Frequency Response

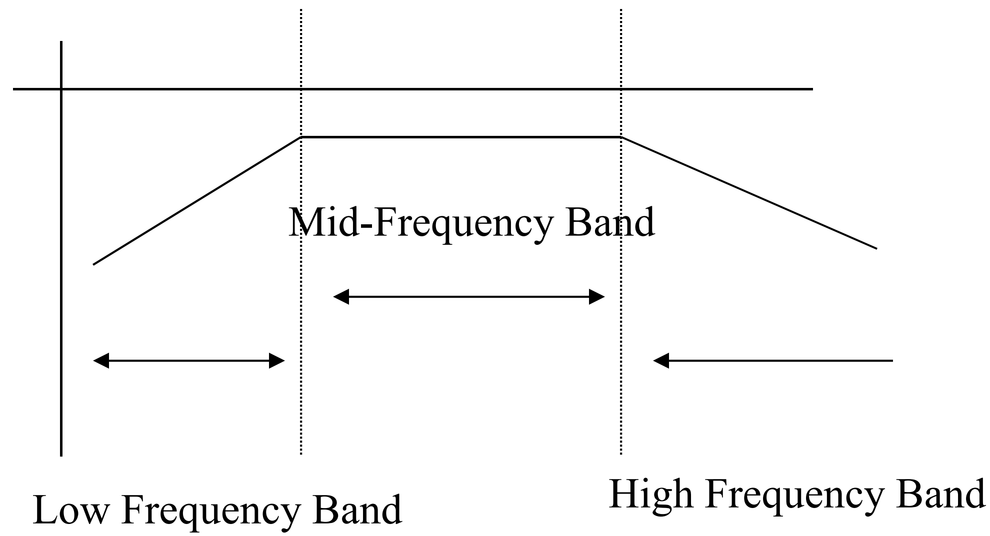
Lesson #12

Small Signal Equivalent Circuits for the BJT

Section 8.4-8.8

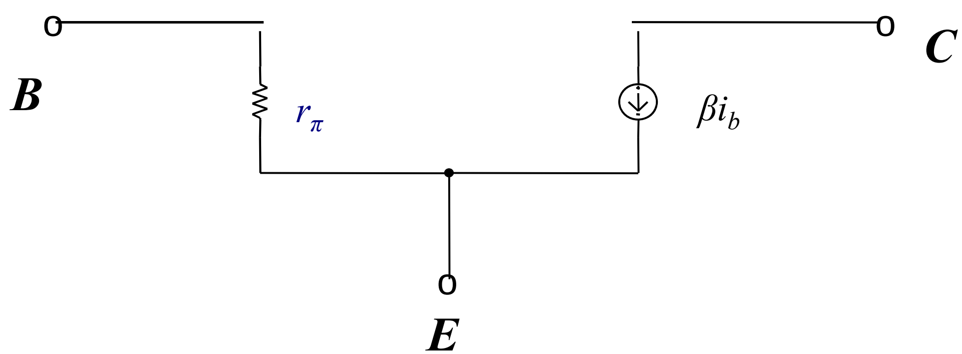
Frequency Response

- The gain of an amplifier is affected by the capacitance associated with its circuit. This capacitance reduces the gain in both the low and high frequency ranges of operation.
- The Bode Plot may look something like this where there is a low frequency band, a midfrequency band and a high frequency band.
- The reduction of gain in the low frequency band is due to the coupling and bypass capacitors selected. They are essentially short circuits in the mid and high bands.
- The reduction of gain in the high frequency band is due to the internal capacitance of the amplifying device, e.g., BJT, FET, etc.. This capacitance is represented by capacitors in the small signal equivalent circuit for these devices. They are essentially open circuits in the low and mid bands.
- First, let's continue to study the small signal equivalent circuits.



Small Signal Equivalent Circuits and Parameters for the BJT
r_π-β *Model*

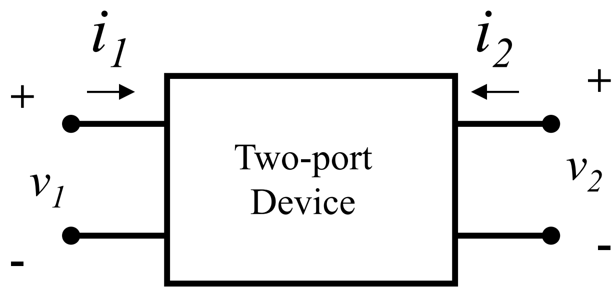
- When the AC Portion of the input is small around the Q point ($\ll V_T$ in value*)* then we can approximate the operation of transistor by an equivalent circuit consisting of a resistor, $r_\pi = V_T/I_{BQ}$ and a current source, βi_b, where i_b is the small signal component of the base current:



- A more thorough Equivalent Circuit may be needed to specify the performance of the BJT

Two-Port Devices and the Hybrid Model

- Generalized model for two-port devices



$$v_1 = h_{11} i_1 + h_{12} v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2$$

11 $\Rightarrow$ input relationship; $h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0}$ = input resistance with output shorted-circuited

22 $\Rightarrow$ output relationship; $h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1=0}$ = output conductance with input open-circuited

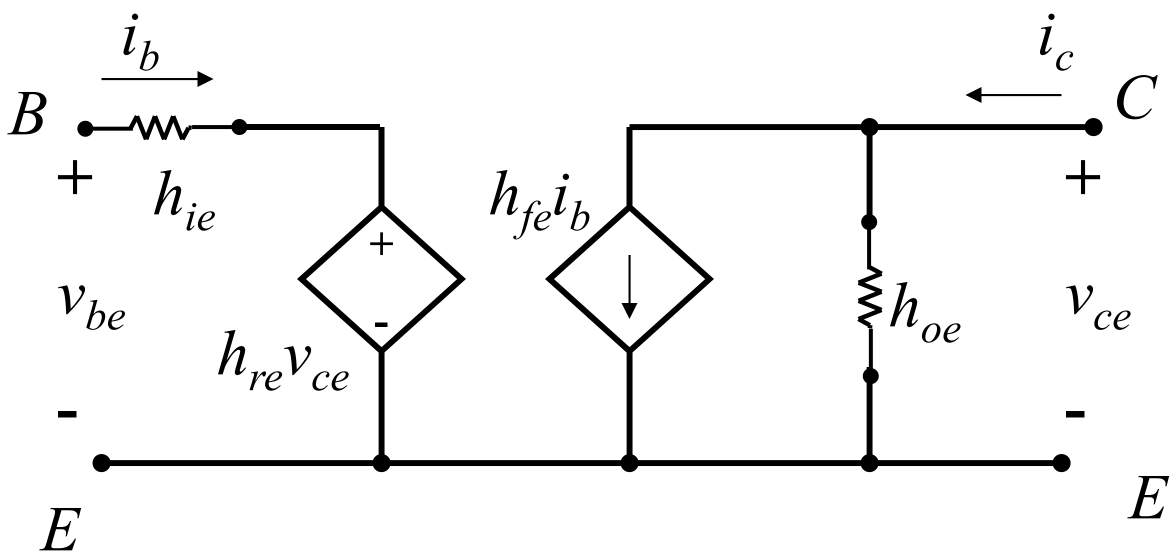
21 $\Rightarrow$ forward transfer relationship; $h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2=0}$ = forward current transfer (or gain) with output short-circuited

(short-ciruit current gain)

12 $\Rightarrow$ reverse transfer relationship; $h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1=0}$ = reverse voltage transfer (or gain) with input open-circuited

(reverse-open-circuit voltage gain)

Hybrid-Parameter Model for the Common Emitter BJT



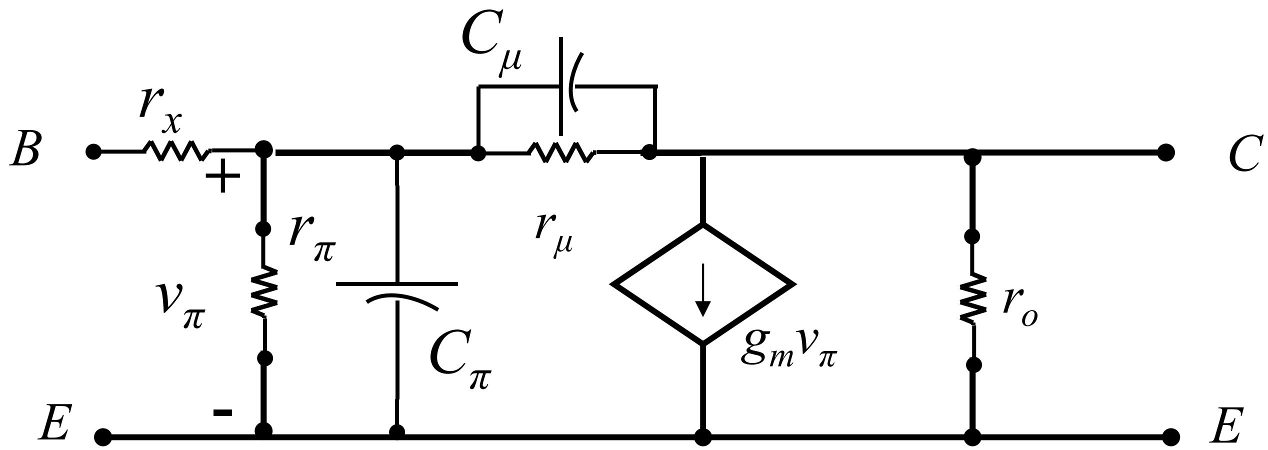
$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$
$$i_c = h_{fe} i_b + h_{oe} v_{ce}$$

The parameters defined by this equivalent circuit as usually provided by transistor manufacturers to describe the performance of the BJT. For example, β and h_{fe} are typically given in BJT data sheets.

