} \)

• Since the derivative [and integral] of \( Ae^{st} = sAe^{st} \)
\[ \Rightarrow \frac{(1/s)Ae^{st}}{s} \], we can define the impedance of a circuit
element as \( Z(s) = \frac{V}{I} \) where \( Z \) is only a function of \( s \) since
the time dependency drops out:

For an inductor, let's assume \( I(t) = Ae^{st} \); then \( V(t) = L \frac{dI(t)}{dt} = LsAe^{st} \); \( Z(s) = \frac{V}{I} = \frac{sLAe^{st}}{Ae^{st}} = sL \)

For a capacitor, let's assume \( V(t) = Ae^{st} \); then \( I(t) = C \frac{dV(t)}{dt} = CsAe^{st} \); \( Z(s) = \frac{V}{I} = \frac{Ae^{st}}{sCAe^{st}} = \frac{1}{sC} \)

For a resistor, let's assume \( I(t) = Ae^{st} \); then \( V(t) = RI(t) = RAe^{st} \); \( Z(s) = \frac{V}{I} = \frac{RAe^{st}}{Ae^{st}} = R \)
Sinusoidal Steady State

- A special case of our special case is for sinusoidal inputs, where \( s = j \omega \)
- If \( V(t) = A \cos (\omega t + \theta) \), then we can represent \( V(t) \) as \( \text{Re}\{A e^{j(\omega t + \theta)}\} = \text{Re}\{A e^{j\theta} e^{j\omega t}\} \) since \( A e^{j(\omega t + \theta)} = \cos(\omega t + \theta) + j \sin(\omega t + \theta) \)
- What is \( A e^{j(\omega t + \theta)} \)?
  - First, it is a complex function since it is a function of a complex number. If we plot on the complex plane, it has a magnitude of \( A \) and angle of \( \omega t + \theta \). It can be viewed as a vector which rotates in time around the origin of the complex plane at angular velocity \( \omega \) and at \( t=0 \) is at \( \theta \) degrees from the real axis.

- We can represent this function by a PHASOR in terms of rectangular coordinates or polar coordinates \( \Rightarrow \text{MAG} \angle \phi \) (phasor notation) or in this case \( V = A \angle \theta \)
Sinusoidal Steady State Continued

• So for the SSS, the impedances become:
  
  - \( Z_L = j\omega L = \omega L \angle 90^\circ \); here we say that the voltage across an inductor leads its current by 90°
  
  - \( Z_C = 1/j\omega C = 1/\omega C \angle -90^\circ \); here we say that the voltage across a capacitor lags its current by 90°
  
  - \( Z_R = R = R \angle 0^\circ \); here we say that the voltage across a resistor is in phase with the current
Sinusoidal Steady State Continued

For \( V(t) = A \cos \omega t \), using phasor notation for \( V(t) \Rightarrow V = A \angle 0^\circ \) and \( I(t) \Rightarrow I \), our equation can be re-written:

\[
V(t) = I(t)R_1 + L_1 \frac{dI(t)}{dt} + \frac{1}{C_1} \int I(t)dt
\]

Converting to Phasor representation

\[
V = A \angle 0 = IR_1 + j\omega L_1I + \frac{1}{j\omega C_1}I
\]

\[
I = \frac{A \angle 0}{R_1 + j\omega L_1 + \frac{1}{j\omega C_1}}
\]

\[
= \frac{A \angle 0}{R_1 + j(\omega L_1 - \frac{1}{\omega C_1})} = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \frac{1}{\omega C_1})^2}} \angle -\tan^{-1}\left[\frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1}\right]
\]

Converting back to the time representation,

\[
I(t) = \frac{A}{\sqrt{R_1^2 + (\omega L_1 - \frac{1}{\omega C_1})^2}} \cos(\omega t - \tan^{-1}\left[\frac{\omega L_1 - \frac{1}{\omega C_1}}{R_1}\right])
\]
**Voltage Division**

• The voltage across impedances in series divides in proportion to the impedances.

