Frequency Response

Lesson #10
FET
Section 8.2
**Equivalent Circuit Of An FET**

\[ i_d(t) = g_m v_{gs}(t) + \frac{v_{ds}(t)}{r_d} \]

where

\[ g_m = \frac{i_d}{v_{gs}} \bigg|_{v_{ds}=0} \approx \frac{\Delta i_D}{\Delta v_{GS}} \bigg|_{v_{DS}=v_{DSQ}} \]

and

\[ r_d = \frac{i_d}{v_{ds}} \bigg|_{v_{gs}=0} \approx \frac{\Delta i_D}{\Delta v_{DS}} \bigg|_{v_{GS}=v_{GSQ}} \]

\[ g_m = \frac{\partial i_D}{\partial v_{GS}} \bigg|_{Q-point} \]

\[ r_d = \frac{\partial i_D}{\partial v_{DS}} \bigg|_{Q-point} \]
Metal Oxide Semiconductor Field Effect Transistor
MOSFET (NMOS) Enhancement Mode

- Consists of Four terminals
  - Drain which is $n$-doped material
  - Source also $n$-doped material
  - Base which is $p$-doped material
  - Gate is a metal and is insulated from the Drain, Source and Base by a thin layer of silicon dioxide ~ .05-.1µm thick
- Basically, an electric current flowing from drain to source, $i_D$, is controlled by the amount of voltage (electric field) appearing between the gate and base (note that the base and source are usually tied together and therefore, it is referred to as the gate to source voltage or gate voltage), $v_{GS}$.
- $i_D$ flows through a channel of $n$-type material which is induced by $v_{GS}$. The amount of $i_D$ is a function of the thickness of the channel and the voltage between drain and source, $v_{DS}$.
- However, the thickness of channel is controlled by the level of gate voltage. (The width, .5 to 500 µm, and length, .2 to 10 µm, of the channel is shown in the diagram.)
FET Equivalent Circuit Updated for High Frequencies

\[ C_{gs} \text{ and } C_{gd} \text{ account for the capacitance effects between the gate and source and between the gate and drain.} \]
An Example

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Example Continued

Gate Nodal Equation using KCL
\[ \frac{V_{gs}(s) - V_{sig}(s)}{R_{sig}} + \frac{V_{gs}(s)}{1/(sC_{gs})} + \frac{V_{gs}(s) - V_0(s)}{1/(sC_{gd})} = 0 \]

Drain Nodal Equation using KCL
\[ \frac{V_o(s) - V_{gs}(s)}{1/(sC_{gd})} + \frac{V_o(s)}{R'_L} + g_mV_{gs}(s) = 0 \]

Solving these two equations for \( V_{gs} \) and combining:
\[ A_v(s) = \frac{V_o}{V_{sig}} = -g_m R'_L \times \frac{1 - s(C_{gd}/g_m)}{1 + s[C_{gs} R_{sig} + C_{gd}(R_{sig} + g_m R'_L R_{sig} + R'_L)] + s^2 C_{gs} C_{gd} R_{sig} R'_L} \]

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Breakpoint Frequencies

\[ f_z = \frac{g_m}{2\pi C_{gd}} \]

\[ f_{p1} = \frac{1}{2\pi \left\{ C_{gd} \left( R'_L + R_{\text{sig}} + R_{\text{sig}}' R'_L g_m \right) + C_{gs} R_{\text{sig}} \right\}} \]

\[ f_{p2} = \frac{\left\{ C_{gd} \left( R'_L + R_{\text{sig}} + R_{\text{sig}}' R'_L g_m \right) + C_{gs} R_{\text{sig}} \right\}}{2\pi (C_{gs} C_{gd} R_{\text{sig}} R'_L)} \]

Assume that \( f_{p2} > f_z >> f_{p1} \), then the bode plot would look like this:

Then the 3dB point (half-power point) frequency is \( f_{p1} \), and the Bandwidth is defined as \( BW = f_{p1} - 0 = f_{p1} \)

\( A_{v_{\text{mid}}} \) is defined as the value of \( A_v(f) \) when the effects of frequency are small (i.e., when \( |A_v(f)| > A_v(f_{3\text{dB}}) \))
Bandwidth

\[ BW = \frac{1}{2\pi \left\{ C_{gd} \left( R'_L + R_{sig} + R_{sig} R'_L g_m \right) + C_{gs} R_{sig} \right\}} \]