Hybrid-π Model for the BJT

- Another model typically specified by BJT manufacturers and is used for frequency analysis
- Includes
  - Resistance to model the base-emitter junction, the base to collector junction, and the collector to emitter path
  - Capacitance to model base-emitter junction and the base to collector junction
  - A dependent (forward) current source in the collector

Hybrid-π Model for the BJT (Continued)



- r_x called the base spreading resistance and represents the resistance of the base-emitter junction
- r_π represents the dynamic resistance for small signal analysis and depends on the Q-point of the design - $r_\pi = V_T/I_{BQ}$
- r_μ represents the feedback from the collector to the base and is related to the hybrid parameter $h_{re} = r_\pi/(r_\pi + r_\mu)$
- r_o represents the resistance from the collector to the emitter and is related to the hybrid parameter $h_{oe} \approx 1/r_o$ and is also related to the EARLY Voltage by VA/I_{CQ}
- C_μ is the depletion capacitance of the collector-base junction
- C_π is the capacitance of the base-emitter junction and depends on the Q-point
- $g_m v_\pi$ is the amplification factor and is equal to $\beta\, i_b$.
- Transition frequency, $f_t = \beta/[2\pi r_\pi(C_\mu + C_\pi)]$, when $|\mathbf{I}_c/\mathbf{I}_b|$ is unity when the collector is grounded for ac.

Example

- The Hybrid-π parameters of a 2N2222A:
  - Q-point => I_{CQ}=10 mA, V_{CEQ}= 10V
  - Assume V_T = 26 mV
  - Average β => 225
  - h_{re} => 4x10^{-4}
  - h_{oe} => *25* mS ~ *200* mS
  - C_μ => 8 pF
  - f_t => 300 MHz (transition frequency)
  - Collector-base time constant:

    $r_x C_\mu$=150 x 10^{-12}

$$f_t \cong \frac{\beta}{2\pi r_\pi (C_\mu + C_\pi)}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{10\text{m}}{26\text{m}} = 0.385\,\text{S},$$

$$r_\pi = \frac{\beta}{g_m} = \frac{225}{.385} = 585\,\Omega,$$

$$r_\mu \cong \frac{r_\pi}{h_{re}} = \frac{585}{4\times 10^{-4}} = 1.5\,\text{M}\Omega,$$

$$r_o \cong \frac{1}{h_{oe}} \Rightarrow 5\,\text{k}\Omega \sim 40\,\text{k}\,\Omega$$

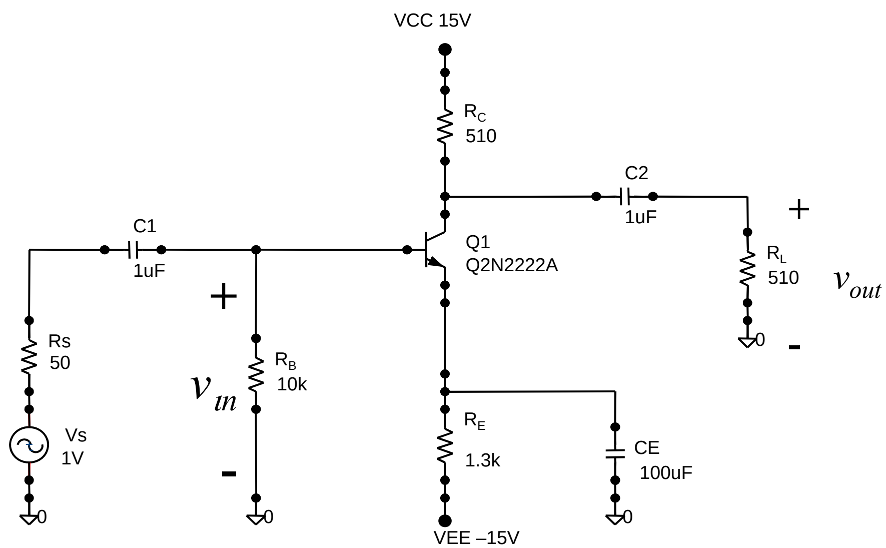
$$r_{o\ \text{AVG}} = 22.5\,\text{k}\Omega,$$

$$C_\pi \cong \frac{\beta}{2\pi r_\pi f_t} - C_\mu = \frac{225}{2\pi \times 585 \times 300 \times 10^6} - 8\text{pF}$$

$$\cong 196\text{pF}$$

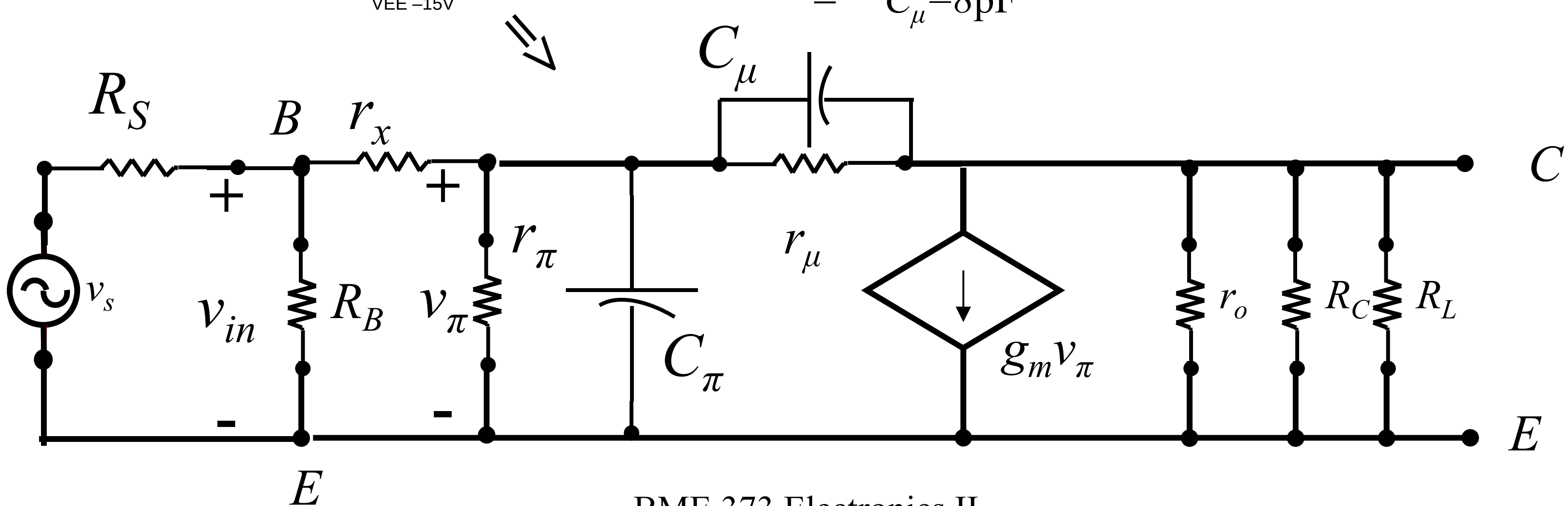
$$r_x = \frac{150\times 10^{-12}}{C_\mu} = \frac{150\times 10^{-12}}{8} = 19\,\Omega$$

