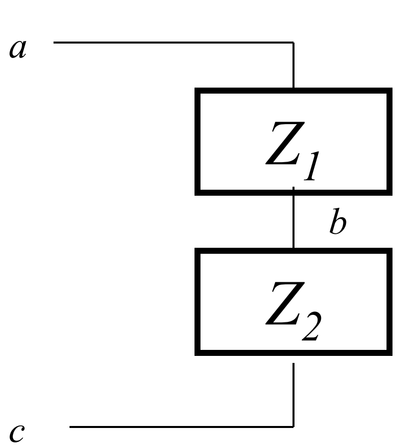
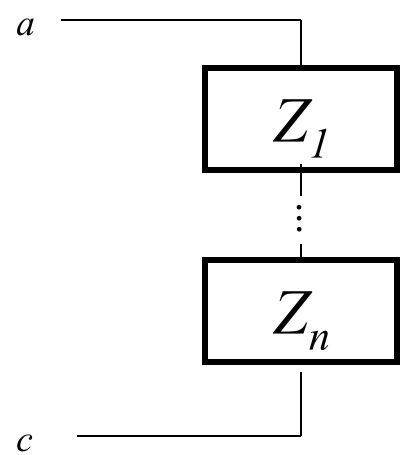
Circuit Analysis

Lecture 2

Voltage Division

- The voltage across impedances in series divides in proportion to the impedances.





$$\mathbf{V}_{ac} = \mathbf{V}_{ab} + \mathbf{V}_{bc} = \mathbf{I}(Z_1 + Z_2); \text{KVL} + \text{Ohm's Law}$$

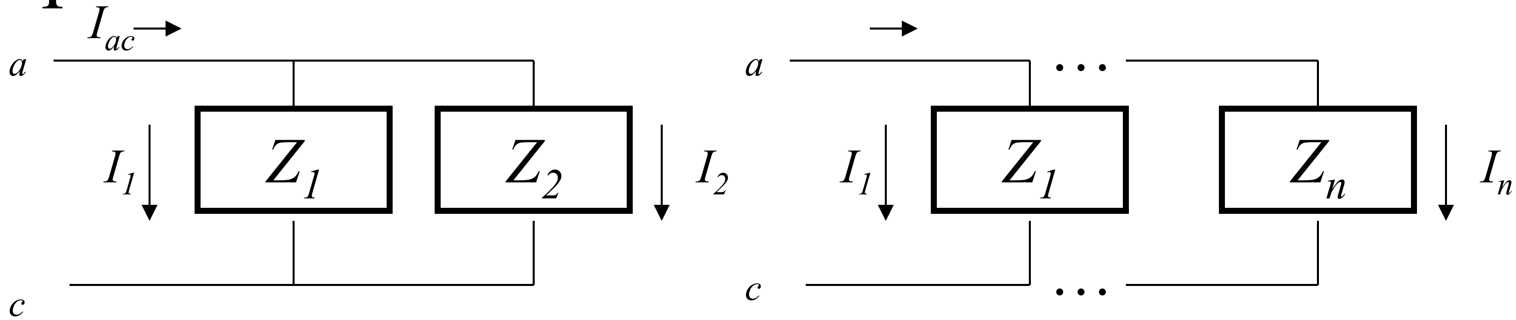
$$\mathbf{V}_{bc} = \mathbf{I}Z_2$$

$$\frac{\mathbf{V}_{bc}}{\mathbf{V}_{ac}} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbf{V}_i}{\mathbf{V}_{ac}} = \frac{Z_i}{Z_1 + Z_2 + \cdots Z_n}$$

Current Division

- The current into impedances in parallel divides in proportion to the inverse of the impedances.



$$\mathbf{I}_{ac} = \mathbf{I}_1 + \mathbf{I}_2 = \mathbf{V}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right); \text{KCL} + \text{Ohm's Law}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{Z_1}$$

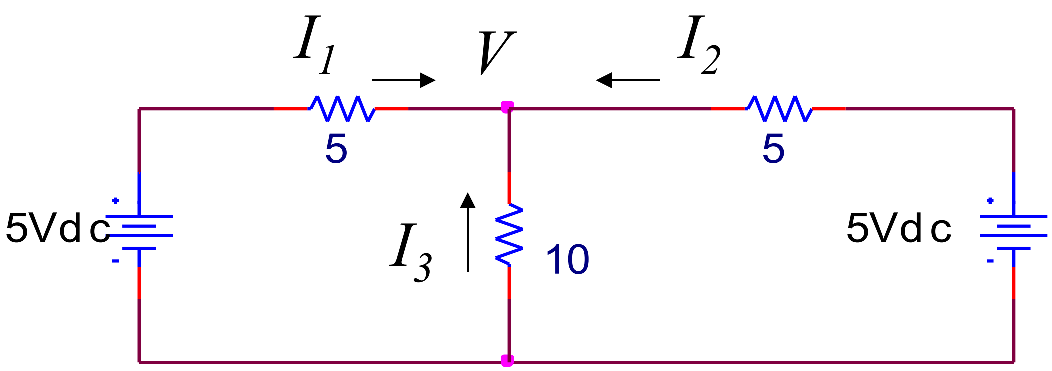
$$\frac{\mathbf{I}_1}{\mathbf{I}_{ac}} = \frac{1/Z_1}{(1/Z_1) + (1/Z_2)} = \frac{Z_2}{Z_1 + Z_2}$$

