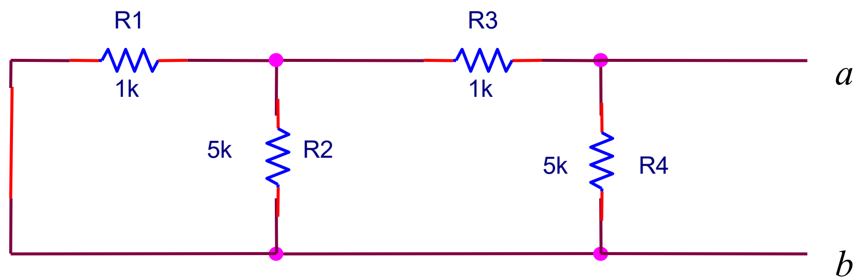
BME 372 Electronics I

Circuit Analysis

Lesson #1

Homework



Find the total resistance R_{ab}

Method #1

$$R_x = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

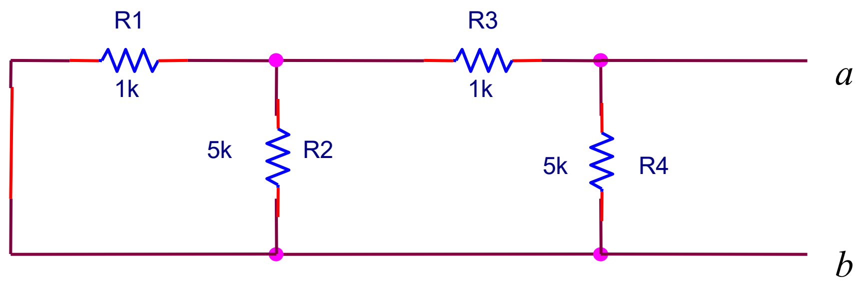
$$R_y = R_x \Leftrightarrow R_3 = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2}$$

$$R_{ab} = R_y \| R_4 = \frac{R_y R_4}{R_y + R_4} = \frac{\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2} R_4}{\frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2} + R_4} = \frac{R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4}{R_1 R_2 + R_1 R_3 + R_2 R_3 + R_1 R_4 + R_2 R_4}$$

$$R_1 = R_3 = R = 1k; R_2 = R_4 = 5R = 5k$$

$$R_{ab} = \frac{25R^3 + 5R^3 + 25R^3}{R^2 + 5R^2 + 5R^2 + 5R^2 + 25R^2} = \frac{55}{41} R = \frac{55}{41} k = 1.34k$$

Homework



Find the total resistance

Method #2

$$R_x = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{1k \times 5k}{6k} = \frac{5}{6}k$$

$$R_y = R_x \Leftrightarrow R_3 = \frac{5}{6}k + 1k = \frac{11}{6}k$$

$$R_{ab} = R_y \parallel R_4 = \frac{R_y R_4}{R_y + R_4} = \frac{\frac{11}{6}k \times 5k}{\frac{11}{6}k + 5k} = \frac{\frac{55}{6}k}{\frac{41}{6}} = \frac{55}{41}k = 1.34k$$