\[ V_{ac} = V_{ab} + V_{bc} = I(Z_1 + Z_2); \text{KVL + Ohm's Law} \]

\[ V_{bc} = IZ_2 \]

\[ \frac{V_{bc}}{V_{ac}} = \frac{Z_2}{Z_1 + Z_2} \]

\[ \frac{V_i}{V_{ac}} = \frac{Z_i}{Z_1 + Z_2 + \ldots Z_n} \]
Current Division

- The current into impedances in parallel divides in proportion to the inverse of the impedances.

\[
I_{ac} = I_1 + I_2 = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right); \text{KCL + Ohm's Law}
\]

\[
I_1 = \frac{V}{Z_1}
\]

\[
\frac{I_1}{I_{ac}} = \frac{1/Z_1}{(1/Z_1)+(1/Z_2)} = \frac{Z_2}{Z_1+Z_2}
\]

\[
\frac{I_i}{I_{ac}} = \frac{(1/Z_i)}{(1/Z_1)+(1/Z_2)+\cdots+(1/Z_n)}
\]
Mesh Analysis

1. Define a current in each mesh (loop) of a network. For example, in a 5 mesh network, define 5 current unknowns.

2. Using KVL, write an equation for each mesh using the unknown currents. In our 5 mesh example, you’ll have 5 equations and 5 unknown currents.

3. Solve for the unknown currents and now apply these currents to the network to find the voltages for each impedance in the network.
**Nodal Analysis**

1. Define a voltage at each node (junction point) of a network. For example, in a 5 node network, define 5 voltage unknowns.

2. Using KCL, write an equation for each node using the unknown voltages. In our 5 node example, you’ll have 5 equations and 5 unknown voltage.

3. Solve for the unknown voltages and now apply these voltages to the network to find the currents for each impedance in the network.
Mesh Analysis Example

Mesh 1
0 = I₁ 5 + 10(I₁ – I₂) – 5; \(5 = 15I₁ - 10I₂\)

Mesh 2
0 = I₂ 5 + 5 + 10(I₂ – I₁); \(-5 = 15I₂ - 10I₁\)

\(I₁ = \frac{1}{5}; I₂ = -\frac{1}{5}\)
Nodal Analysis Example

Node 1
\[ I_1 + I_2 + I_3 = 0 \]
\[ \frac{5-V}{5} + \frac{5-V}{5} - \frac{V}{10} = 0; \quad 2 = \frac{V}{2} \]

\[ V = 4 \]
Superposition

- Used to analyze a circuit with multiple sources.
- Steps:
  1. Set all sources except for one to zero (voltage sources are shorted-circuited, current sources are open-circuited)
  2. Solve for the currents and voltages for all of the circuit elements
  3. Repeat steps 1-2 for the remaining sources.
  4. Add each of the solutions to obtain the solution for the entire circuit
Superposition Analysis Example

\[ R_p = 10 \parallel 5 = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = \frac{10}{3} = 3.33 \Omega \]

\[ I_{is1} = \frac{5}{5 + \frac{10}{3}} = \frac{15}{25} = \frac{3}{5} \]

\[ I_{ls1} = .6 \]

\[ I_{2s1} = -.4 \]

\[ I_{3s1} = -.2 \]
Superposition Analysis Example

\[ R_p = 10 \parallel 5 = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = \frac{10}{3} = 3.33 \Omega \]

\[ I_{2s2} = \frac{5}{5 + \frac{10}{3}} = \frac{15}{25} = \frac{3}{5} \]

\[ I_{1s2} = -\frac{10}{15} \times \frac{3}{5} = -\frac{2}{5} = -0.4 \]

\[ I_{3s2} = -\frac{5}{15} \times \frac{3}{5} = -\frac{1}{5} = -0.2 \]
Superposition Analysis Example

