Operational Amplifiers

Lecture 4

Ideal Amplifiers

- Parameters:

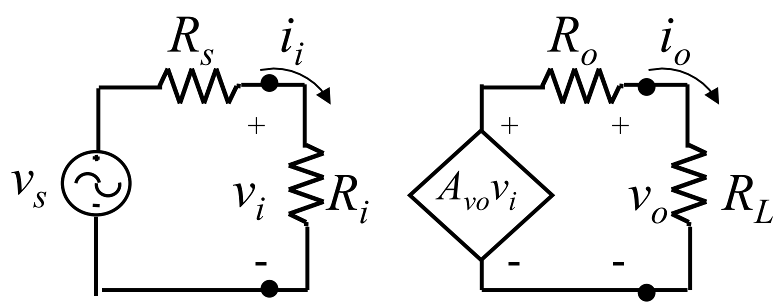
$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} A_{vo} \frac{R_L}{R_L + R_O}$$

This states the gain of the amplifier depends on the external components.

This is BAD!!!!!

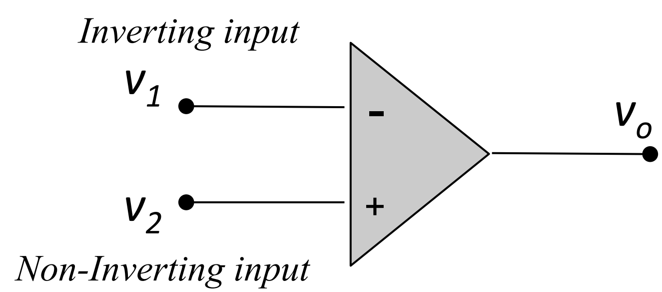
However, if $R_i \rightarrow \infty$ and $R_O \rightarrow 0$; then $\frac{R_i}{R_i + R_s} \rightarrow 1$ and $\frac{R_L}{R_L + R_O} \rightarrow 1$.

Therefore, $\frac{v_o}{v_s} \rightarrow A_{vo}$ and the gain of the amplifier is independent of the external components.



Operational Amplifiers

- An operational Amplifier is an ideal differential with the following characteristics:
  - Infinite input impedance
  - Infinite gain for the differential signal
  - Zero gain for the common-mode signal
  - Zero output impedance
  - Infinite Bandwidth



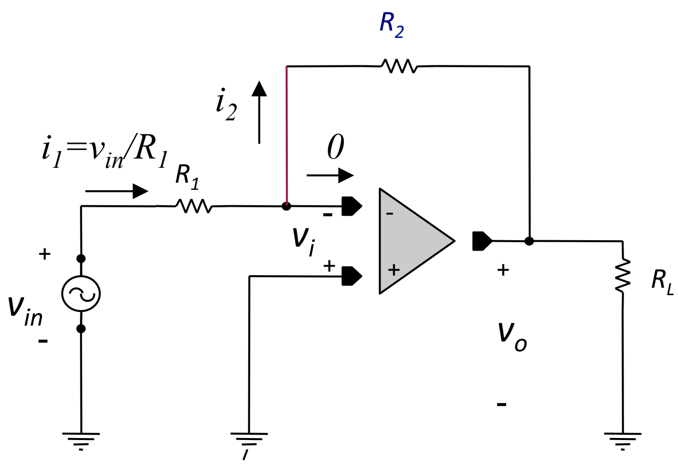
Operational Amplifier Feedback

- Operational Amplifiers are used with negative feedback
- Feedback is a way to return a portion of the output of an amplifier to the input
  - Negative Feedback: returned output opposes the source signal
  - Positive Feedback: returned output aids the source signal
- For Negative Feedback
  - In an Op-amp, the negative feedback returns a fraction of the output to the inverting input terminal forcing the differential input to zero.
  - Since the Op-amp is ideal and has infinite gain, the differential input will exactly be zero. This is called a virtual short circuit
  - Since the input impedance is infinite the current flowing into the input is also zero.
  - These latter two points are called the summing-point constraint.

Operational Amplifier Analysis Using the Summing Point Constraint

- In order to analyze Op-amps, the following steps should be followed:
  1. Verify that negative feedback is present
  2. Assume that the voltage and current at the input of the Op-amp are both zero (Summing-point Constraint
  3. Apply standard circuit analyses techniques such as Kirchhoff's Laws, Nodal or Mesh Analysis to solve for the quantities of interest.

Example: Inverting Amplifier



1. Verify Negative Feedback: Note that a portion of v_o is fed back via R_2 to the inverting input. So if v_i increases and, therefore, increases v_o, the portion of v_o fed back will then have the affect of reducing v_i (i.e., negative feedback).
2. Use the summing point constraint.
3. Use KVL at the inverting input node for both the branch connected to the source and the branch connected to the output

$$v_{in} = i_1 R_1 + 0 \text{ since } v_i \text{ is zero due to the summing-point constraint}$$

$$i_1 = i_2 \text{ due to the summing-point constraint}$$

$$v_0 = -i_2 R_2 + 0 \text{ since } v_i \text{ is zero}$$

$$= -\frac{R_2}{R_1} v_{in} \text{ which is independent of } R_L \text{ (note that the output is opposite to the input : inverted)}$$

$$Z_{in} = \frac{v_{in}}{i_1} = \frac{i_1 R_1}{i_1} = R_1$$

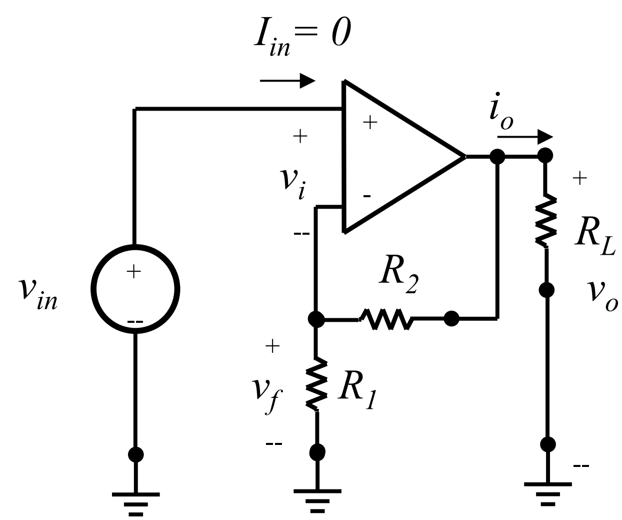
Op-amp

- Because we assumed that the Op-amp was ideal, we found that with negative feedback we can achieve a gain which is:

1. Independent of the load
2. Dependent only on values of the circuit parameter
3. We can choose the gain of our amplifier by proper selection of resistors.

Non-inverting Amp

1. First check: negative feedback?
2. Next apply, summing point constraint
3. Use circuit analysis



$$v_{in} = v_i + v_f = 0 + v_f = v_f$$

$$v_f = \frac{R_1}{R_1 + R_2} v_o = v_{in};$$

$$A_v = \frac{v_o}{v_{in}} = \frac{R_2 + R_1}{R_1} = 1 + \frac{R_2}{R_1}$$

$$\text{Since } i_{in} = 0; Z_{in} = \frac{v_{in}}{i_{in}} = \infty$$

Note:

1. The gain is always greater than one
2. The output has the same sign as the input

Non-inverting Amp Special Case

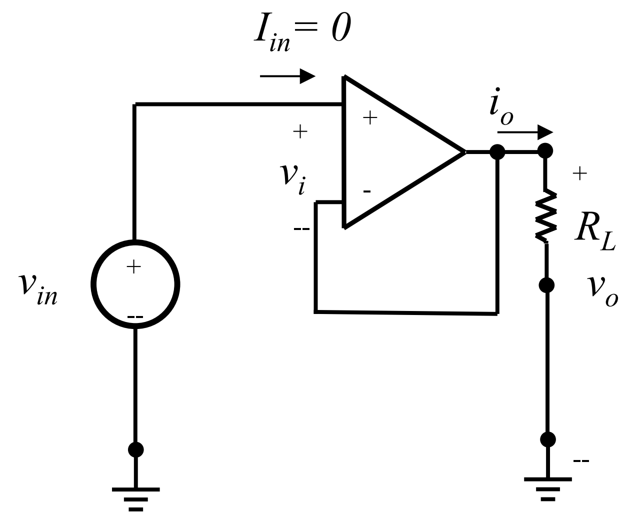
What happens if $R_2 = 0$?