Analysis of CE at High Frequencies

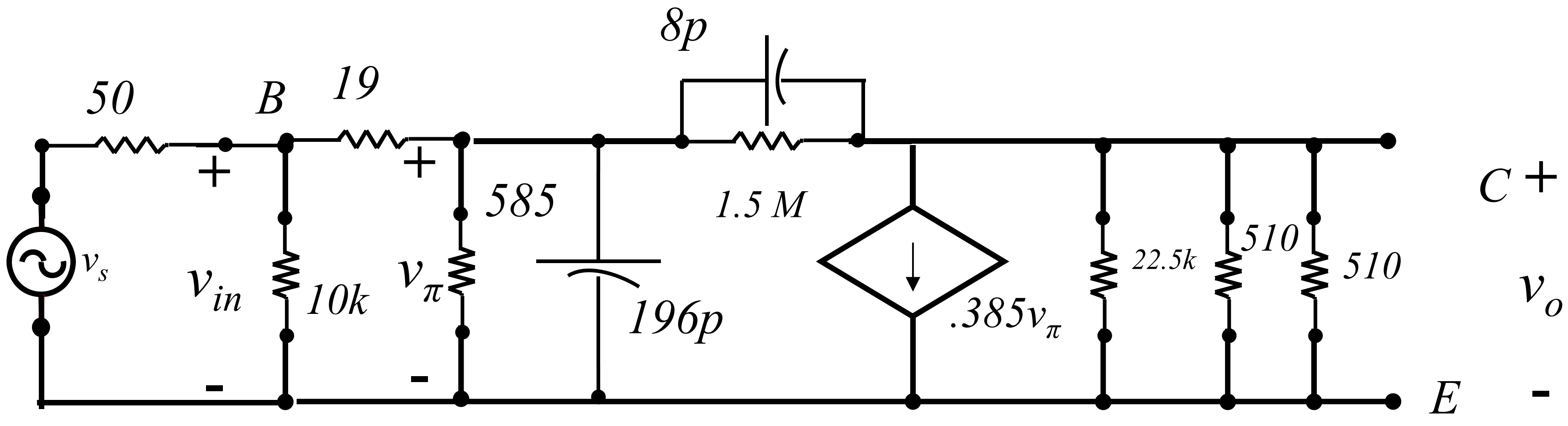


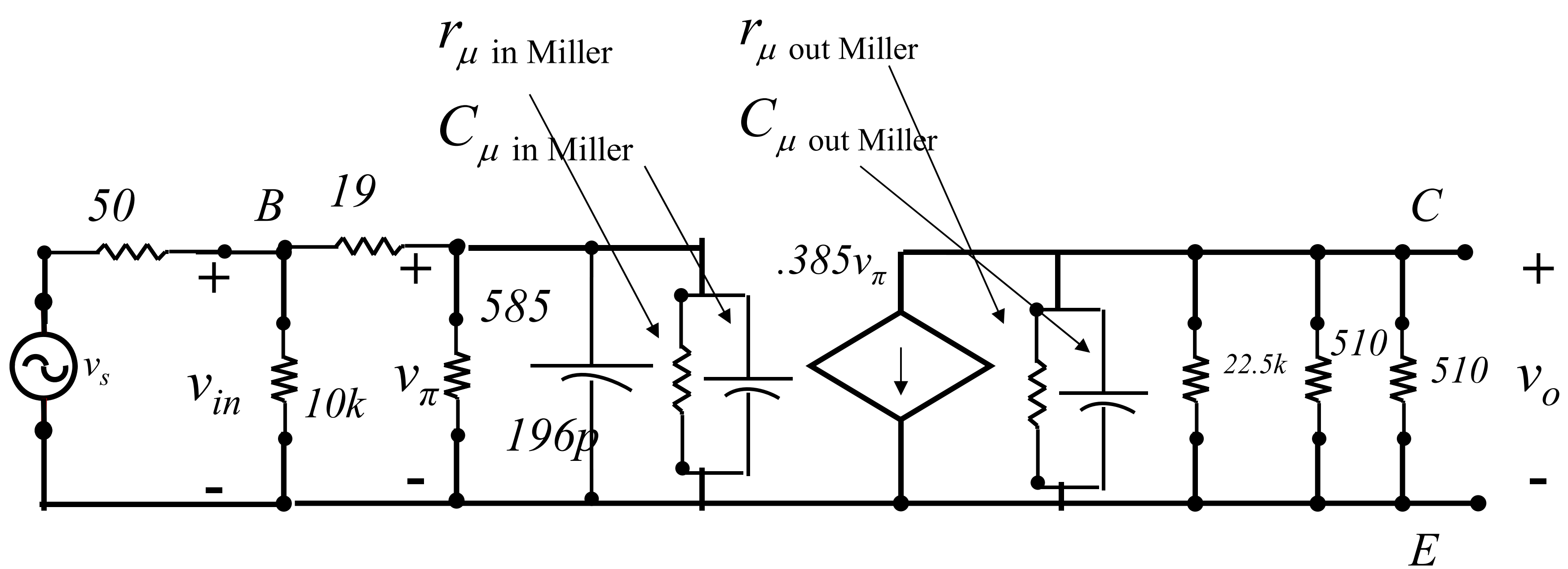
The Hybrid-π parameters :

- Q-point => I_{CQ}=10 mA, V_{CEQ}= 10V
- Assume V_T = 26 mV
- g_m=.385 S
- r_x=19 Ω
- r_π=585 Ω
- r_o=22.5k Ω
- r_μ=1.5 M Ω
- C_π=196pF
- C_μ=8pF