• Summing the results of each solution:

\[ I_1 = I_{1s1} + I_{1s2} = 0.6 - 0.4 = 0.2 \]
\[ I_2 = I_{2s1} + I_{2s2} = -0.4 + 0.6 = 0.2 \]
\[ I_3 = I_{3s1} + I_{3s2} = -0.2 + -0.2 = -0.4 \]
Thevenin and Norton Equivalent Circuits

• Thevenin’s Theorem: Any circuit consisting of passive and active components can be represented by a voltage source in series with an equivalent set of passive components
  – The value of the voltage source equals the voltage seen at the output terminal without any load connected to it, i.e., the open-circuit voltage
  – The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.
Thevenin and Norton Equivalent Circuits

• Norton’s Theorem: Any circuit consisting of passive and active components can be represented by a current source in parallel with an equivalent set of passive components
  – The value of the current source equals the current seen at the output terminal shorted and without any load connected to it, i.e., the short-circuit current
  – The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.

• Note that the Thevenin and Norton Equivalents Circuits are equivalent to each other when the value of the Thevenin’s voltage source equals the product of the equivalent impedance times the Norton’s current source.
Thevenin and Norton Examples

Open Circuit Voltage at terminals : ab

$$V_{abOC} = \frac{15}{5+15} \times 10 = 7.5v$$

Output Impedance

$$R_o = 5 \parallel 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75\Omega$$
Thevenin and Norton Examples

Short Circuit Voltage at terminals : ab

\[ I_{abSC} = \frac{10}{5} = 2a \]

Output Impedance

\[ R_o = 5 || 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75 \Omega \]
**Input and Output Impedance of a Circuit**

- Input impedance of a circuit is the impedance looking into the input terminals of the circuit with any load connected to the output and all internal sources are set to zero.

- Output impedance of a circuit is the impedance looking into the output terminals of the circuit without any load connected to the output and all internal and input sources are set to zero.
Examples

Find the currents and voltages in these circuits

\[ I_1 = I_2 = \frac{2k}{4k} \times 4 = 2a \]

\[ V_{kBRANCH} = 1k \times 2a = 2mV \]

\[ V_{kSOURCEBRANCH} = 1k \times 4a = 4mV \]

\[ V_{CURRENTSOURCE} = 2 \times V_{kBRANCH} + V_{kSOURCEBRANCH} = 8mV \]
Examples

\[ I_{2s2} = I_{1s2} = \frac{5}{3} = 1.67 \, A \]

\[ I_{3s2} = 0 \]

\[ I_2 = -1 \]

\[ I_3 = 4 \, \text{Adc} \]

\[ I_1 = 1.33 + 1.67 = 3 \, A \]

\[ I_2 = -2.67 + 1.67 = -1 \, A \]

\[ I_3 = 0 + 4 = 4 \, A \]

\[ V_{R1} = V_{R2} = 3 \times 1 = 3 \, V \]

\[ V_{R3} = -1 \times 1 = -1 \, V \]

\[ V_{R4} = 4 \times 1 = 4 \, V \]

\[ V_{\text{CURRENTSOURCE}} = V_{R4} + V_{R2} + V_{R1} = 4 + 3 + 3 = 10 \, V \]
**Examples**

Find the voltage $V_{ab}$ in terms of $V$

\[ V_{ab} = \frac{Z_{c2}}{Z_{R2} + Z_{c2}} \times V_{cb}; \text{ (1) using voltage division} \]

\[ V_{cb} = \frac{Z_{cb}}{Z_{cb} + Z_{R1}} \times V; \text{ (2) using voltage division} \]

\[ Z_{cb} = Z_{c1} \text{ in parallel with the series combination of } Z_{c1} + Z_{R2} \]

\[ = Z_{c1} \parallel (Z_{c2} + Z_{R2}) = \frac{Z_{c1} \times (Z_{c2} + Z_{R2})}{Z_{c1} + Z_{c2} + Z_{R2}} \]

\[ V_{cb} = \frac{Z_{c1} \times (Z_{c2} + Z_{R2})}{Z_{c1} \times (Z_{c2} + Z_{R2}) + Z_{R1} \times (Z_{c1} + Z_{c2} + Z_{R2})} \times V \]