$$v_{in} = v_i + v_f = 0 + v_f = v_f$$

$$v_f = \frac{R_1}{R_1 + 0} v_o = v_o = v_{in};$$

$$Av = \frac{v_o}{v_{in}} = \frac{0 + R_1}{R_1} = 1$$

$$\text{Since } i_{in} = 0; Z_{in} = \frac{v_{in}}{i_{in}} = \infty$$

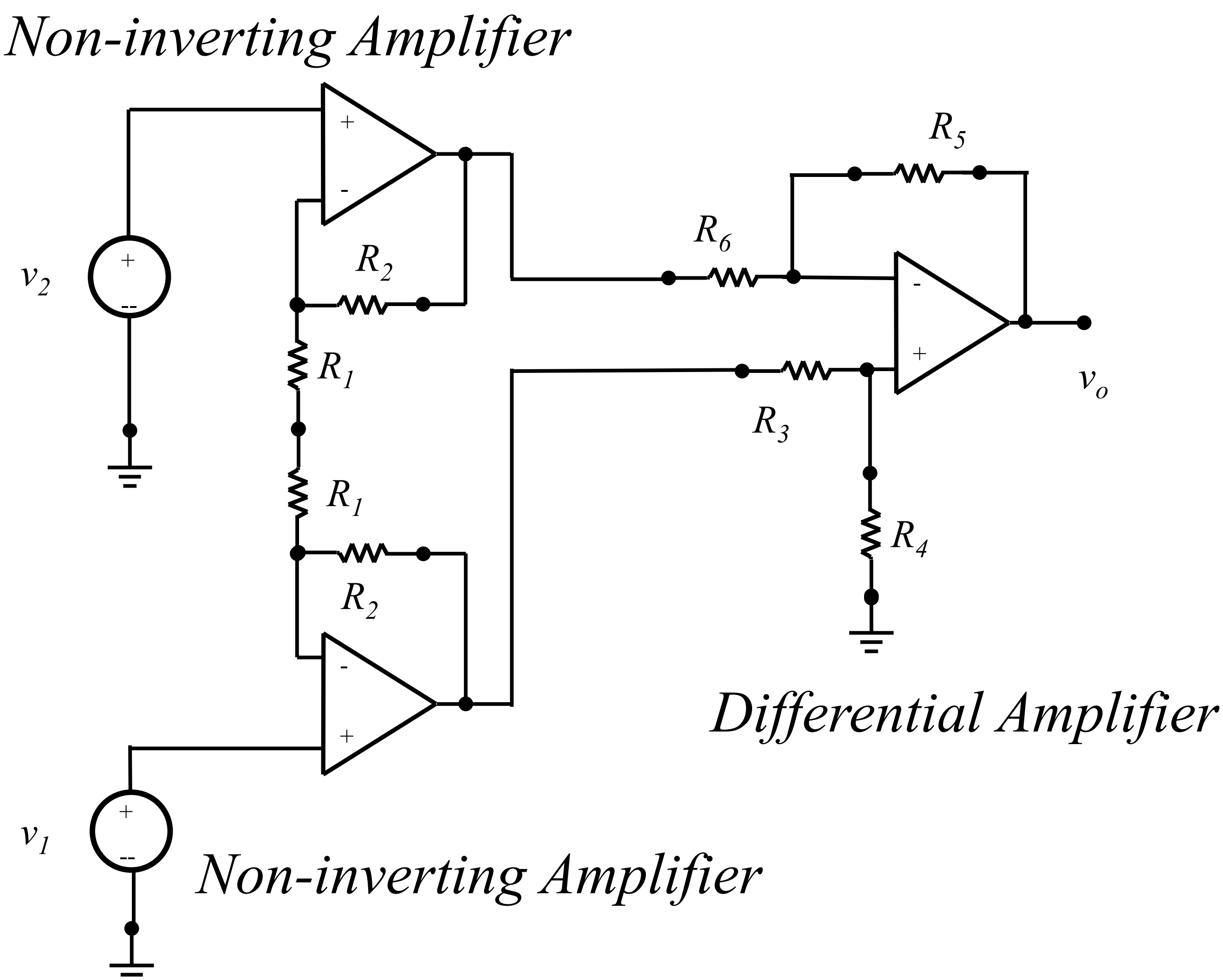


This is a unity gain amplifier and is also called a voltage follower.

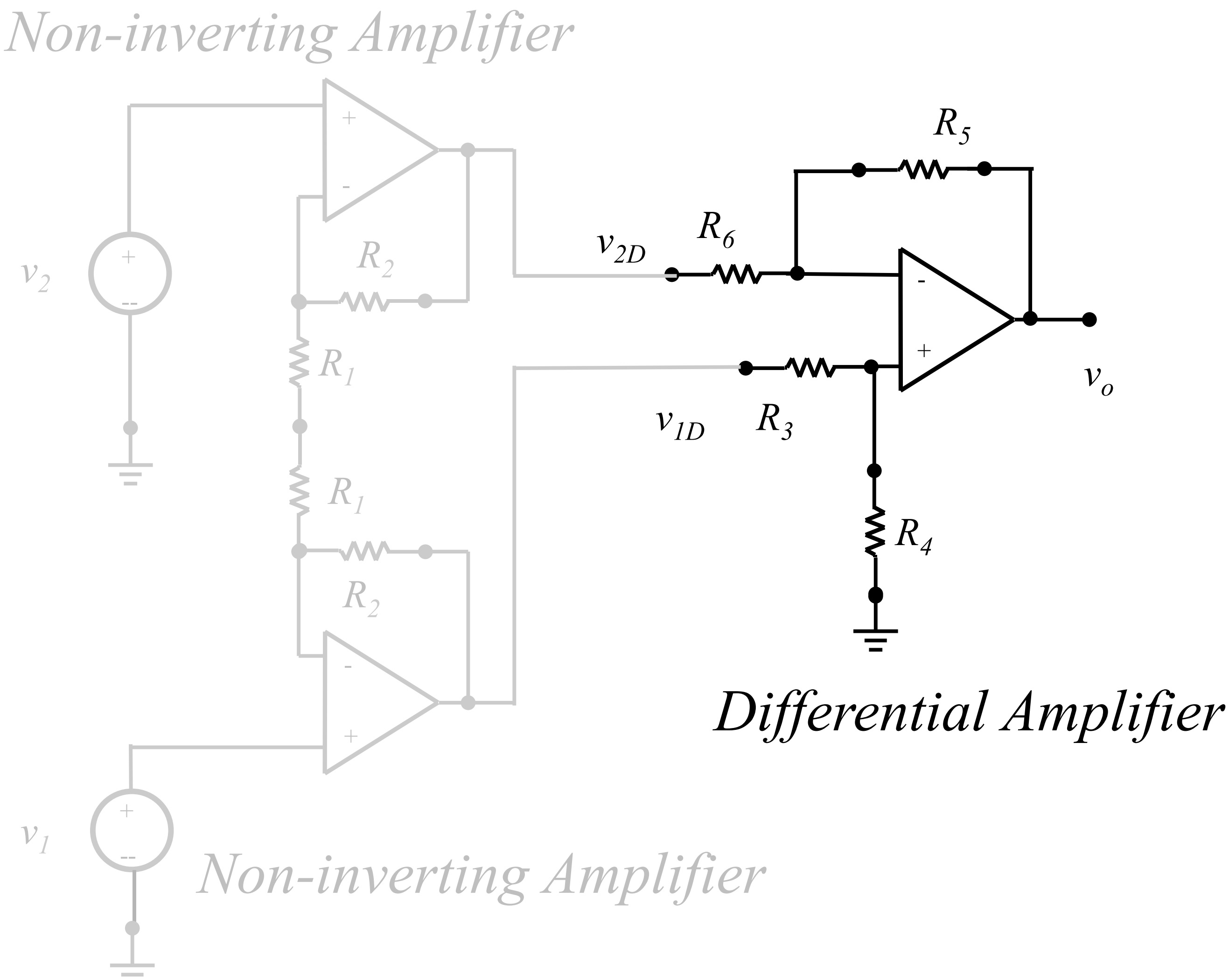
Special Amplifiers

- Summer (Homework Problem)
- Instrumentation Amplifier
  - Uses 3 Op-amps
  - One as a differential amplifier
  - Two Non-inverting Amps using for providing gain