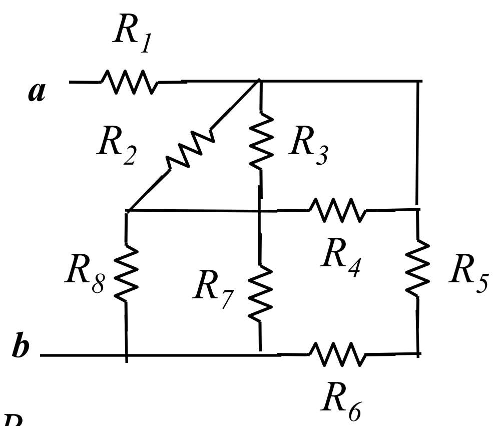
Homework

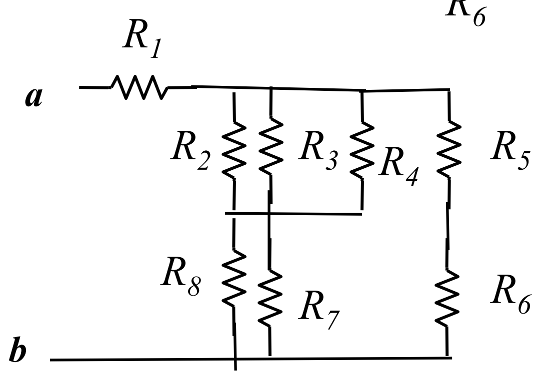
Find the total resistance R_{ab} where

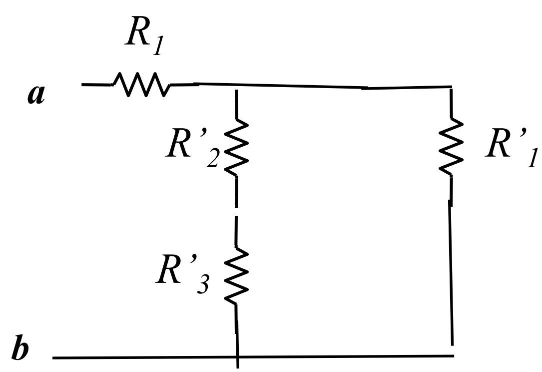
$R_1 = 3\Omega$, $R_2 = 6\Omega$, $R_3 = 12\Omega$,

$R_4 = 4\Omega$, $R_5 = 2\Omega$, $R_6 = 2\Omega$,

$R_7 = 4\Omega$, $R_8 = 4\Omega$







$$R_5 + R_6 = 2 + 2 = 4 = R'_1$$

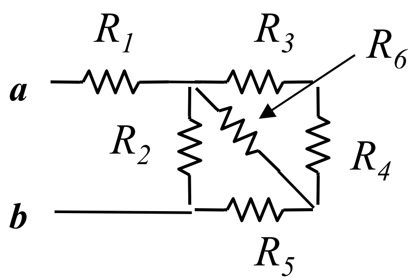
$$R_2 \parallel R_3 \parallel R_4 = \frac{1}{\frac{1}{6} + \frac{1}{12} + \frac{1}{4}} = 2 = R'_2$$

$$R_7 \parallel R_8 = \frac{1}{\frac{1}{4} + \frac{1}{4}} = 2 = R'_3$$

$$R'_2 + R'_3 = 2 + 2 = 4 = R'_4$$

$$R'_4 \parallel R'_1 = \frac{1}{\frac{1}{4} + \frac{1}{4}} = 2 = R'_5$$

$$R'_5 + R_1 = 2 + 3 = 5 = R_{ab}$$

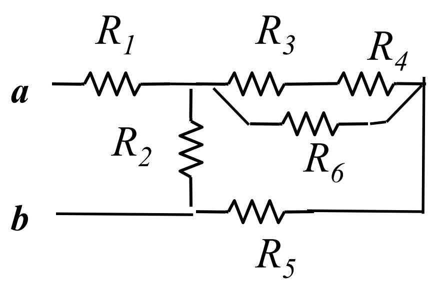


Homework

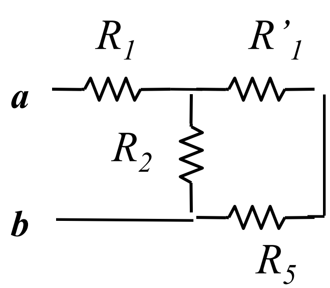
Find the total resistance R_{ab} where

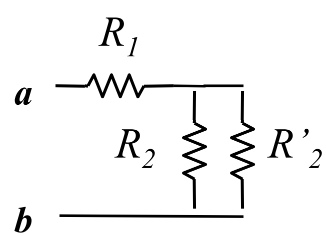
$R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 2\Omega$,