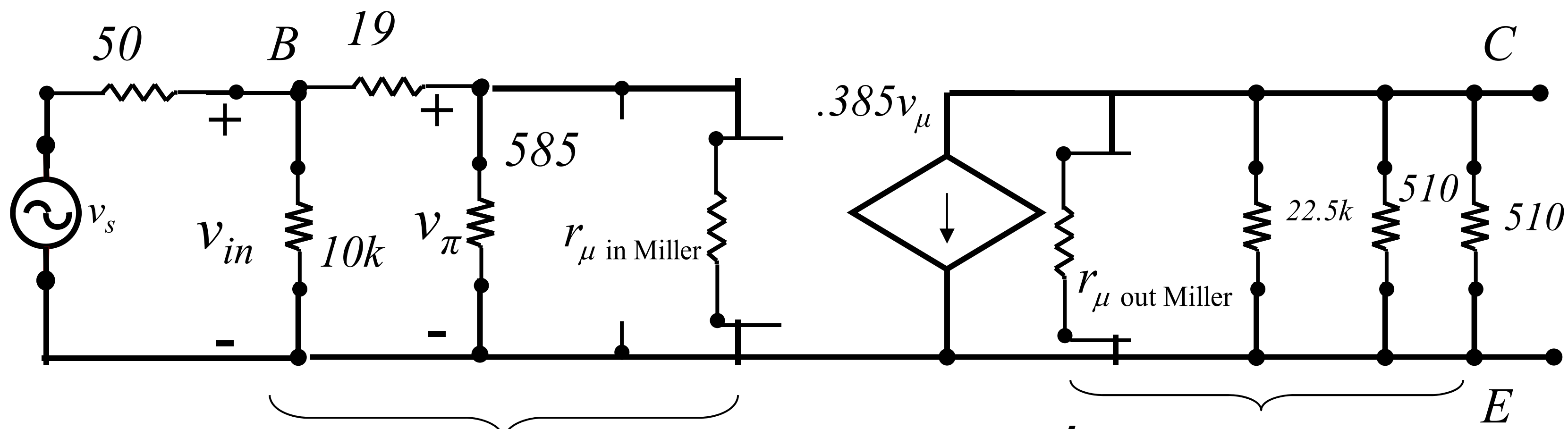


Example Using the Miller Effect





Example Calculation of the Miller Parameters and the Midband Gain



R'_s=[10k||(19+r_π||$r_{\mu INMILLER}$)]=550

R'_L=22.5k||510||510||$r_{\mu OUTMILLER}$=252

Miler Effect Parameters

$r_{\mu out\ MILLER}$=1.5M*A_v/(A_v-1) ≈ 1.5M

A_v= v_o /v_π=-$g_m R'_L$=-.385*252=-97

$r_{\mu in\ MILLER}$=1.5M/(1-A_v)=1.5M/(1+$g_m R'_L$) ≈ 15.k

Calculations

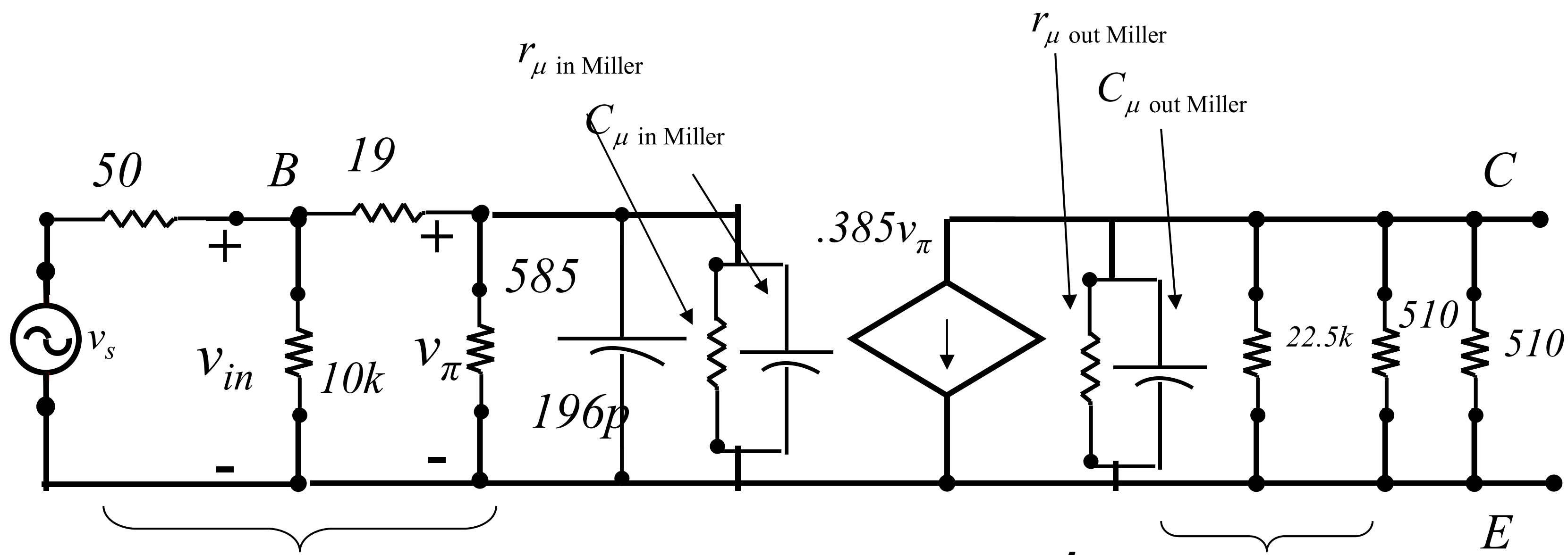
r_π||$r_{\mu in\ MILLER}$ = 563

v_π/v_{in} =r_π||$r_{\mu in\ MILLER}$ / ((r_π||$r_{\mu in\ MILLER}$)+r_x) =.97

v_{in}/v_s=R'_s/(R'_s+50)=0.92

A_{vs}= v_o/v_s= (v_o/v_π)(v_π/v_{in}) (v_{in}/v_s)=-97(.92)(.97)=-86.1

Example Calculation of the Break Frequencies



$R'_s=[(50||10k)+19]||585||r_{\mu INMILLER}=61.3$

$R'_L=22.5k||510||510||r_{\mu OUTMILLER}=252$

Miler Effect Parameters

$r_{\mu in\ MILLER}=15.k$

$r_{\mu out\ MILLER}= 1.5M$

$C_{\mu in\ MILLER}=8p*(1-A_v) \approx 784\ pF$

$C_{\mu out\ MILLER}=8p*(A_v-1)/Av \approx 8.08\ pF$

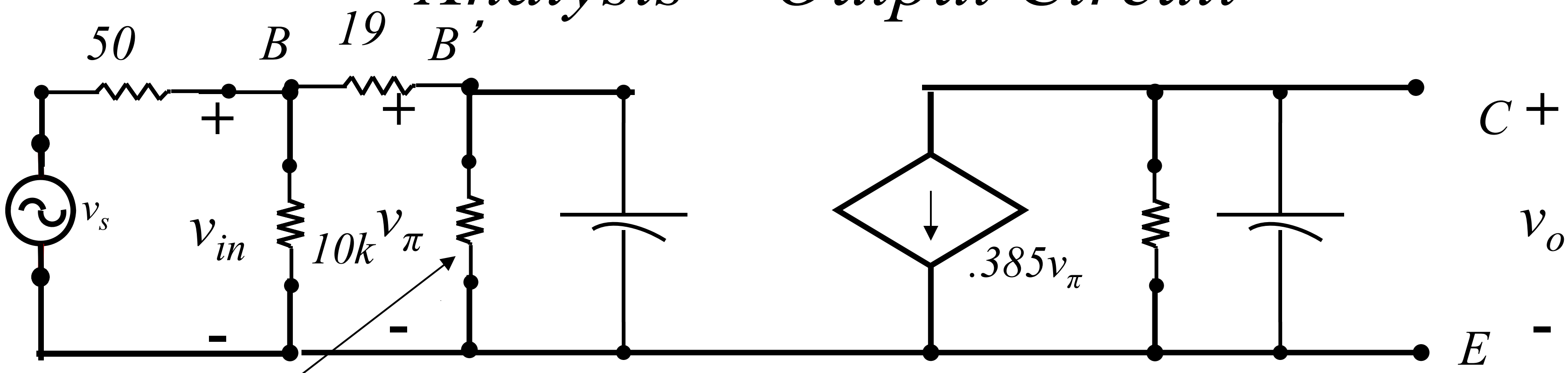
Calculations

$C_T=196+C_{\mu inMiller}= 980\ pF$

$f_{b\ in}=1/(2\pi R'_s C_T)=1/(2\pi*61.3*980x10^{-12})=2.65\ MHz$

$f_{b\ out}=1/(2\pi R'_L C_{\mu out\ Miller})=1/(2\pi*252*8.08x10^{-12})=78.1\ MHz$

Example Alternative Method Using Circuit Analysis - Output Circuit



$R_{B'}=r_{\mu INMILLER} \| r_\pi$ = 15k‖585 =563

$C_T=C_{\mu inmiller}+C_{gs}$
$=784p+196p=980pf$

$R'_L=252$ $C_{\mu outmiller}=8p$

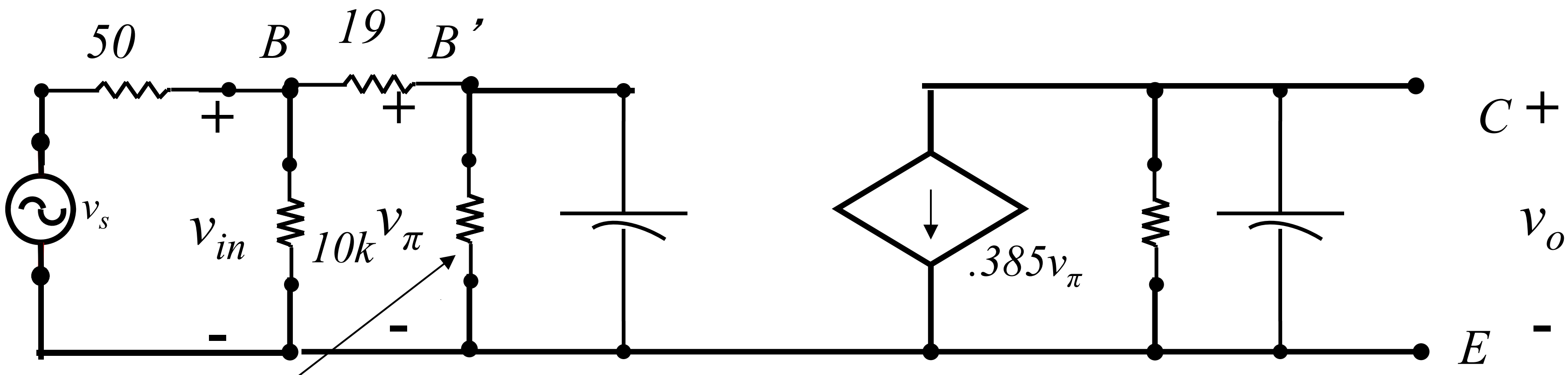
Output Pole Frequency

$$\frac{v_o}{v_\pi} = -g_m R'_L \| Z_{C_{outmiller}} = -.385 \frac{252 \times \frac{1}{j\omega 8\times 10^{-12}}}{252 + \frac{1}{j\omega 8\times 10^{-12}}}$$

$$= -.385 \frac{252}{1 + j\omega 8\times 10^{-12}\times 252} = -.385 \frac{252}{1+j\omega 2.03\times 10^{-9}}$$

$$f_{bout} = \frac{1}{2\pi 2.03\times 10^{-9}} = 78.1 MHz$$

Example Alternative Method Using Circuit Analysis - Input Circuit



$R_{B'} = r_{\mu INMILLER} || r_\pi$ = 15k||585=563

$C_T = C_{\mu inmiller} + C_{gs}$

$= 784p + 196p = 980pf$

R'_L=252 $C_{\mu outmiller} = 8p$

Input Pole Frequency

$$\frac{v_\pi}{v_S} = \frac{v_\pi}{v_B} \times \frac{v_B}{v_s}$$

$$\frac{v_\pi}{v_B} = \frac{R_{B'} \,||\, Z_{C_T}}{R_{B'} \,||\, Z_{C_T} + r_x}$$

$$R_{B'} \,||\, Z_{C_T} = \frac{R_{B'} \times \frac{1}{j\omega C_T}}{R_{B'} + \frac{1}{j\omega C_T}} = \frac{R_{B'}}{1 + j\omega C_T R_{B'}}$$

$$\frac{v_\pi}{v_B} = \frac{\frac{R_{B'}}{1 + j\omega C_T R_{B'}}}{\frac{R_{B'}}{1 + j\omega C_T R_{B'}} + r_x} = \frac{R_{B'}}{r_x(1 + j\omega C_T R_{B'}) + R_{B'}}$$

$$= \frac{R_{B'}}{r_x + R_{B'} + j\omega C_T R_{B'} r_x} = \frac{R_{B'}}{r_x + R_{B'}} \frac{1}{1 + j\omega C_T \frac{R_{B'} r_x}{r_x + R_{B'}}}$$