$$\frac{\mathbf{I}_i}{\mathbf{I}_{ac}} = \frac{(1/Z_i)}{(1/Z_1) + (1/Z_2) + \cdots + (1/Z_n)}$$

Nodal Analysis

1. Define a voltage at each node (junction point) of a network. For example, in a 5 node network, define 5 voltage unknowns.
2. Using KCL, write an equation for each node using the unknown voltages. In our 5 node example, you'll have 5 equations and 5 unknown voltage.
3. Solve for the unknown voltages and now apply these voltages to the network to find the currents for each impedance in the network.

Nodal Analysis Example

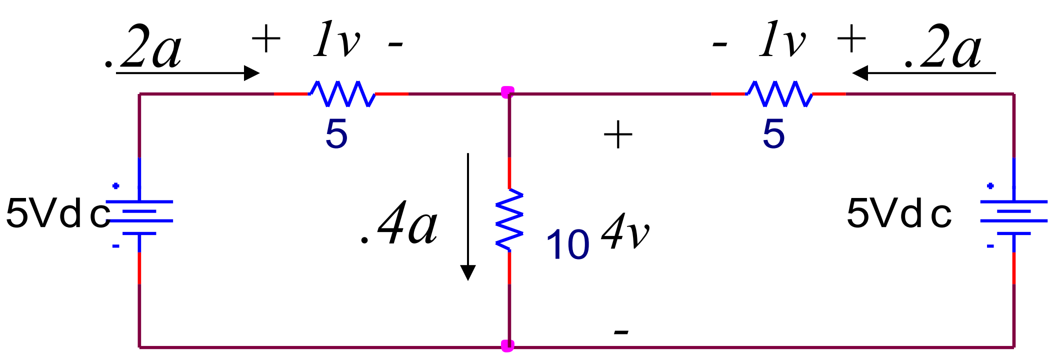


Node 1

$$I_1 + I_2 + I_3 = 0$$

$$\frac{5-V}{5} + \frac{5-V}{5} - \frac{V}{10} = 0;\ 2 = \frac{V}{2}$$

$$V = 4$$



$$I_1 = \frac{5-V}{5} = \frac{1}{5}$$

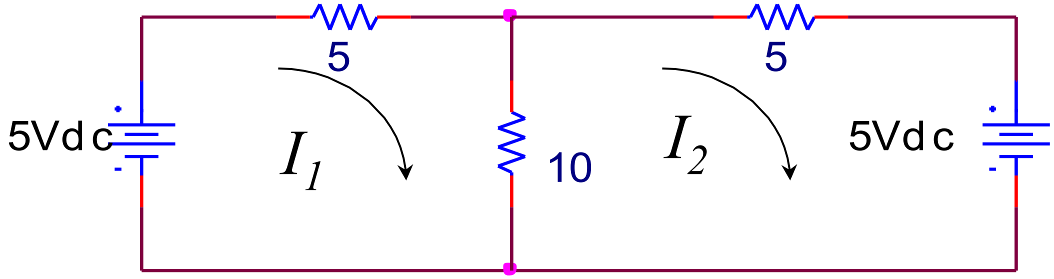
$$I_2 = \frac{5-V}{5} = \frac{1}{5}$$

$$I_3 = \frac{-V}{10} = -\frac{4}{10} = -\frac{2}{5}$$

Mesh Analysis

1. Define a current in each mesh (loop) of a network. For example, in a 5 mesh network, define 5 current unknowns.
2. Using KVL, write an equation for each mesh using the unknown currents. In our 5 mesh example, you'll have 5 equations and 5 unknown currents.
3. Solve for the unknown currents and now apply these currents to the network to find the voltages for each impedance in the network.