Note that to increase the bandwidth we can:

1. Reduce device capacitances: \( C_{gs} \) & \( C_{gd} \)
2. Reduce source and load resistances
3. Reduce the capacitance which has the biggest effect. In this case \( C_{gd} g_m R'_L R_{sig} \) \((= C_{gd} |A_{mid}| R_{sig})\) dominates, so reducing \( C_{gd} \) as opposed to \( C_{gs} \) would have a bigger payoff.
4. Reducing \( |A_{mid}| = |g_m R'_L| \) will also increase the bandwidth.

NOTE increasing the BW will reduce the gain. We define Gain-BW product.
The Algebra

\[ V_{gs} \left( \frac{1}{R_{sig}} + s(C_{gs} + C_{gd}) \right) = V_{sig} \frac{1}{R_{sig}} + V_a sC_{gd} \]

\[ V_{gs} (g_m - sC_{gd}) + V_a (sC_{gd} + \frac{1}{R_L}) = 0 \]

\[ V_{gs} = -V_a \left( \frac{sC_{gd} + \frac{1}{R_L}}{g_m - sC_{gd}} \right) \]

\[ V_a \left( sC_{gd} + \frac{1}{R_L} \right) \left[ \frac{1}{R_{sig}} + s(C_{gs} + C_{gd}) \right] = V_{sig} \frac{1}{R_{sig}} + V_a sC_{gd} \]

\[ V_a \{ sC_{gd} + \frac{1}{R_L} \} \left[ \frac{1}{R_{sig}} + s(C_{gs} + C_{gd}) \right] = -V_{sig} \frac{1}{R_{sig}} \]

\[ V_a \{ sR_{sig} C_{gd} + \frac{1}{R_L} \} \left[ sRC_{gd} + \frac{1}{R_{sig}} \right] \left[ 1 + sR_{sig} (C_{gs} + C_{gd}) \right] = -V_{sig} \]

\[ V_a \{ sR_{sig} C_{gd} + \frac{1}{R_L} \} \left[ 1 + sR_{sig} (C_{gs} + C_{gd}) \right] = -V_{sig} \left( 1 - \frac{C_{gd}}{g_m} \right) \]

\[ V_{sig} = -R_L g_m s^2 R_{sig} R' L C_{gs} C_{gd} + s(R_{sig} C_{gs} R' L g_m + R' L C_{gd} + R' L C_{gd} + R_{sig} C_{gs} + R_{sig} C_{gd}) + 1 \]

\[ s_{12} = \frac{-R_{sig} C_{gd} R' L g_m + R' L C_{gd} + R_{sig} C_{gs} + R_{sig} C_{gd} \pm \sqrt{(R_{sig} C_{gd} R' L g_m + R' L C_{gd} + R_{sig} C_{gs} + R_{sig} C_{gd})^2 - 4R_{sig} R' L C_{gs} C_{gd}}}{2R_{sig} R' L C_{gs} C_{gd}} \]

\( (s + s_1)(s + s_2) = s^2 + (s_1 + s_2)s + s_1s_2 = s^2 + bs + c \)

\( s_1 + s_2 = b; s_1s_2 = c \)

if \( s_1 >> s_2 \) then \( s_1 + s_2 \approx s_1 = b; s_2 = \frac{c}{s_1} = \frac{c}{b} \)

\[ s_1 = \frac{1}{R_{sig} R' L C_{gs} C_{gd}} \]

\[ s_2 = \frac{1}{R_{sig} R' L C_{gs} C_{gd}} \]

\[ f_1 = \frac{1}{2\pi R_{sig} R' L C_{gs} C_{gd}} \]

\[ f_2 = \frac{1}{2\pi (R_{sig} C_{gd} R' L g_m + R' L C_{gd} + R_{sig} C_{gs} + R_{sig} C_{gd})} \]

and \( f_1 >> f_2 \) and Bandwidth is \( f_2 \)
Homework

- FET
  - Problems: 8.12 \((r_d=40k)\), 13, 14, 16, 17