Input Pole Frequency

$$\frac{v_B}{v_s} = \frac{R_B \,||\, (r_x + R_{B'} \,||\, Z_{C_T})}{R_B \,||\, (r_x + R_{B'} \,||\, Z_{C_T}) + R_S}$$

$$(r_x + R_{B'} \,||\, Z_{C_T}) = r_x + \frac{R_{B'}}{1 + j\omega C_T R_{B'}} = \frac{r_x(1 + j\omega C_T R_{B'}) + R_{B'}}{1 + j\omega C_T R_{B'}} = \frac{r_x + R_{B'} + j\omega C_T R_{B'} r_x}{1 + j\omega C_T R_{B'}}$$

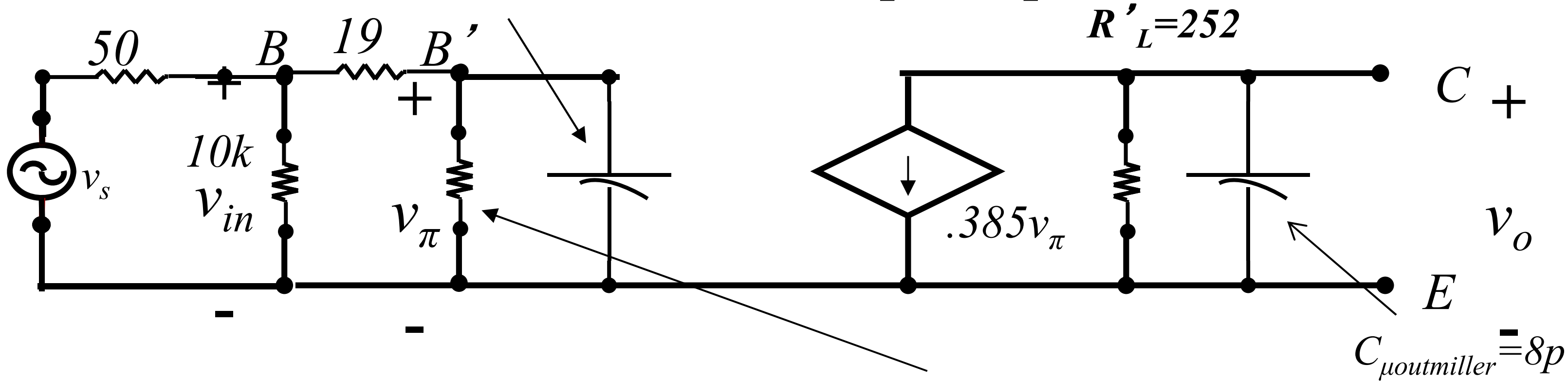
$$R_B \,||\, (r_x + R_{B'} \,||\, Z_{C_T}) = R_B \,||\, \frac{r_x + R_{B'} + j\omega C_T R_{B'} r_x}{1 + j\omega C_T R_{B'}} = \frac{R_B \times \frac{r_x + R_{B'} + j\omega C_T R_{B'} r_x}{1 + j\omega C_T R_{B'}}}{R_B + \frac{r_x + R_{B'} + j\omega C_T R_{B'} r_x}{1 + j\omega C_T R_{B'}}}$$

$$= \frac{R_B \times (r_x + R_{B'} + j\omega C_T R_{B'} r_x)}{R_B(1 + j\omega C_T R_{B'}) + (r_x + R_{B'} + j\omega C_T R_{B'} r_x)} = \frac{R_B (r_x + R_{B'})(1 + j\omega C_T \frac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B + r_x + R_{B'} + j\omega C_T (R_{B'} R_B + R_{B'} r_x)}$$

$C_T = C_{\mu inmiller} + C_{gs}$

$= 784p + 196p = 980pf$

Example Input Circuit Cont'd



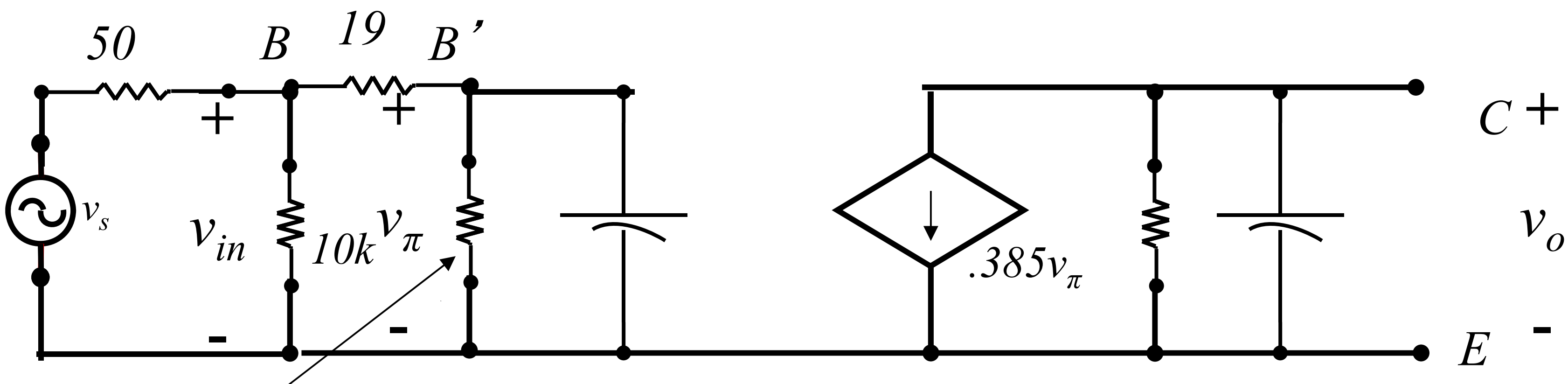
$R_{B'} = r_{\mu INMILLER} \| r_\pi = 15k \| 585 = 563$

$$\frac{v_B}{v_s} = \frac{R_B \| (r_x + R_{B'} \| Z_{C_T})}{R_B \| (r_x + R_{B'} \| Z_{C_T}) + R_S} = \frac{\dfrac{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B + r_x + R_{B'} + j\omega C_T (R_{B'} R_B + R_{B'} r_x)}}{\dfrac{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B + r_x + R_{B'} + j\omega C_T (R_{B'} R_B + R_{B'} r_x)} + R_S}$$

$$= \frac{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})}) + R_S (R_B + r_x + R_{B'} + j\omega C_T (R_{B'} R_B + R_{B'} r_x))} = \frac{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B(r_x + R_{B'}) + j\omega C_T R_{B'} r_x R_B) + R_S (R_B + r_x + R_{B'} + j\omega C_T (R_{B'} R_B + R_{B'} r_x))}$$

$$= \frac{R_B(r_x + R_{B'})(1 + j\omega C_T \dfrac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B(r_x + R_{B'}) + R_S (R_B + r_x + R_{B'}) + j\omega C_T (R_{B'} r_x R_B + R_S (R_{B'} R_B + R_{B'} r_x))}$$