Mesh Analysis Example



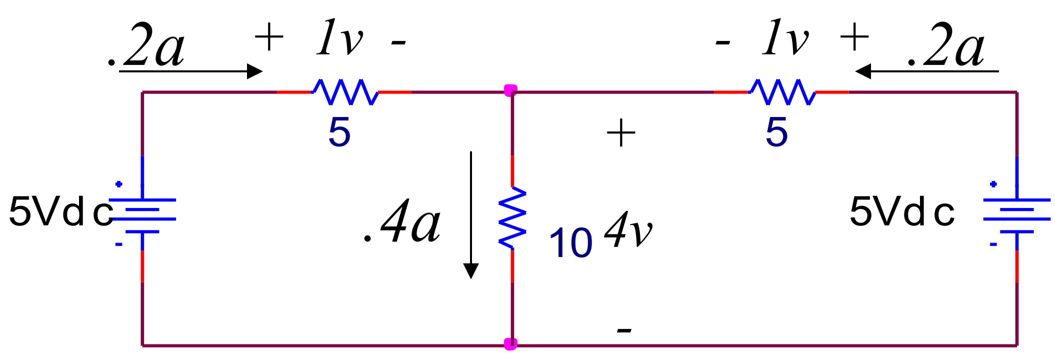
Mesh 1

$$0 = I_1 5 + 10(I_1 - I_2) - 5;\ 5 = 15I_1 - 10I_2$$

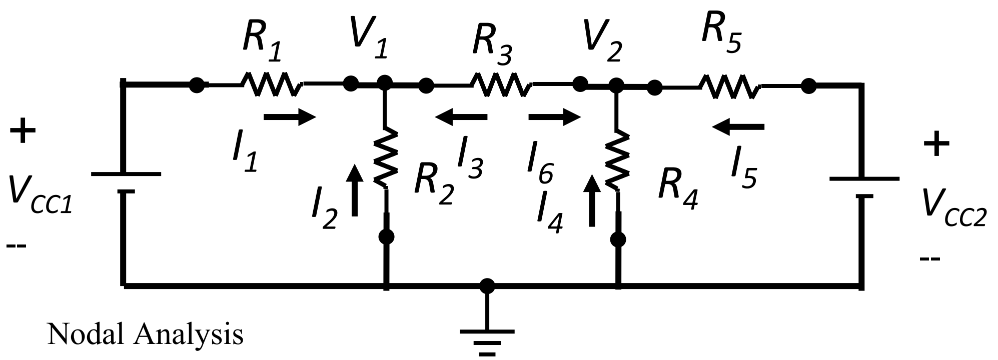
Mesh 2

$$0 = I_2 5 + 5 + 10(I_2 - I_1);\ -5 = 15I_2 - 10I_1$$

$$I_1 = \frac{1}{5}; I_2 = -\frac{1}{5}$$



Comparing Nodal and Mesh Analyses



Nodal Analysis

3 Nodes:

Bottom Node is reference node

Node 1 (left hand node) has a voltage with repect to the reference node V_1

Node 2 (right hand node) has a voltage with repect to the reference node V_2

Therefore, 2 Nodal Equations: NOTE: $I_3 = -I_6$

Node 1 equation with respect to V_1

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_{cc1} - V_1}{R_1} + \frac{-V_1}{R_2} + \frac{V_2 - V_1}{R_3} = 0$$

Node 2 equation with respect to V_2

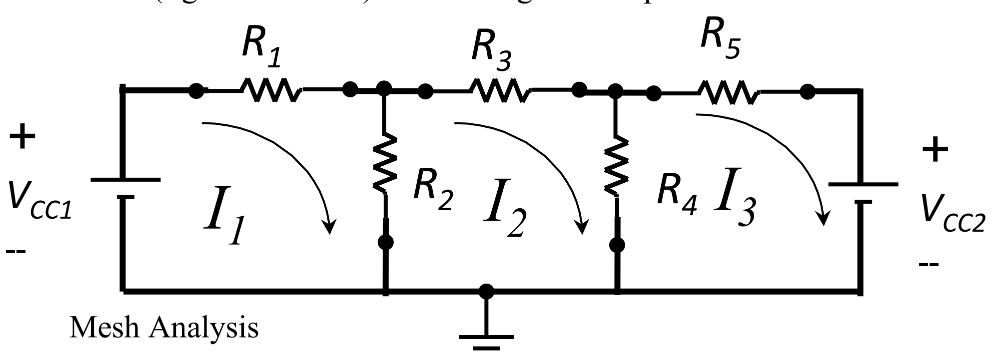
$$I_6 + I_4 + I_5 = 0$$

$$\frac{V_1 - V_2}{R_3} + \frac{-V_2}{R_4} + \frac{V_{cc2} - V_2}{R_5} = 0$$

2 simultaneous equations

$$\frac{V_{cc1}}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_2\frac{1}{R_3}$$

$$\frac{V_{cc2}}{R_5} = \frac{-V_1}{R_3} + V_2\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)$$



Mesh Analysis

3 Meshes:

Mesh 1 has a current I_1

Mesh 2 has a current I_2

Mesh 3 has a current I_3

Therefore, 3 Mesh Equations:

Mesh 1 equation with respect to I_1

$$V_{cc1} = I_1 R_1 + (I_1 - I_2) R_2$$

Mesh 2 equation with respecnt to I_2

$$0 = (I_2 - I_1) R_2 + I_2 R_3 + (I_2 - I_3) R_4$$

Mesh 2 equation with respecnt to I_2

$$-V_{cc1} = (I_3 - I_2) R_4 + I_3 R_5$$

3 simultaneous equations

$$V_{cc1} = I_1(R_1 + R_2) + -I_2 R_2 + 0$$

$$0 = -I_1 R_2 + I_2(R_2 + R_3 + R_4) - I_3 R_4$$

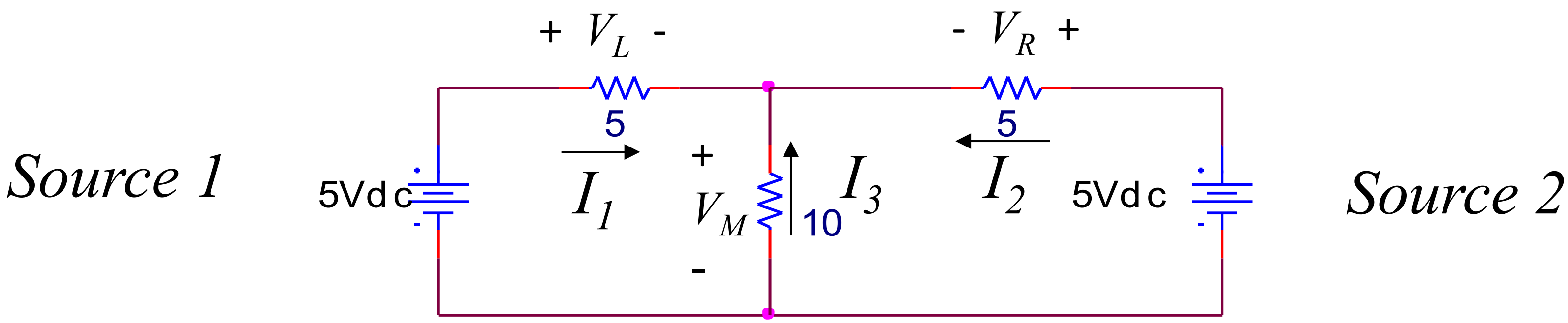
$$-V_{cc1} = 0 - I_2 R_4 + I_3(R_4 + R_5)$$

Superposition

- Used to analyze a circuit with multiple sources.
- Steps:
  1. Set all sources except for one to zero (voltage sources are shorted-circuited, current sources are open-circuited)
  2. Solve for the currents and voltages for all of the circuit elements
  3. Repeat steps 1-2 for the remaining sources.
  4. Add each of the solutions to obtain the solution for the entire circuit

Superposition Analysis Example

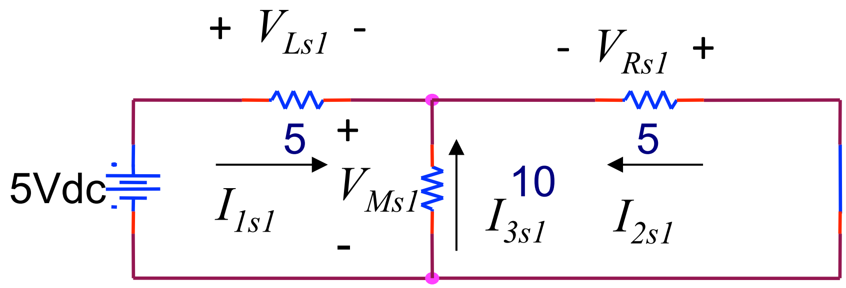
- Define all of the voltages and currents in the circuit



Superposition Analysis Example

Source 1

Currents are with respect to Source 1

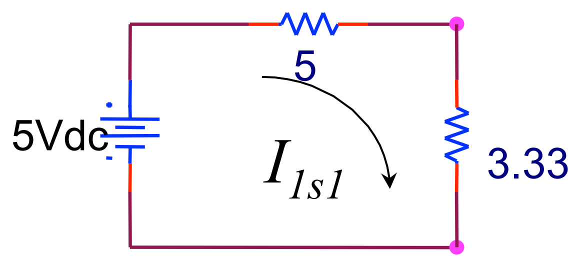


Source 2 becomes a short circuit

1. Simplify circuit and calculate I_{1s1}

$$R_p = 10 \| 5 = \frac{10 * 5}{10 + 5} = \frac{50}{15} = \frac{10}{3} = 3.33\Omega$$