$R_4 = 2\Omega$, $R_5 = 2\Omega$, $R_6 = 4\Omega$



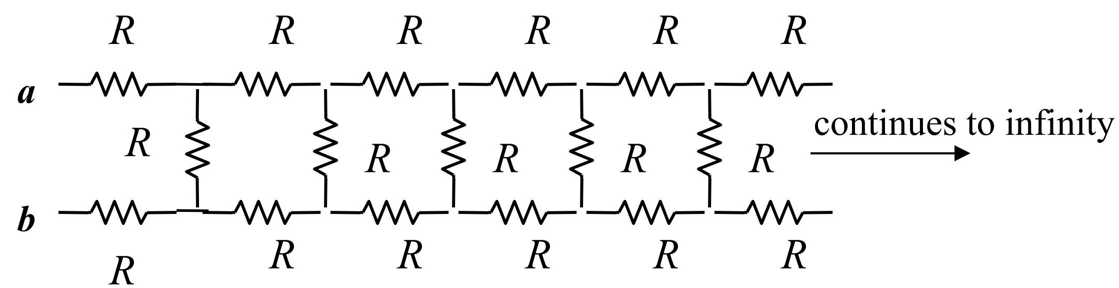
$$(R_3 + R_4) \| R_6 = (2+2) \| 4 = (4) \| 4 = 2 = R'_1$$
$$R_5 + R'_1 = 2 + 2 = 4 = R'_2$$
$$(R'_2 \| R_2) + R_1 = (4 \| 4) + 2 = 2 + 2 = 4 = R_{ab}$$

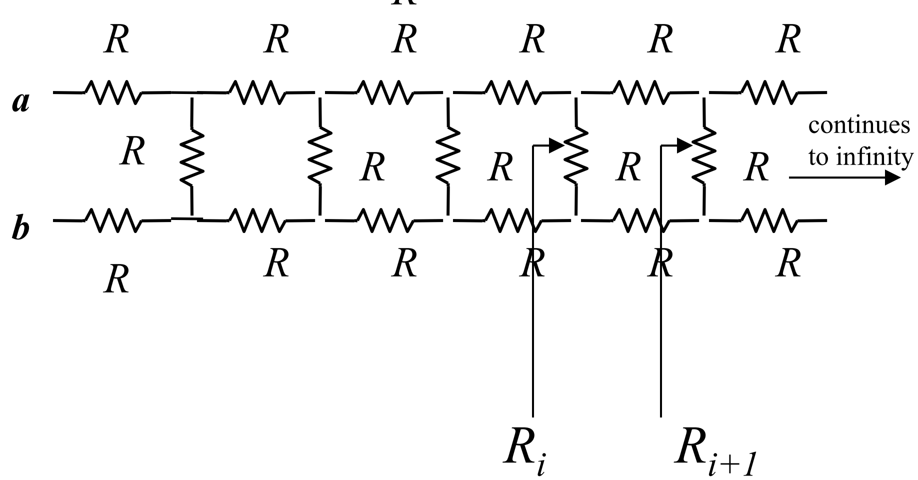




Homework

Find the total resistance R_{ab} for this infinite resistive network





METHOD #1 Iterative

$$R_i = (R_{i+1} + 2R) \parallel R = \frac{(R_{i+1} + 2R)R}{R_{i+1} + 3R}$$

But since the chain is infinite, shouldn't $R_i = R_{i+1} = r$

$$r = \frac{(r + 2R)R}{r + 3R} \Rightarrow r^2 + 3Rr = Rr + 2R^2 \Rightarrow r^2 + 2Rr - 2R^2 = 0$$

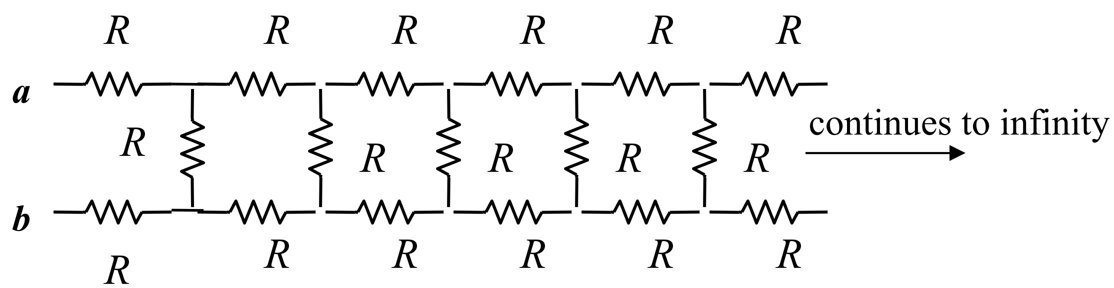
$$r = \frac{-2R \pm \sqrt{(2R)^2 - 4(-2R^2)}}{2} = \frac{-2R \pm R\sqrt{4+8}}{2} = \frac{-2R \pm R2\sqrt{3}}{2}$$