Example Input Circuit Cont'd



$R_{B'}$=$r_{\mu INMILLER}$|| r_π = 15k||585 =563

C_T=$C_{\mu inmiller}$ +C_{gs} =784p+196p=980pf

R'$_L$=252 *$C_{\mu outmiller}$=8p*

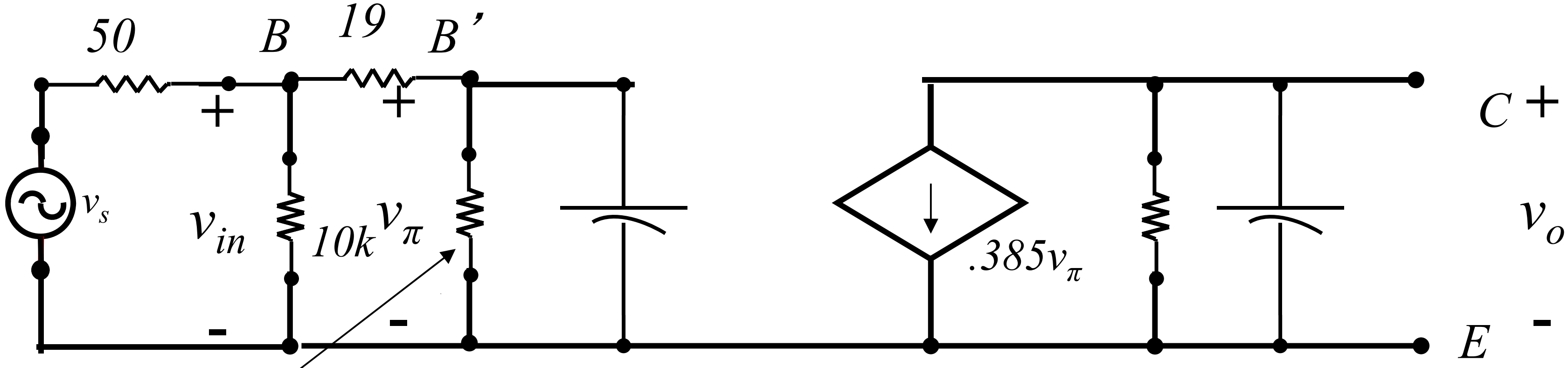
Input Pole Frequency

$$\frac{v_\pi}{v_S} = \frac{v_\pi}{v_B} \times \frac{v_B}{v_s} = \frac{R_{B'}}{r_x + R_{B'}} \frac{1}{1 + j\omega C_T \frac{R_{B'} r_x}{r_x + R_{B'}}} \times \frac{R_B(r_x + R_{B'})(1 + j\omega C_T \frac{R_{B'} r_x}{(r_x + R_{B'})})}{R_B(r_x + R_{B'}) + R_S(R_B + r_x + R_{B'}) + j\omega C_T (R_{B'} r_x R_B + R_S(R_{B'} R_B + R_{B'} r_x))}$$

$$= \frac{R_{B'} R_B}{R_B(r_x + R_{B'}) + R_S(R_B + r_x + R_{B'}) + j\omega C_T (R_{B'} r_x R_B + R_S(R_{B'} R_B + R_{B'} r_x))} = \frac{\frac{R_B R_{B'}}{R_B r_x + R_B R_{B'} + R_S R_B + R_S r_x + R_S R_{B'}}}{1 + j\omega C_T (\frac{R_{B'} r_x R_B + R_S R_{B'} R_B + R_{B'} r_x R_S}{R_B r_x + R_B R_{B'} + R_S R_B + R_S r_x + R_S R_{B'}}))}$$

$$f_{bin} = \frac{1}{2\pi 6 \times 10^{-8}} = 2.65MHz$$

Example Input Circuit using Thevenin's Theorem



$R_{B'}=r_{\mu INMILLER} \| r_\pi$ = 15k‖585

=563

$C_T=C_{\mu nmiller}+C_{gs}$

=784p+196p=980pf

R'$_L$=252 $C_{\mu outmiller}$=8p

Thevenin's method for Input Pole

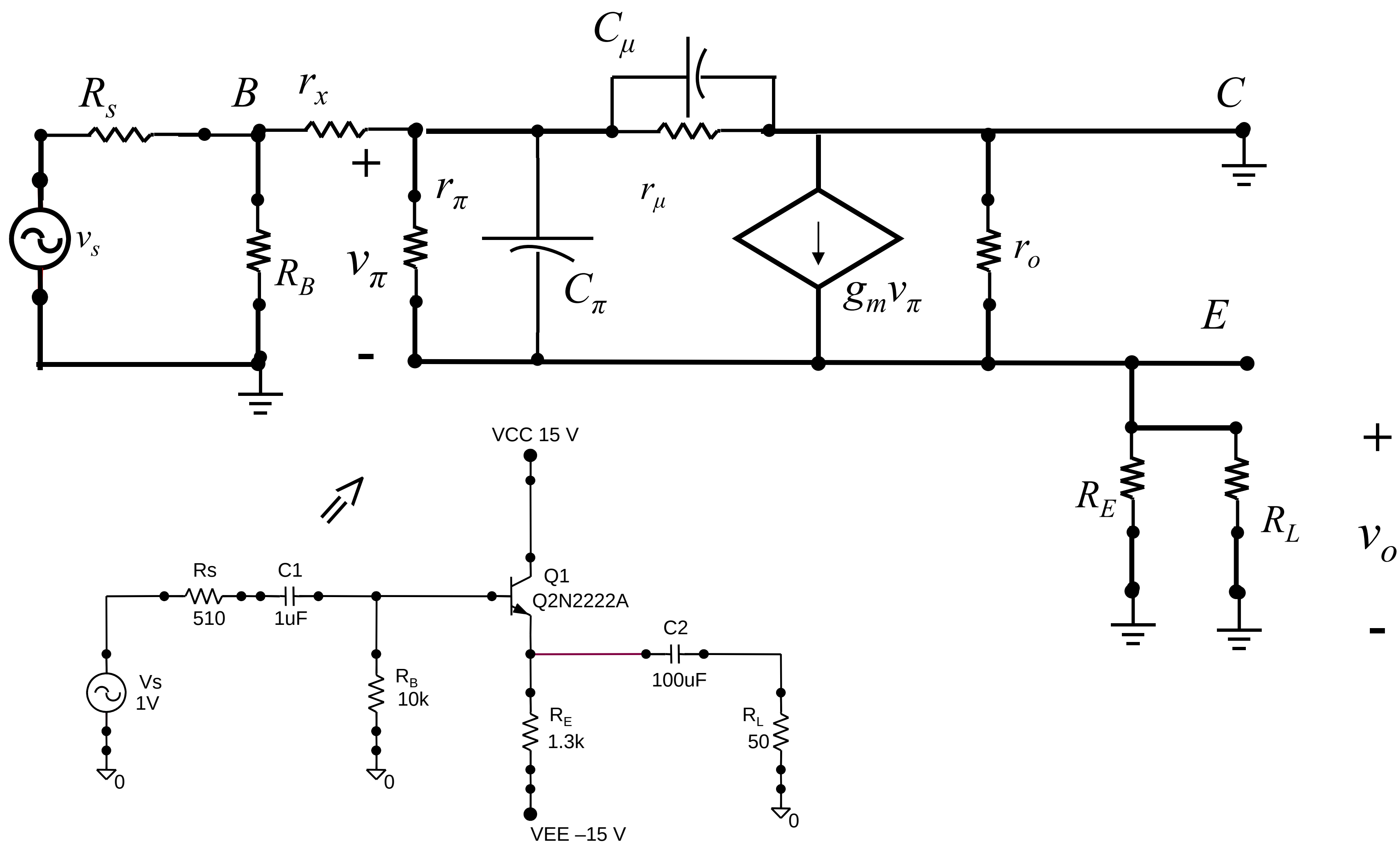
$$R_T = (R_S \| R_B + r_x) \| R_{B'}$$

$$= (\frac{R_S R_B}{R_S + R_B} + r_x) \| R_{B'} = (\frac{R_S R_B + r_x R_S + r_x R_B}{R_S + R_B}) \| R_{B'}$$