$$I_{1s1} = \frac{5}{5 + \frac{10}{3}} = \frac{15}{25} = \frac{3}{5}$$



Superposition Analysis Example

2. Use current division to calculate the remaining currents and voltages

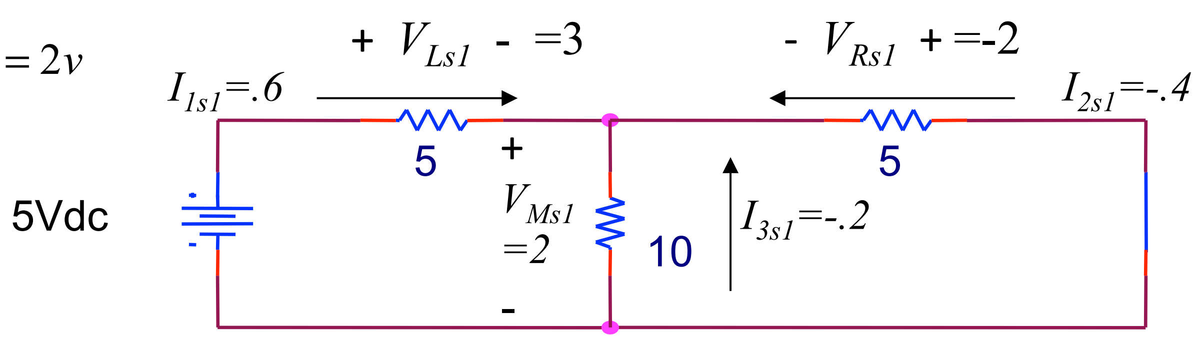
$$I_{2s1} = -\frac{10}{15} \times I_{1s1} = -\frac{10}{15} \times \frac{3}{5} = -\frac{2}{5} = -.4$$

$$I_{3s1} = -\frac{5}{15} \times I_{1s1} = -\frac{5}{15} \times \frac{3}{5} = -\frac{1}{5} = -.2$$

$$V_{Ls1} = I_{1s1} \times 5 = .6 \times 5 = 3v$$

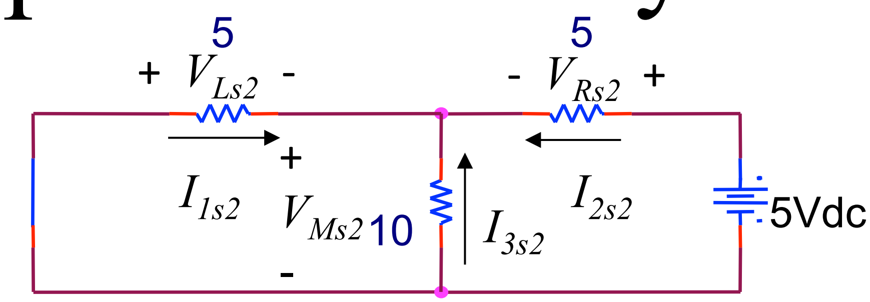
$$V_{Rs1} = I_{2s1} \times 5 = -.4 \times 5 = -2v$$

$$V_{Ms1} = -I_{3s1} \times 10 = -(-.2) \times 10 = 2v$$



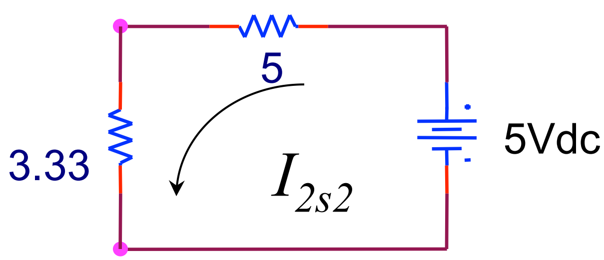
Superposition Analysis Example

Now Source 1 becomes a short circuit



Source 2

Currents are with respect to Source 2



1. Repeat for Source 2

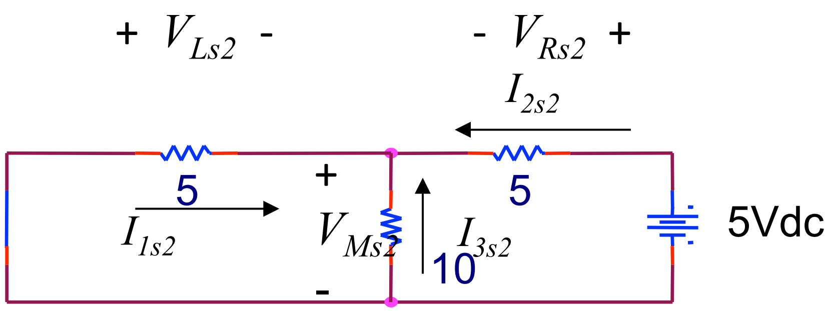
$$R_p = 10 \,||\, 5 = \frac{10*5}{10+5} = \frac{50}{15} = \frac{10}{3} = 3.33\Omega$$