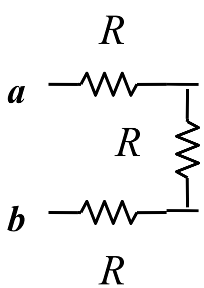
$$r = R(\sqrt{3} - 1)$$

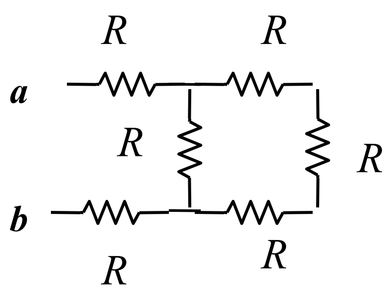
$$R_{ab} = 2R + r = (1 + \sqrt{3})R = 2.732R$$

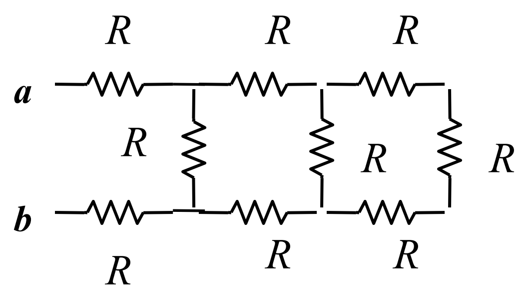
Homework

Find the total resistance R_{ab} for this infinite resistive network









METHOD #2

Numerical Iteration

Iteration 1

$R_{ab}^{1} = 3R$

Iteration 2

$R_{ab}^{2} = 2R + R \parallel 3R = 2R + R \parallel R_{ab}^{1}$

$R_{ab}^{2} = 2R + \frac{R(3R)}{4R} = 2R + .75R = 2.75R$

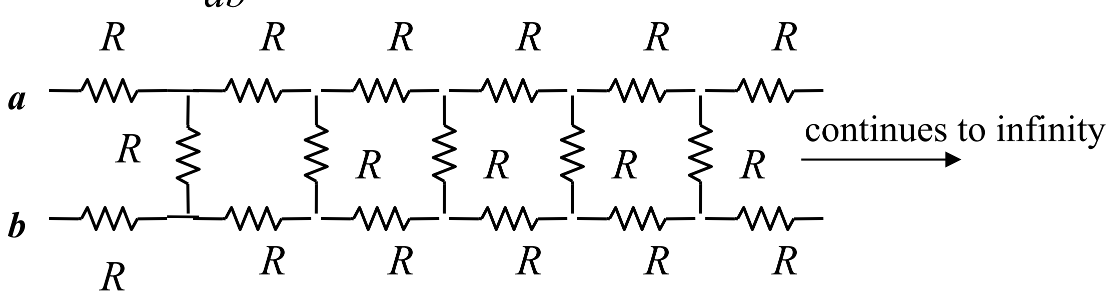
Iteration 3

$R_{ab}^{3} = 2R + R \parallel (2R + R \parallel 3R) = 2R + R \parallel (R_{ab}^{2})$

$R_{ab}^{3} = 2R + \frac{R(2.75R)}{R + (2.75R)} = 2R + \frac{2.75R}{3.75} = 2R + .733R = 2.733R$