Medical Instrumentation Amplifier

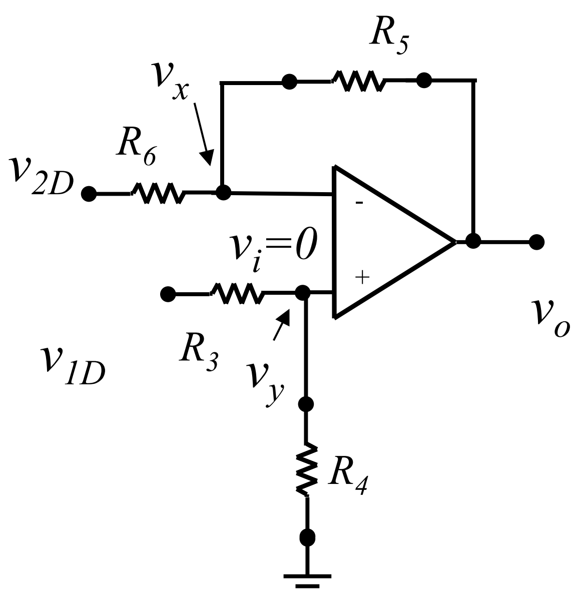


Medical Instrumentation Amplifier



Medical Instrumentation Amplifier

Differential Amplifier



Chose $\frac{R_5+R_6}{R_5}\frac{R_4}{R_3+R_4}=1$

$$R_4R_5+R_4R_6=R_5R_3+R_5R_4$$

$$R_4R_6=R_5R_3$$

$$\frac{R_6}{R_5}=\frac{R_3}{R_4}$$

$$\frac{v_{2D}-v_x}{R_6}=\frac{v_x-v_o}{R_5}$$

$$\frac{v_{2D}}{R_6}-v_x\left(\frac{1}{R_6}+\frac{1}{R_5}\right)=\frac{-v_o}{R_5}$$

$$\frac{v_{2D}}{R_6}-v_x\left(\frac{R_5+R_6}{R_6R_5}\right)=\frac{-v_o}{R_5}$$

$$v_y=\frac{R_4}{R_3+R_4}v_{1D}=v_x-v_i=v_x$$

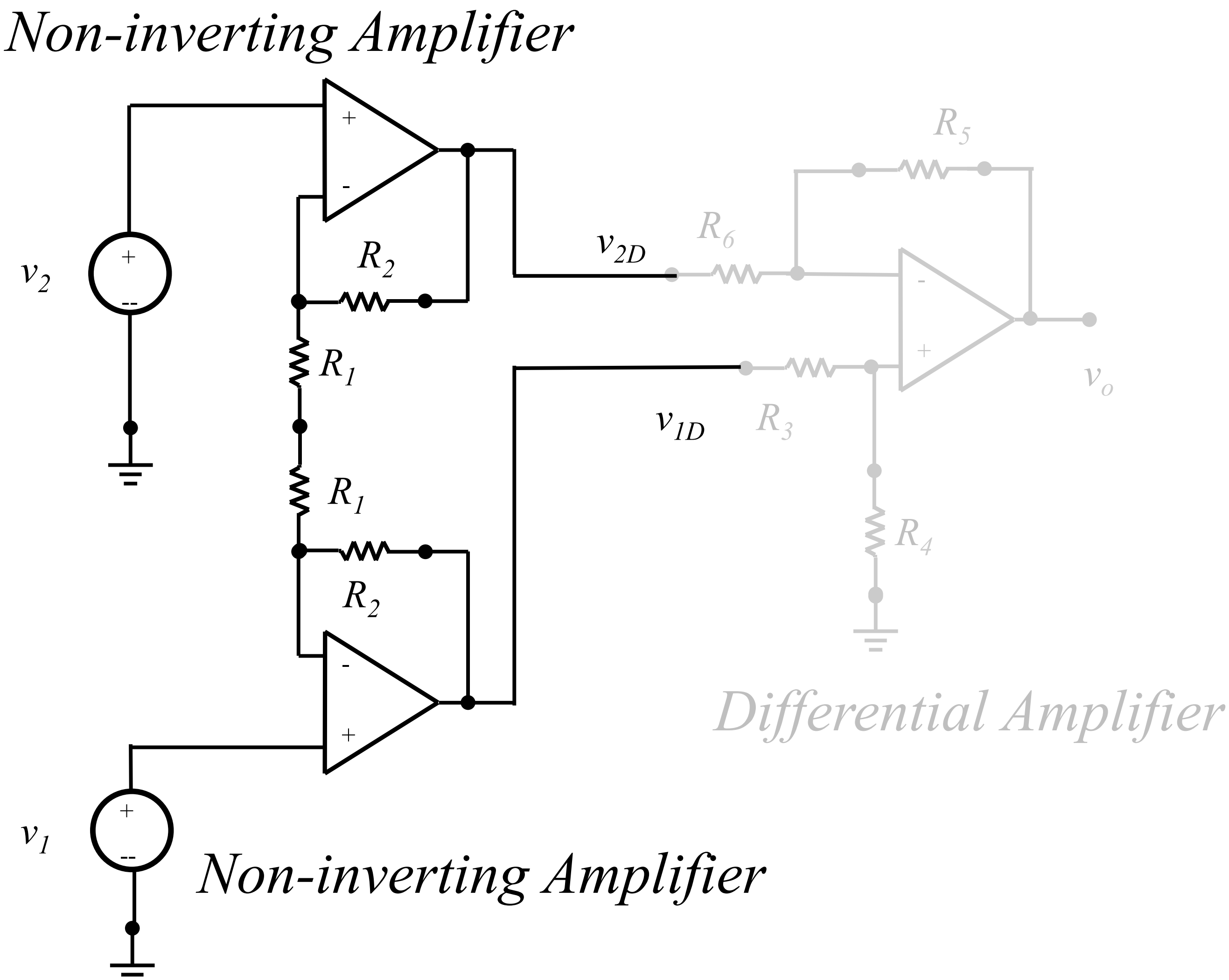
$$\frac{v_{2D}}{R_6}-\frac{R_4}{R_3+R_4}v_{1D}\left(\frac{R_5+R_6}{R_6R_5}\right)=\frac{-v_o}{R_5}$$

$$v_o=\frac{R_5}{R_6}\left(v_{1D}\frac{R_5+R_6}{R_5}\frac{R_4}{R_3+R_4}-v_{2D}\right)$$