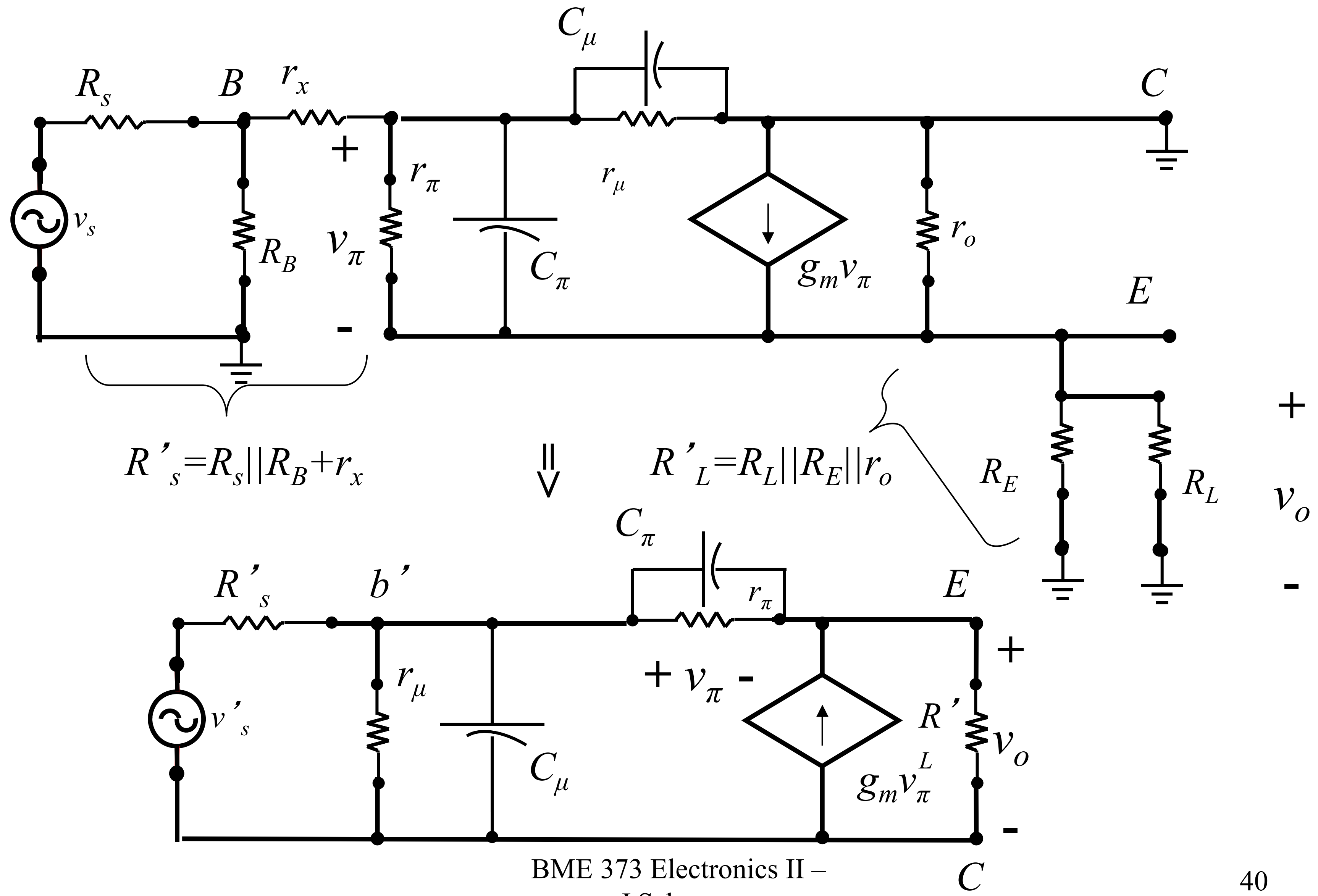
$$= \frac{(\frac{R_S R_B + r_x R_S + r_x R_B}{R_S + R_B}) R_{B'}}{(\frac{R_S R_B + r_x R_S + r_x R_B}{R_S + R_B}) + R_{B'}} = \frac{R_S R_B R_{B'} + r_x R_S R_{B'} + r_x R_B R_{B'}}{R_S R_B + r_x R_S + r_x R_B + R_{B'} R_S + R_{B'} R_B}$$

$$f_{bin} = \frac{1}{2\pi C_T R_T} = 2.65MHz$$

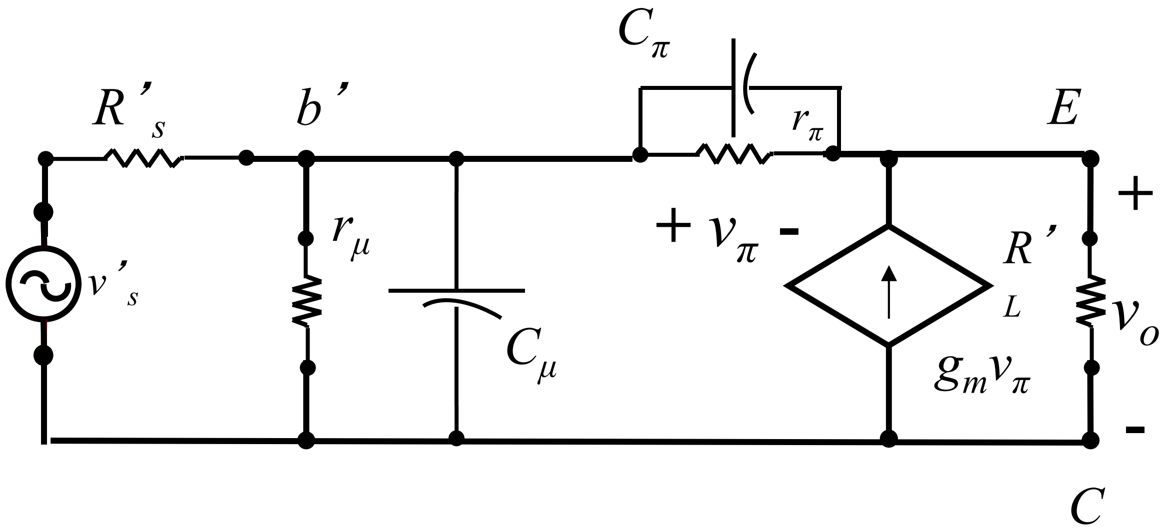
Emitter Follower



Emitter Follower (Continued)



Emitter Follower (Continued)



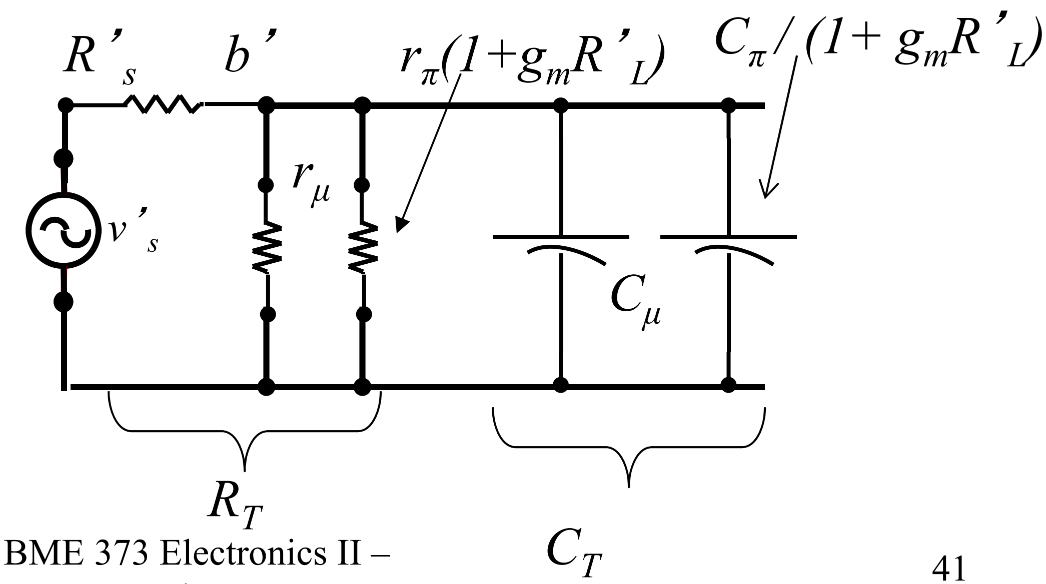
Miller Effect Calculations for the Input

$$v_o = g_m R'_L v_\partial$$

$$v_{b'} = v_\partial + v_o = (1 + g_m R'_L) v_\partial$$

$$I_{fIN} = \frac{V_f}{Z_f} = \frac{v_\pi}{Z_f} = \frac{v_{b'}}{Z_f(1 + g_m R'_L)}$$

$$Z_{\text{in Miller}} = Z_f (1 + g_m R'_L)$$



Example

The Hybrid-π parameters :

- Q-point => I_{EQ}=10.6 mA, V_{CEQ}= 15V
- Assume V_T = 26 mV
- $\beta=225$
- $g_m=.385\ S$
- $r_x=19\ \Omega$
- $r_\pi=585\ \Omega$
- $r_o=22.5k\ \Omega$
- $r_\mu=1.5\ \mathrm{M}\ \Omega$
- $C_\pi=196$pF
- $C_\mu=8$pF

The Circuit parameters :

- $R_S=510$
- $R_B=10\ k$
- $R_E=1.3\ k$
- $R_L=50$

Calculations:

- $R'_s=R_s||R_B+r_x= 510||10k+19=504$
- $R'_L=R_L||R_E||r_o= 50||1.3k||22.5k=48$
- $R_T=R'_S||r_\mu||r_\pi(1+g_mR'_L)=483$
- $C_T=C_\mu+C_{p\pi}/(1+g_mR'_L)= 18.1\ pF$
- $f_b=1/(2\pi R_TC_T)=18.2\ MHz$