Homework

Find the total resistance R_{ab} for this infinite resistive network

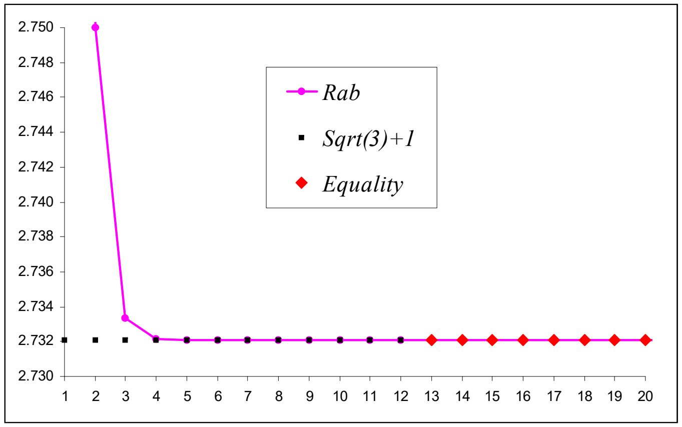


Iteration N

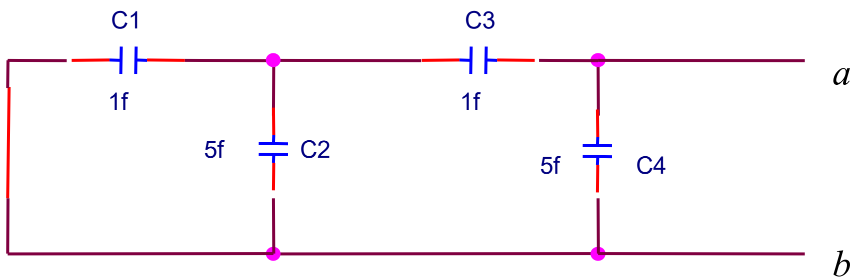
$$R_{ab}^{N} = 2R + R \,||\, R_{ab}^{N-1}$$

Continuing this way we see that R_{ab} approaches 2.732R where 2.732 is the sqrt(3)+1

Iteration	Rab
1	3
2	2.75
3	2.733333
4	2.732143
5	2.732057
6	2.732051
7	2.732051
8	2.732051
9	2.732051
10	2.732051
11	2.732051
12	2.732051
13	2.732051
14	2.732051
15	2.732051
16	2.732051
17	2.732051
18	2.732051
19	2.732051
20	2.732051



Homework



Find the total capacitance

Method #1

$$C_x = C_1 \,||\, C_2 = C_1 + C_2$$

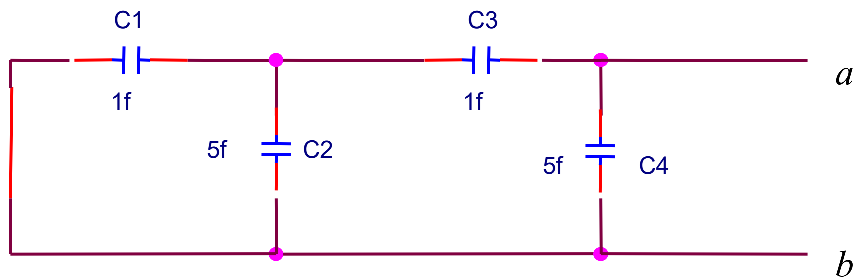
$$C_y = C_x \Leftrightarrow C_3 = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{C_1C_3 + C_2C_3}{C_1 + C_2 + C_3}$$

$$C_{ab} = C_y \,||\, C_4 = \frac{C_1C_3 + C_2C_3}{C_1 + C_2 + C_3} + C_4 = \frac{C_1C_3 + C_2C_3 + C_4C_1 + C_4C_2 + C_4C_3}{C_1 + C_2 + C_3}$$