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Examples

Find the voltage \( V_{ab} \) in terms of \( V \)

Substituting (2) into (1)

\[
V_{ab} = \frac{Z_{c2}}{Z_{R2} + Z_{c2}} \times \frac{Z_{c1} \times (Z_{c2} + Z_{R2})}{Z_{c1} \times (Z_{c1} + Z_{R1}) + Z_{R1} \times (Z_{c1} + Z_{c2} + Z_{R2})} \times V
\]

\[
= \frac{Z_{c1} Z_{c2}}{Z_{c1} Z_{c1} + Z_{c1} Z_{R2} + Z_{R1} Z_{c1} + Z_{R1} Z_{c2} + Z_{R1} Z_{R2}} \times V
\]

\[
V_{ab} = \frac{1}{j \omega C_1} \times \frac{1}{j \omega C_2} \times \frac{R_2}{j \omega C_2} + \frac{R_1}{j \omega C_1} + \frac{R_1}{j \omega C_2} + R_1 R_2 \times V = \frac{V}{1 - \omega^2 C_1 C_2 R_1 R_2 + j \omega (R_1 C_1 + R_1 C_2 + R_2 C_2)}
\]
Homework

• Voltage and Current division
  – How does the voltage divide across two capacitors in series? Show your results.
  – How does the current divide among two capacitors in parallel? Show your results.

• Calculate the Currents and Voltages for the following circuits:
Homework

• Repeat the analysis of this circuit using Mesh and Nodal Analysis
**Basic Amplifier Types**

- An amplifier produces an output signal with the same wave shape as the input signal but usually with a larger amplitude.

\[ v_o(t) = A_v v_i(t) \]

- \( A_v \) is called the voltage gain and if \(<0\) then the amplifier is inverting; otherwise non-inverting.
Voltage-Amplifier Model

• Parameters:
  – Open Circuit Voltage Gain $A_{vo}$
  – Input and Output impedance $R_i=v_i/i_i; R_o=v_o/i_o$ (after removing $R_L$ and replacing all sources with their internal resistance)
  – Current Gain $A_i=i_o/i_i=(v_o/R_L)/(v_i/R_i)=A_vR_i/R_L$
  – Power Gain $G=P_o/P_i=v_o i_o/v_i i_i=A_vA_i=(A_v)^2R_i/R_L$

Example:
$v_s=10 \text{ mv}, R_s=2M\Omega, R_L=10\Omega$
$A_v=100, R_i=2M\Omega, R_o=5\Omega$

$v_i=2/(2+2)10\text{ mv}=5\text{ mv}$,
$A_{vo}v_i=0.5 \, v, v_o=10/(10+5)0.5=0.33\, v$

$A_v=.33/5x10^{-3}=66.7; A_{vs}=.33/10x10^{-3}=33.3$

$A_i=66.7*2x10^6/10=1.33e^7; G=1.33x10^7*66.7=8.9x10^8$
Decibel Notation

• A logarithmic scale is sometimes easier to use:
  – \( G_{dB} = 10 \log G \)
  – And if \( G = G_1 G_2 \); then \( G_{dB} = G_1_{dB} + G_2_{dB} \)

• And to convert voltage and current to dBs.
  – \( A_{v_{dB}} = 20 \log |A_v| \)
  – \( A_{i_{dB}} = 20 \log |A_i| \)
Other Amplifier Models

• Current-Amplifier
  – Short Circuit ($R_L=0$) Current Gain $i_o = A_{isc} i_i$

• Transconductance-Amplifier
  – Short Circuit Transconductance Gain $i_{osc} = G_{msc} v_i$

• Transresistance – Amplifier
  – Open Circuit Transresistance Gain $v_{osc} = R_{moc} i_i$
# Ideal Amplifiers