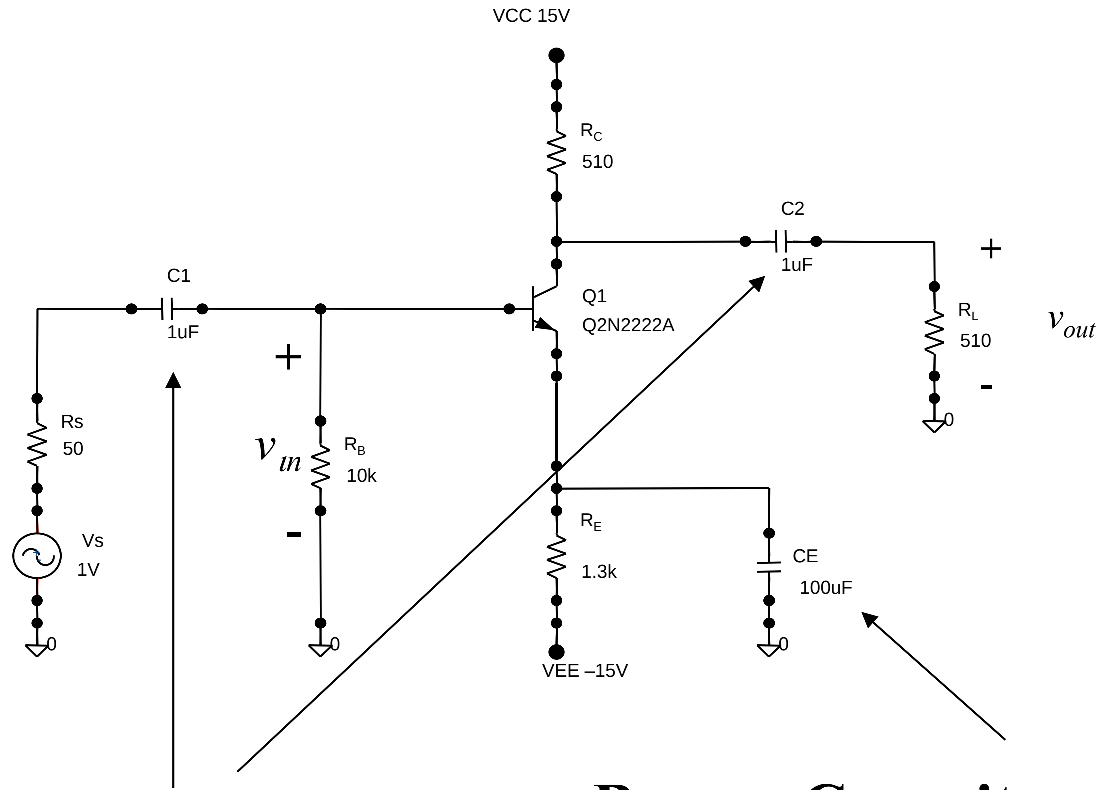
Recall for an emitter follower:

- $A_v=(\beta+1)R'_L/[r_\pi+(\beta+1)R'_L]=.949$
- $R_{in}=R_B||[r_\pi+(\beta+1)R'_L]= 5.33\ k\Omega$
- $A_{vs}=A_vR_{in}/(R_{in}+R_S)= 8.66$

Low Frequency Response of RC-Coupled Amplifiers

- Coupling Capacitors
  - To couple the various stages of a multi-stage amplifier
    - For AC performance essentially a short circuit and AC current flows from one stage to the next stage
  - To support the biasing of each stage individually:
    - For DC performance: open circuit and no biasing current flows from one stage to another
- ByPass Capacitors
  - To support the addition of a resistor for biasing purposes only
    - For DC performance: open circuit and current flows through the biasing resistor
  - Short-circuit the biasing resistor for AC performance.
    - For AC performance: short circuit and no current flows through the resistor (shorted out/bypassed)

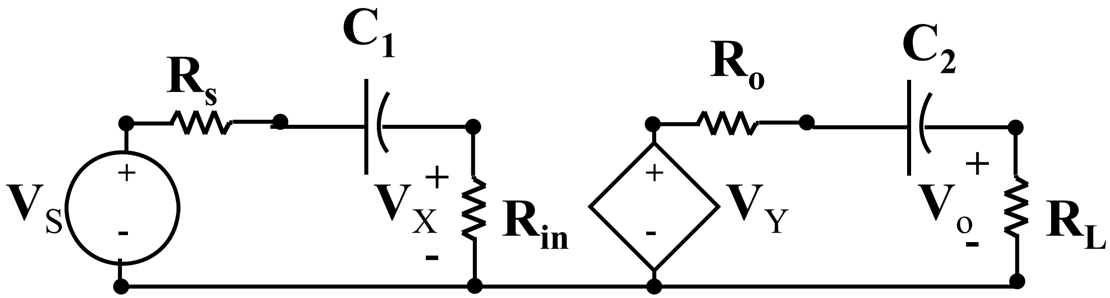
Capacitors



Coupling Capacitors

Bypass Capacitors

Coupling Capacitors



$V_Y = A_{vo}\, V_X$

$$A_{vs} = \frac{\mathbf{V}_O}{\mathbf{V}_S} = \frac{\mathbf{V}_X}{\mathbf{V}_S} \times \frac{\mathbf{V}_Y}{\mathbf{V}_X} \times \frac{\mathbf{V}_O}{\mathbf{V}_Y}$$

$$\frac{\mathbf{V}_X}{\mathbf{V}_S} = \frac{R_{in}}{R_{in} + R_S + \frac{1}{j\omega C_1}} = \frac{j\omega C_1 R_{in}}{1 + j\omega C_1 (R_{in} + R_S)}$$

$$= \frac{R_{in}}{R_{in} + R_S} \times \frac{j(f/f_1)}{1 + j(f/f_1)},$$

$$f_1 = \frac{1}{2\pi (R_S + R_{in}) C_1}$$

$$\frac{\mathbf{V}_O}{\mathbf{V}_Y} = \frac{R_L}{R_L + R_o + \frac{1}{j\omega C_2}} = \frac{R_L}{R_L + R_o} \times \frac{j(f/f_2)}{1 + j(f/f_2)},$$

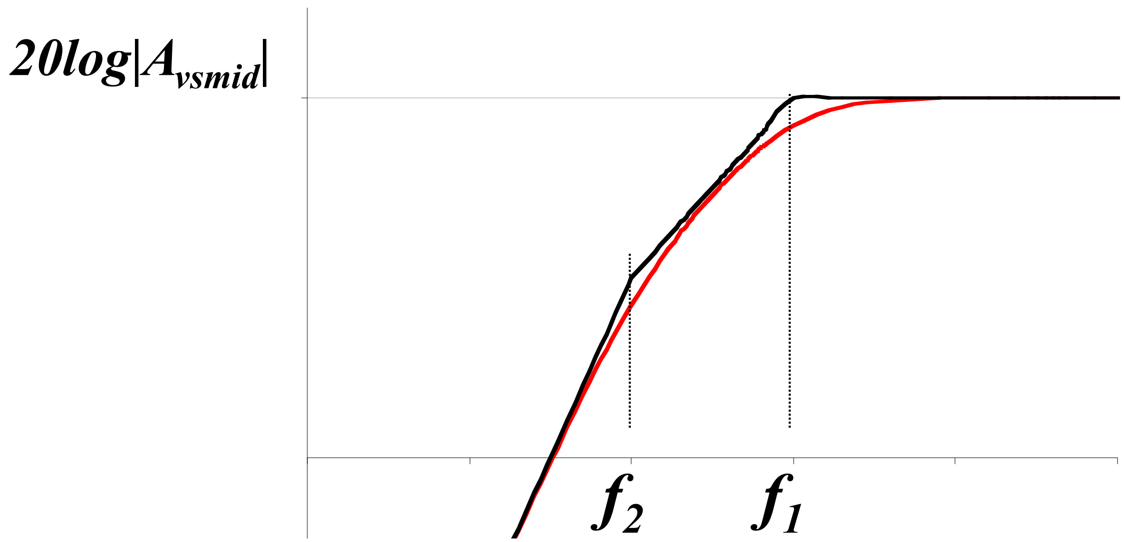
$$f_2 = \frac{1}{2\pi (R_o + R_L) C_2},$$

$$\frac{\mathbf{V}_X}{\mathbf{V}_S} = A_{vo},$$

$$A_{vs} = \frac{R_{in}}{R_{in} + R_S} \times \frac{j(f/f_1)}{1 + j(f/f_1)} \times A_{vo} \times \frac{R_L}{R_L + R_o} \times \frac{j(f/f_2)}{1 + j(f/f_2)}$$

$$A_{vs} = A_{vsmid} \times \frac{j(f/f_1)}{1 + j(f/f_1)} \times \frac{j(f/f_2)}{1 + j(f/f_2)}$$

$$A_{vsmid} = \frac{R_{in}}{R_{in} + R_S} \times A_{vo} \times \frac{R_L}{R_L + R_o}$$



Bypass Capacitors

- The value of a bypass capacitor is chosen to provide a short circuit at a frequency sufficiently low than the band pass of amplifier design
- For a emitter follower, it can be shown

$$f_1=1/(2\pi R'_E C_E)$$

where

R'_E is the equivalent resistance reflected into the emitter circuit

Homework

- Hybrid-π
  - Problems: 8.28,31
- Common Emitter
  - Problems: 8.40-41
- Emitter Follower
  - Problems: 8.56-7
- Low Frequency Response
  - Problems: 8.60-2