$$C_1 = C_3 = C = 1f; C_2 = C_4 = 5f = 5C$$

$$C_{ab} = \frac{C^2 + 5C^2 + 5C^2 + 25C^2 + 5C^2}{7C} = \frac{41}{7}C = 5.86f$$

Homework



Find the total capacitance

Method #2

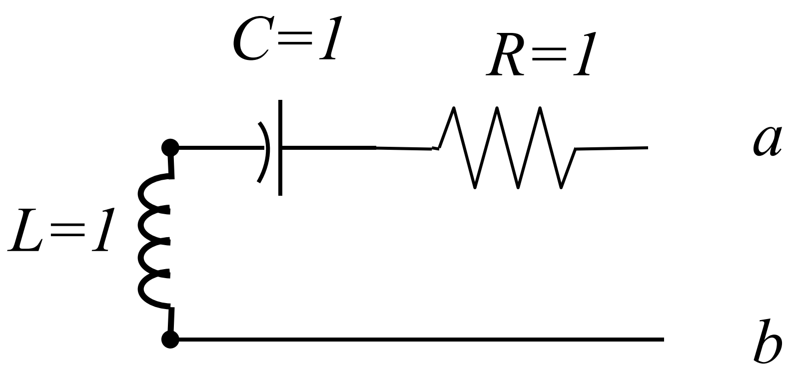
$$C_1 = C_3 = C = 1f; C_2 = C_4 = 5f$$

$$C_x = C_1 \parallel C_2 = 1 + 5 = 6f$$

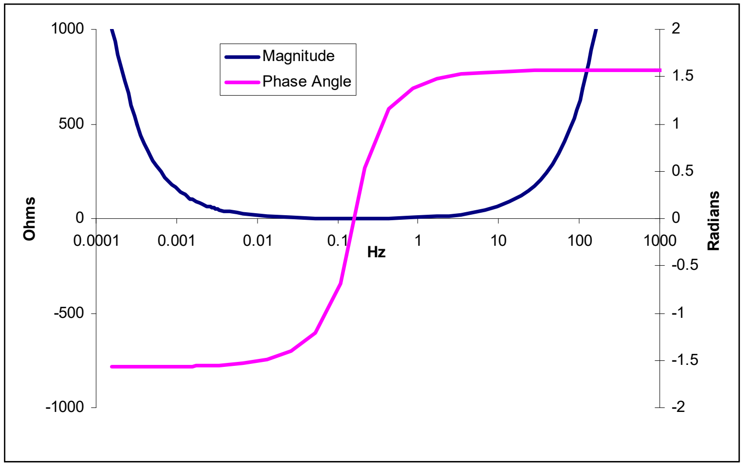
$$C_y = C_x \Leftrightarrow C_3 = \frac{C_x C_3}{C_x + C_3} = \frac{(6)1}{7} = \frac{6}{7}f$$

$$C_{ab} = C_y \parallel C_4 = \frac{6}{7} + 5 = \frac{41}{7} = 5.86f$$

Homework



Find and plot the impedance $Z_{ab}(j\omega)$ as a function of ω



$$Z_{ab}(j\omega) = R + \frac{1}{j\omega C} + j\omega L = R + j(\omega L - \frac{1}{\omega C}) = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \angle \tan^{-1}(\omega \frac{L}{R} - \frac{1}{\omega RC})$$

$$= \sqrt{R^2 + (2\pi fL - \frac{1}{2\pi fC})^2} \angle \tan^{-1}(2\pi f \frac{L}{R} - \frac{1}{2\pi fRC})$$

$$f \to 0; Z_{ab}(j\omega) = \sqrt{R^2 + (2\pi fL - \frac{1}{2\pi fC})^2} \angle \tan^{-1}(2\pi f \frac{L}{R} - \frac{1}{2\pi fRC}) \to \infty \angle -\frac{\pi}{2}$$

Looks like a Capacitor

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi} = 0.159; Z_{ab}(j\omega) = \sqrt{R^2 + (2\pi fL - \frac{1}{2\pi fC})^2} \angle \tan^{-1}(2\pi f \frac{L}{R} - \frac{1}{2\pi fRC}) = R\angle 0 = 1\angle 0$$