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Input Impedance</th>
<th>Output Impedance</th>
<th>Gain</th>
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<tr>
<td>Voltage</td>
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<td>$A_{vo}$</td>
</tr>
<tr>
<td>Current</td>
<td>0</td>
<td>$\infty$</td>
<td>$A_{isc}$</td>
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<tr>
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<td>0</td>
<td>$G_{msc}$</td>
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<tr>
<td>Trans-resistance</td>
<td>0</td>
<td>$\infty$</td>
<td>$R_{msc}$</td>
</tr>
</tbody>
</table>
Frequency Response

- We look at the spectrum (frequencies) which make up the signals being processed by an amplifier.
- Gain is now a complex number since both the amplitude and phase of an input signal can be affected.
- We look at the spectrum in terms of regions or bands:
  - Low, mid, and high frequency bands
- We define parameters associated with the spectrum of the output of an amplifier:
  - Half power frequencies or (3 dB points)
  - Bandwidth (the distance in frequency between 3 dB points)
  - In the time domain we look at the response due to a pulse and define the Rise Time which is a function of the Bandwidth.
Differential Amplifiers

• Output of the amplifier is a function of the difference of the inputs: \( v_o = A_d (v_{i1} - v_{i2}) \)

• However, most real Differential Amplifier are affected by the average of the input and we define the common mode input signal \( v_{icm} = \frac{1}{2}(v_{i1} + v_{i2}) \)

• Therefore, \( v_o = A_d (v_{i1} - v_{i2}) + A_m v_{icm} \) and the ratio of \( A_d \) to \( A_m \) is called the common mode rejection ratio
Operational Amplifiers

• An ideal differential voltage amplifier $R_i=\infty$, $R_o = 0$, common-mode gain=0
• But with infinite open circuit voltage gain

\[ v_o = A_v (v_1 - v_2) \]

\[ v_{id} = v_1 - v_2 \]
Operational Amplifier Usage

• Operational Amplifiers are used with negative feedback
• The differential input to the Op-Amp is assumed to be zero (since its gain is infinite) also called a virtual short circuit.
• Therefore, the input current into the Op-Amp is zero (summing-point constraint)
Example: Inverting Amplifier

Note that a portion of $v_o$ is fed back via $R_2$ to the inverting input. So if $v_i$ increases and, therefore, increases $v_o$, the portion of $v_o$ fed back will then have the affect of reducing $v_i$ (i.e., negative feedback).

Using the summing point constraint:

\[ v_{in} = i_1 R_1 + 0 \text{ since } v_i \text{ is (virtually) zero} \]

\[ i_1 = i_2 \text{ due to the summing - point constraint} \]

\[ v_0 = -i_2 R_2 + 0 \text{ since } v_i \text{ is (virtually) zero} \]

\[ = - \frac{R_2}{R_1} v_{in} \text{ which is independent of } R_L \]
Frequency Analysis

\[ V_{in}(j\omega) = I_1(j\omega)Z_1(j\omega) + 0 \text{ since } v_i \text{ is (virtually) zero} \]

\[ I_1(j\omega) = I_2(j\omega) \text{ due to the summing-point constraint} \]

\[ V_o(j\omega) = -I_2(j\omega)Z_2 + 0 \text{ since } v_i \text{ is (virtually) zero} \]

\[ = -\frac{Z_2}{Z_1} V_{in}(j\omega) \text{ which is independent of } Z_L \]

\[ \frac{V_o(j\omega)}{V_{in}(j\omega)} = -\frac{Z_2}{Z_1} \]

\[ \frac{V_o(j\omega)}{V_{in}(j\omega)} = -\frac{Z_2}{Z_1} \text{ an integrator} \]

\[ \frac{V_o(j\omega)}{V_{in}(j\omega)} = -\frac{Z_2}{Z_1} = -j\omega RC \text{ a differentiator} \]
Homework

• Probs 1.15, 1.17, 1.18, 1.19, 1.30, 1.31, 1.35, 1.36, 1.37, 1.55, 1.56, 1.59, 1.60, 1.61
• Probs 2.2, 2.5, 2.6, 2.10, 2.22, 2.24, 2.25, 2.28