Chose $\frac{R_5+R_6}{R_5}\frac{R_4}{R_3+R_4}=1$

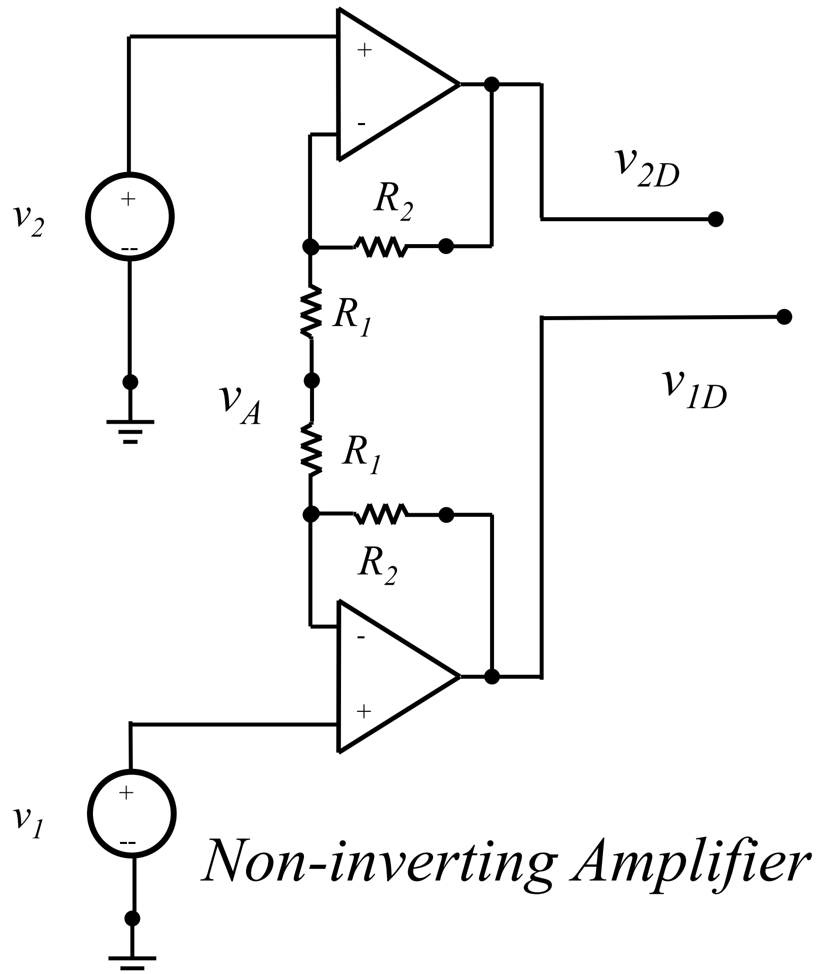
$$v_o=\frac{R_5}{R_6}(v_{1D}-v_{2D})$$

Medical Instrumentation Amplifier



Medical Instrumentation Amplifier

Non-inverting Amplifier



$$\frac{v_{2D} - v_2}{R_2} = \frac{v_2 - v_A}{R_1}$$

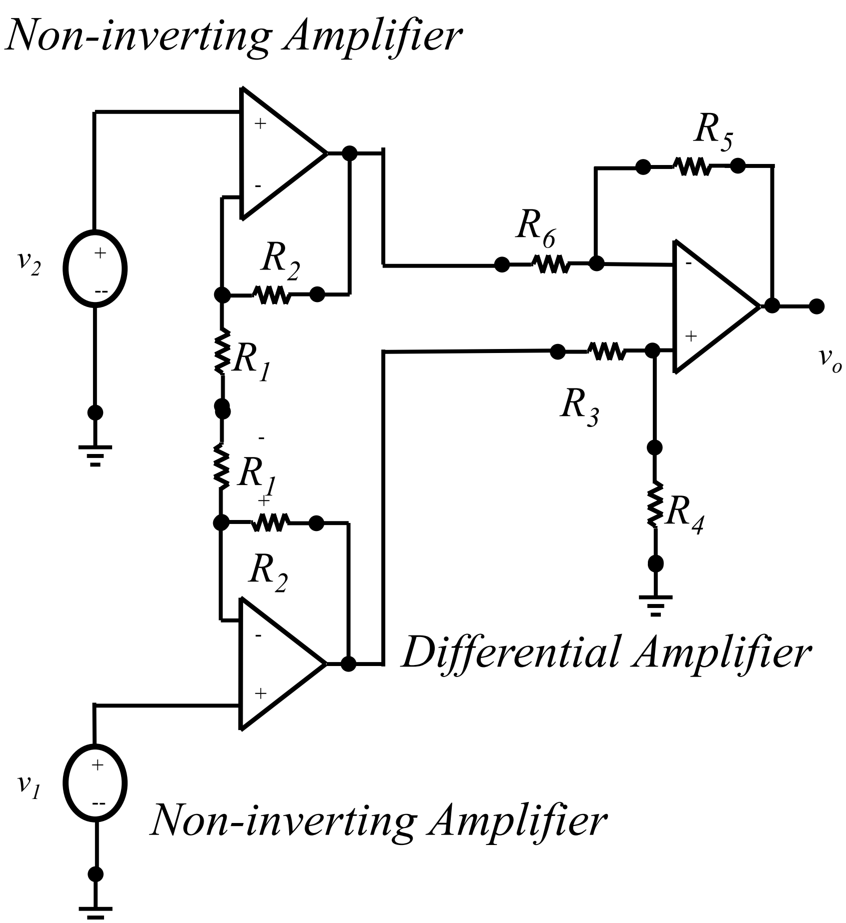
$$v_{2D} = R_2\left(\frac{1}{R_2} + \frac{1}{R_1}\right)v_2 - \frac{R_2}{R_1}v_A$$