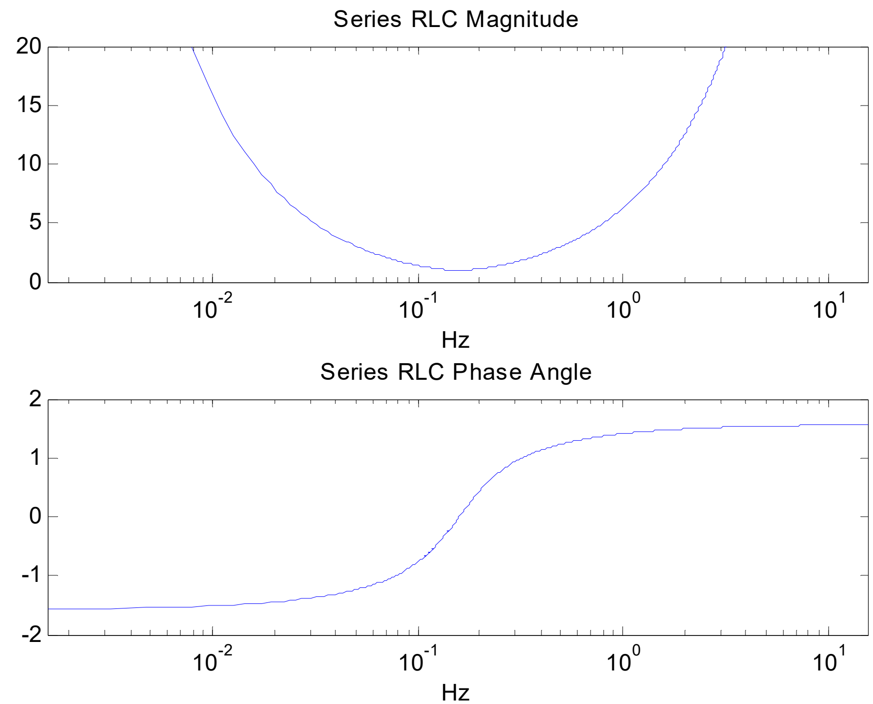
Looks like a Resistor

$$f \to \infty; Z_{ab}(j\omega) = \sqrt{R^2 + (2\pi fL - \frac{1}{2\pi fC})^2} \angle \tan^{-1}(2\pi f \frac{L}{R} - \frac{1}{2\pi fRC}) \to \infty \angle \frac{\pi}{2}$$

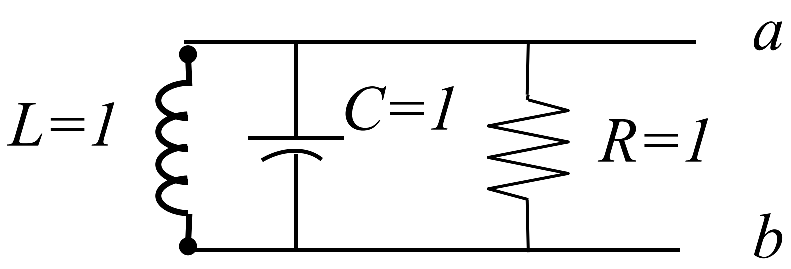
Looks like an Inductor

Matlab Code

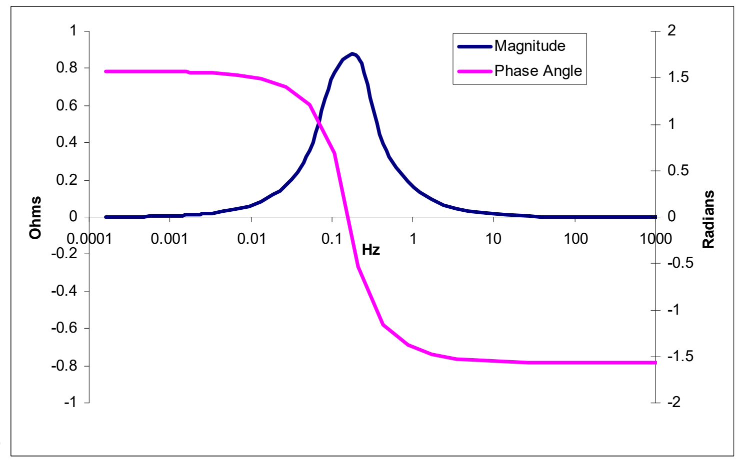
```
clear all;
L=1;C=1;R=1;C=1;
omega=(.01:.01:100);maxomega=length(omega);
for i=1:maxomega
   ZL=complex(0,omega(i)*L);
   ZC=1/complex(0,(omega(i)*C));
   z(i)=R+ZL+ZC;
end
f=omega/(2*pi);
subplot(2,1,1);
semilogx(f,abs(z));
title('Series RLC Magnitude');
xlabel('Hz');
axis([f(1) f(maxomega) 0 20]);
subplot(2,1,2);
semilogx(f,atan2(imag(z),real(z)));
axis([f(1) f(maxomega) -2 2]);
title('Series RLC Phase Angle');
xlabel('Hz');
```