$$I_{2s2} = \frac{5}{5+\frac{10}{3}} = \frac{15}{25} = \frac{3}{5}$$

Superposition Analysis Example

Source 2

Currents are with respect to Source 2



$$I_{1s2} = -\frac{10}{15} \times \frac{3}{5} = -\frac{2}{5} = -.4$$

$$I_{3s2} = -\frac{5}{15} \times \frac{3}{5} = -\frac{1}{5} = -.2$$

$$V_{Ls2} = I_{1s2} \times 5 = -4 \times 5 = -2v$$

$$V_{Rs2} = I_{2s2} \times 5 = .6 \times 5 = 3v$$

$$V_{Ms2} = -I_{3s2} \times 10 = -(-.2) \times 10 = 2v$$

Superposition Analysis Example

- Summing the results of each solution:

$$I_1 = I_{1s1} + I_{1s2} = .6 - .4 = .2$$

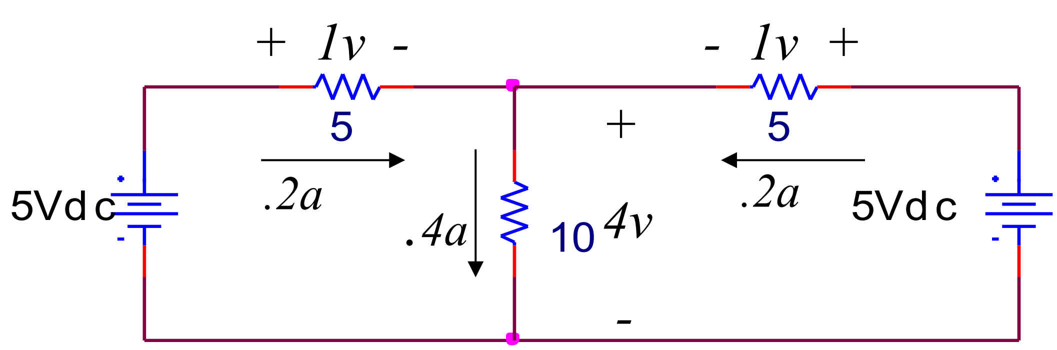
$$I_2 = I_{2s1} + I_{2s2} = -.4 + .6 = .2$$

$$I_3 = I_{3s1} + I_{3s2} = -.2 + -.2 = -.4$$

$$V_L = V_{Ls1} + V_{Ls2} = 3 + (-2) = 1$$

$$V_R = V_{Rs1} + V_{Rs2} = (-2) + 3 = 1$$

$$V_M = V_{Ms1} + V_{Ms2} = 2 + 2 = 4$$



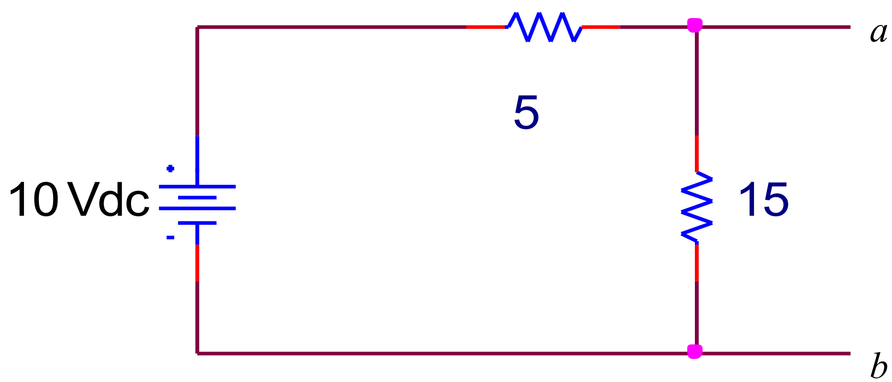
Thevenin and Norton Equivalent Circuits

- Thevenin's Theorem: Any circuit consisting of passive and active components can be represented by a voltage source in series with an equivalent set of passive components
  - The value of the voltage source equals the voltage seen at the output terminal without any load connected to it, i.e., the open-circuit voltage
  - The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.

Thevenin and Norton Equivalent Circuits

- Norton's Theorem: Any circuit consisting of passive and active components can be represented by a current source in parallel with an equivalent set of passive components
  - The value of the current source equals the current seen at the output terminal shorted and without any load connected to it, i.e., the short-circuit current
  - The value of the equivalent set of passive components equals the impedance looking back into the terminals with the sources set to zero, i.e., the output impedance.
- Note that the Thevenin and Norton Equivalents Circuits are equivalent to each other when the value of the Thevenin's voltage source equals the product of the equivalent impedance times the Norton's current source