$$v_{2D} = \frac{R_1 + R_2}{R_1}v_2 - \frac{R_2}{R_1}v_A$$

Likewise

$$v_{1D} = \frac{R_1 + R_2}{R_1}v_1 - \frac{R_2}{R_1}v_A$$

Medical Instrumentation Amplifier



$$v_o = \frac{R_5}{R_6}(v_{1D} - v_{2D})$$

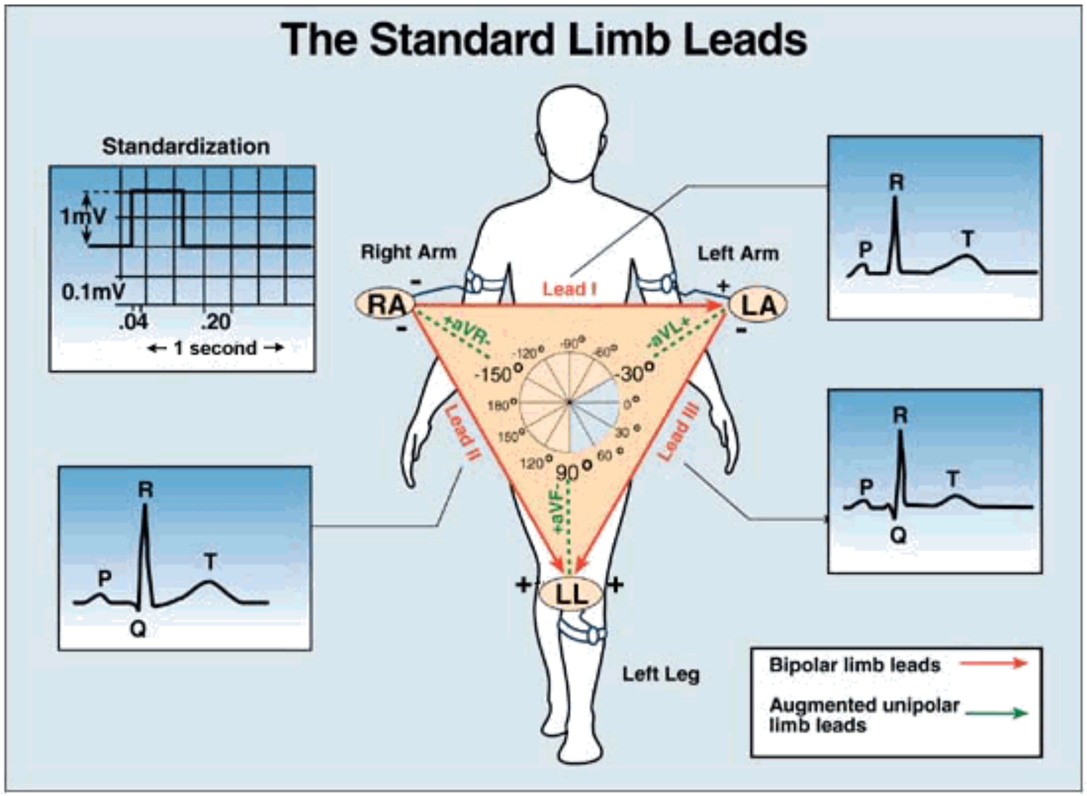
$$v_{2D} = \frac{R_1 + R_2}{R_1} v_2 - \frac{R_2}{R_1} v_A$$

$$v_{1D} = \frac{R_1 + R_2}{R_1} v_1 - \frac{R_2}{R_1} v_A$$

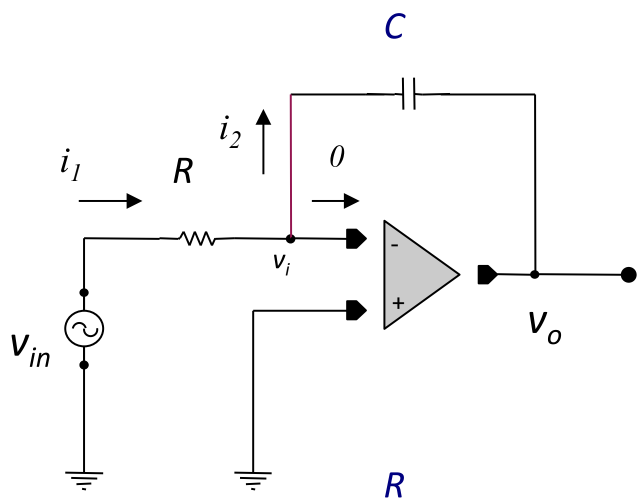
$$v_o = \frac{R_5}{R_6}[\frac{R_1 + R_2}{R_1} v_1 - \frac{R_2}{R_1} v_A - (\frac{R_1 + R_2}{R_1} v_2 - \frac{R_2}{R_1} v_A)]$$

$$v_o = \frac{R_5}{R_6}(\frac{R_1 + R_2}{R_1})(v_1 - v_2) = \frac{R_5}{R_6}(1 + \frac{R_2}{R_1})(v_1 - v_2)$$

Uses of the Differential Amplifier

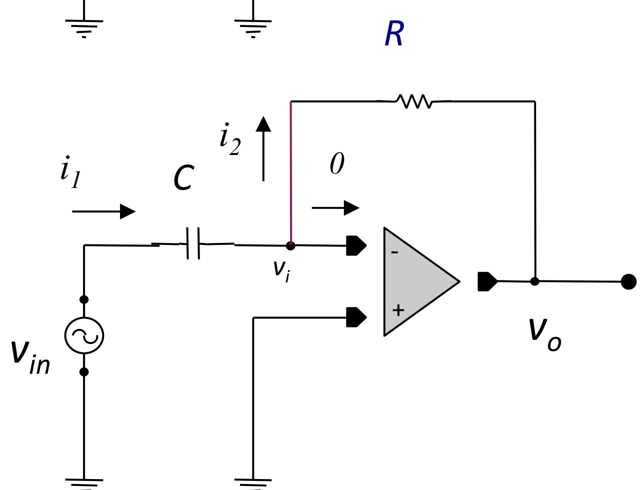


Integrators and Differentiators



$$i_1(t) = \frac{v_{in}(t)}{R} = i_2(t)$$

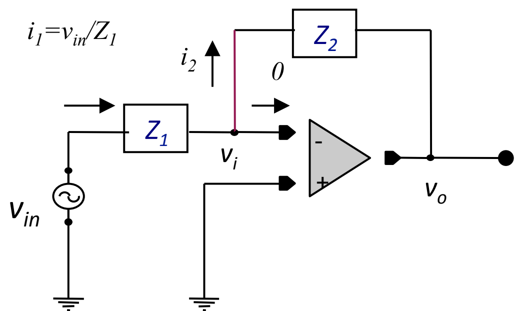
$$v_o = -\frac{1}{C}\int_0^t i_2(x)dx = -\frac{1}{RC}\int_0^t v_{in}(x)dx$$



$$i_1(t) = \frac{C dv_{in}(t)}{dt} = i_2(t)$$

$$v_o = -i_2(t)R = -RC\frac{dv_{in}(t)}{dt}$$

Frequency Analysis



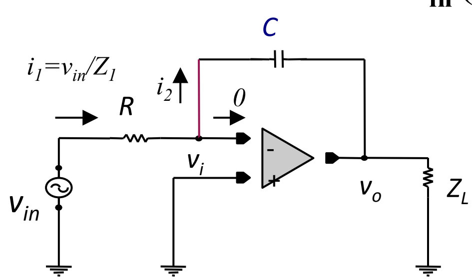
$$\mathbf{V_{in}}(j\omega) = \mathbf{I_1}(j\omega)\mathbf{Z_1}(j\omega) + 0 \text{ since } v_i \text{ is (virtually) zero}$$

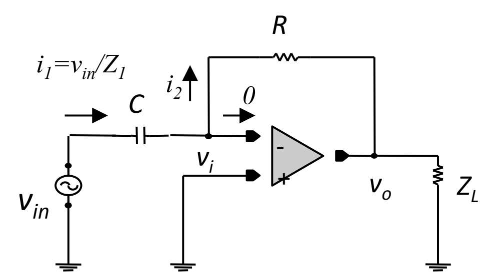
$$\mathbf{I_1}(j\omega) = \mathbf{I}_2(j\omega) \text{ due to the summing-point constraint}$$

$$\mathbf{V_o}(j\omega) = -\mathbf{I}_2(j\omega)\mathbf{Z}_2 + 0 \text{ since } v_i \text{ is (virtually) zero}$$

$$= -\frac{\mathbf{Z}_2}{\mathbf{Z_1}}\mathbf{V_{in}}(j\omega) \text{ which is independent of } \mathbf{Z_L}$$

$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}}$$

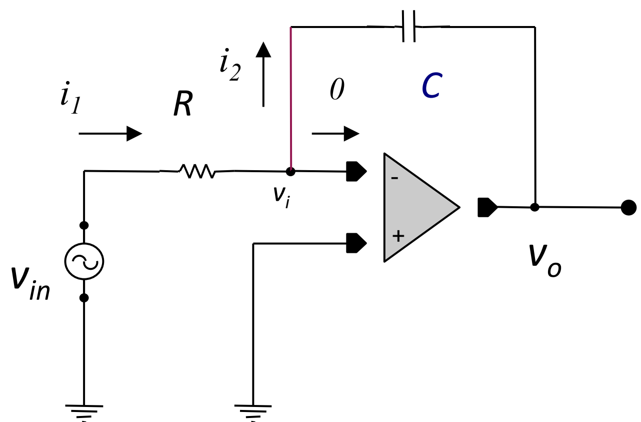




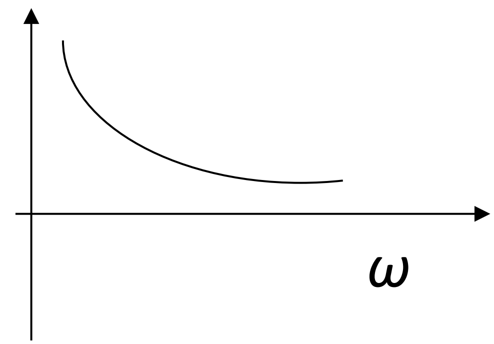
$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}} = -\frac{1}{j\omega RC} \text{ an integrator}$$

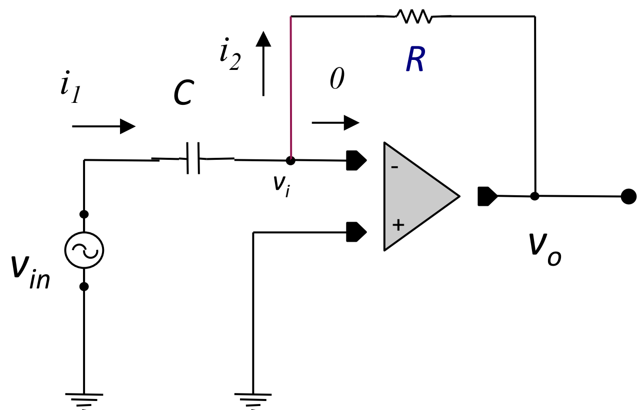
$$\frac{\mathbf{V_o}(j\omega)}{\mathbf{V_{in}}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z_1}} = -j\omega RC \text{ a differeniator}$$