Thevenin and Norton Examples

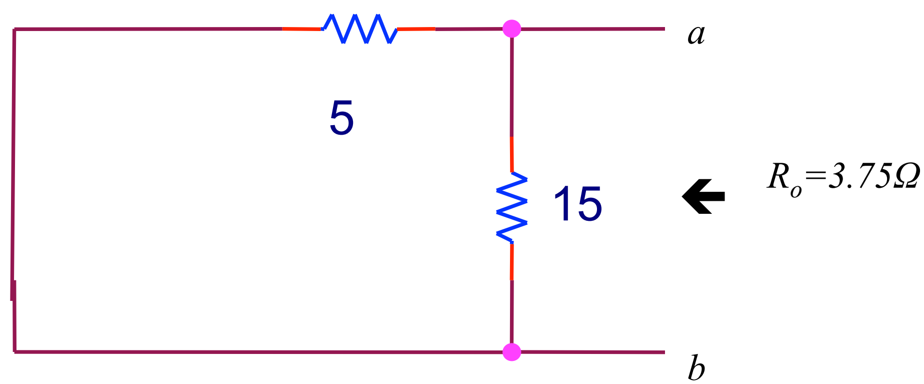


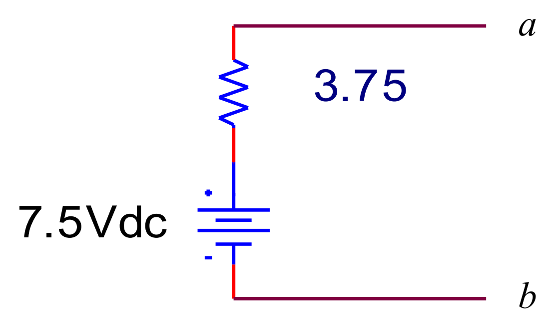
Open Circuit Voltage at terminals : ab

$$V_{abOC} = \frac{15}{5+15} \times 10 = 7.5v$$

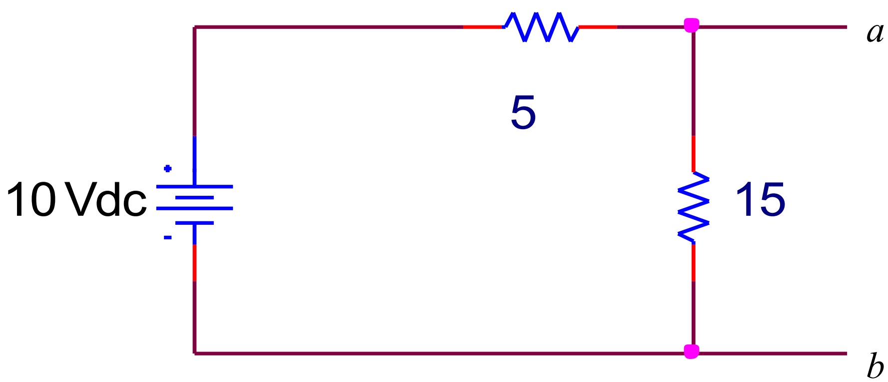
Output Impedance

$$R_o = 5 \,||\, 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75\Omega$$





Thevenin and Norton Examples

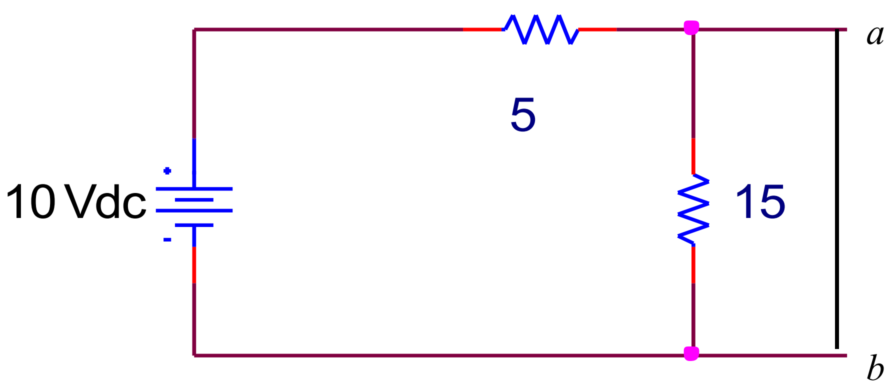


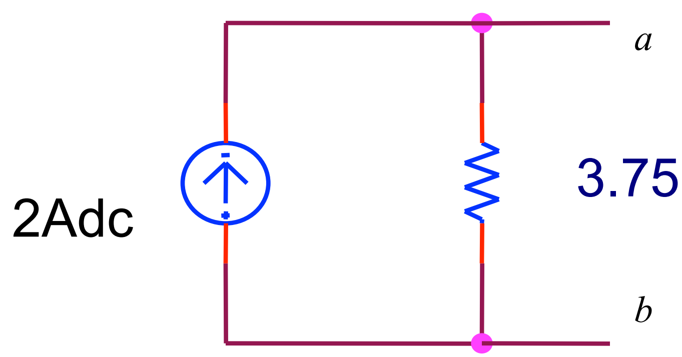
Short Circuit Current at terminals: ab

$$I_{abSC} = \frac{10}{5} = 2a$$

Output Impedance

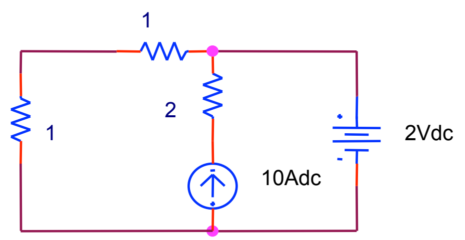
$$R_o = 5 \| 15 = \frac{5 \times 15}{20} = \frac{15}{4} = 3.75\Omega$$

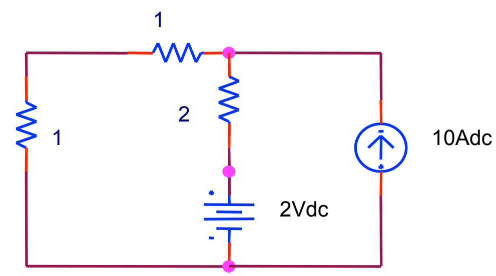


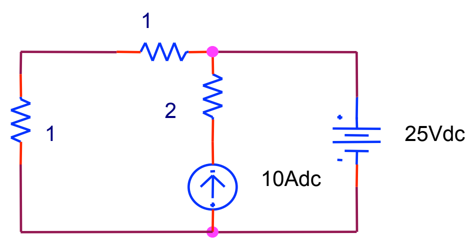


Homework

- Voltage and Current division
  - How does the voltage divide across two capacitors in series? Show your results.
  - How does the current divide among two capacitors in parallel? Show your results.
- Calculate the Currents and Voltages for the following circuits:



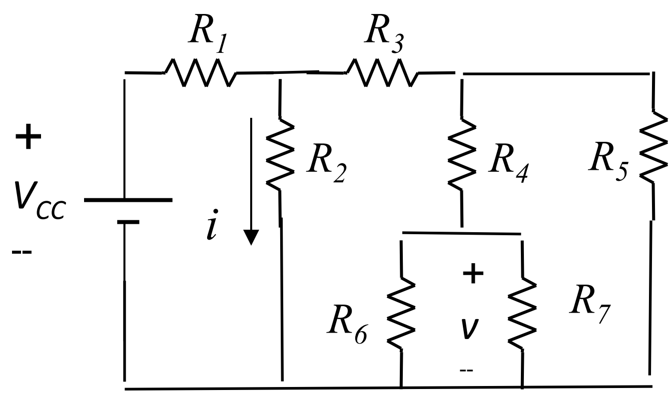




Homework

Calculate the current labeled *i* and the voltage labeled *v* in the following circuit

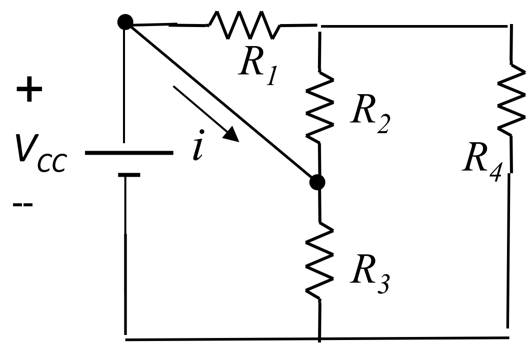
$R_1 = 1\Omega$, $R_2 = 2\Omega$, $R_3 = 1\Omega$, $R_4 = 1\Omega$, $R_5 = 2\Omega$, $R_6 = 2\Omega$, $R_7 = 2\Omega$, $V_{cc} = 4v$



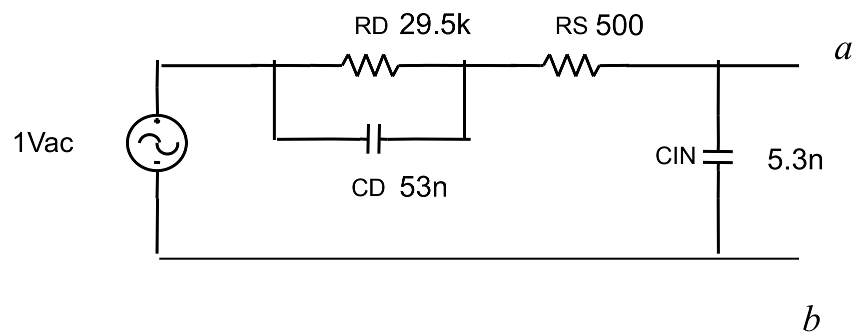
Homework

Calculate the current labe$i_4$led, i.

$R_1 = 2\Omega$, $R_2 = 2\Omega$, $R_3 = 2\Omega$, $R_4 = 3\Omega$, $V_{cc} = 2v$



Homework



An electrode is connected to an oscilloscope which has a purely capacitance input impedance, CIN. Find and plot the output voltage $V_{ab}(j\omega)$ as function of ω. Use Matlab to perform the plot.

Homework

- Repeat the analysis of this circuit using Mesh and Nodal Analysis. That is find and plot V_{ab} as a function of frequency. Use Matlab to calculate the Bode plot.