Frequency Response

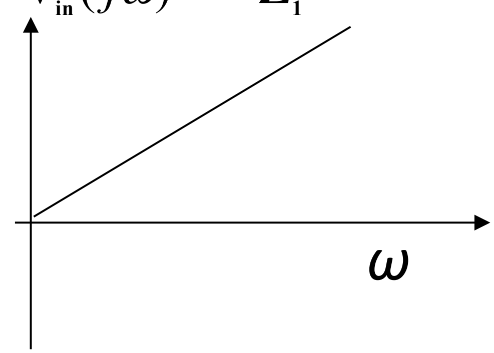


$$\frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_{in}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -\frac{1}{j\omega RC}$$

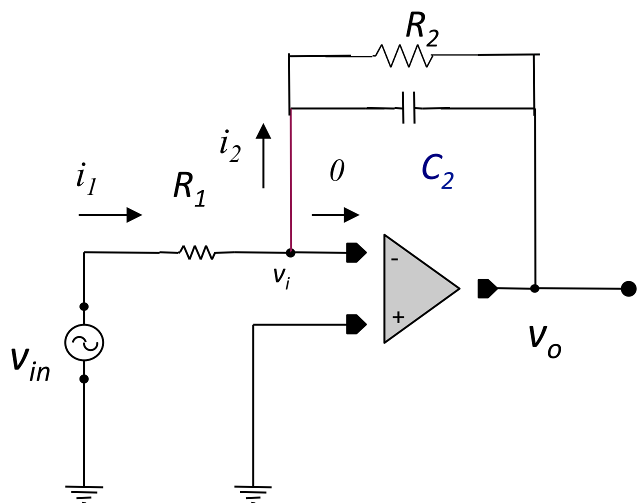




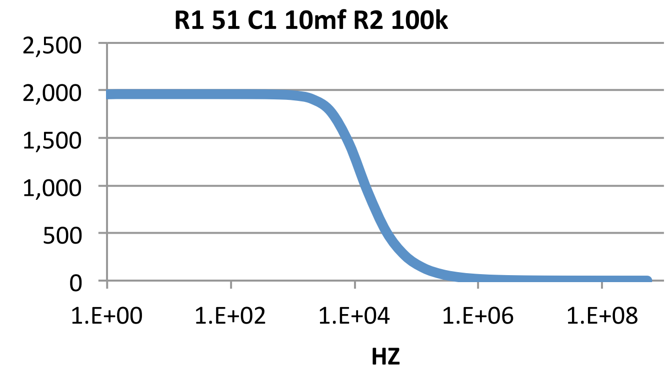
$$\frac{\mathbf{V}_o(j\omega)}{\mathbf{V}_{in}(j\omega)} = -\frac{\mathbf{Z}_2}{\mathbf{Z}_1} = -j\omega RC$$

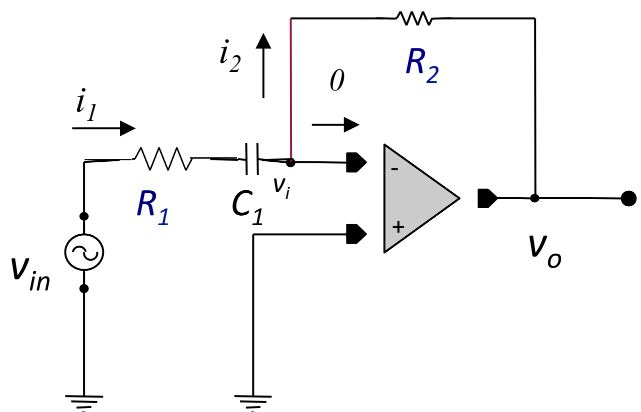


Frequency Response

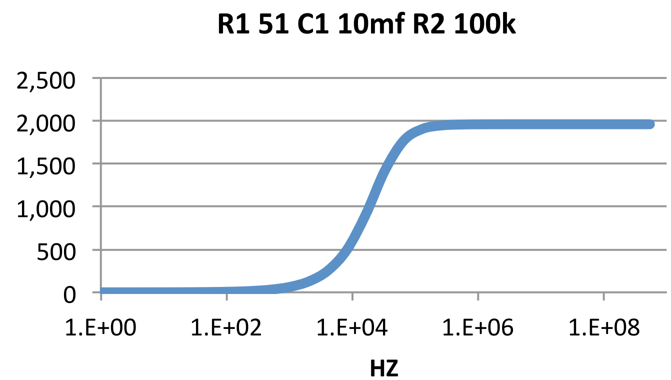


$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\frac{1}{(1+j\omega C_2 R_2)} = \frac{R_2}{R_1}\frac{1}{\sqrt{1+(\omega C_2 R_2)^2}}\angle \pi - \tan^{-1}(\omega C_2 R_2)$$





$$\frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\frac{j\omega C_1 R_1}{(1+j\omega C_1 R_1)} = \frac{R_2}{R_1}\frac{\omega C_1 R_1}{\sqrt{1+(\omega C_1 R_1)^2}}\angle -\frac{\pi}{2} - \tan^{-1}(\omega C_1 R_1)$$



Homework

- Probs 2.2, 2.5, 2.6, 2.10, 2.22, 2.24, 2.25, 2.28
- Calculate and plot the output vs frequency for these circuits. R_1=1k, R_2=3k, C=1μf Use Matlab to calculate the Bode plot