Homework



Find and plot the impedance $Z_{ab}(j\omega)$ as a function of ω



$$Z_{ab}(j\omega)=\frac{1}{\frac{1}{R}+j\omega C+\frac{1}{j\omega L}}=\frac{1}{\frac{1}{R}+j(\omega C-\frac{1}{\omega L})}=\frac{1}{\sqrt{(\frac{1}{R})^2+(\omega C-\frac{1}{\omega L})^2}}\angle-\tan^{-1}(\omega RC-\frac{1}{\omega\frac{L}{R}})$$

$$\omega\to 0; Z_{ab}(j\omega)=\frac{1}{\sqrt{(\frac{1}{R})^2+(\omega C-\frac{1}{\omega L})^2}}\angle-\tan^{-1}(\omega RC-\frac{1}{\omega\frac{L}{R}})\to 0\angle\frac{\pi}{2}$$

Looks like an inductor

$$\omega=\frac{1}{\sqrt{LC}}=1; f=0.159; Z_{ab}(j\omega)=\frac{1}{\sqrt{(\frac{1}{R})^2+(\omega C-\frac{1}{\omega L})^2}}\angle-\tan^{-1}(\omega RC-\frac{1}{\omega\frac{L}{R}})=R\angle 0=1\angle 0$$

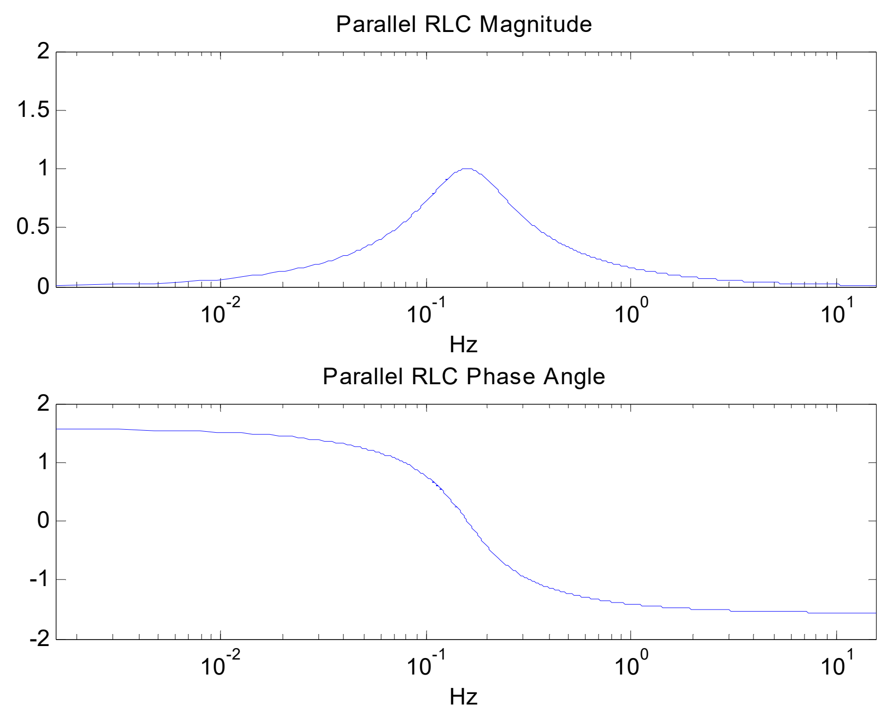
Looks like a resistor

$$\omega\to\infty; Z_{ab}(j\omega)=\frac{1}{\sqrt{(\frac{1}{R})^2+(\omega C-\frac{1}{\omega L})^2}}\angle-\tan^{-1}(\omega RC-\frac{1}{\omega\frac{L}{R}})\to 0\angle-\frac{\pi}{2}$$

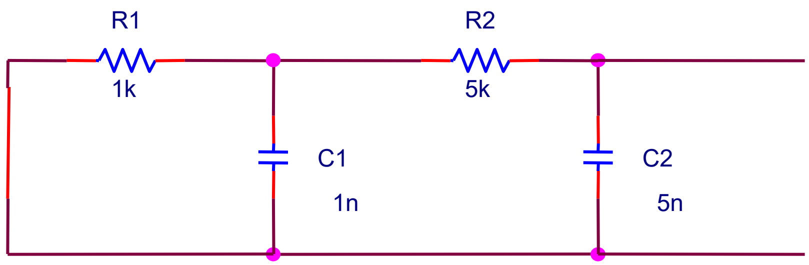
Looks like a Capacitor

Matlab Code

```
clear all;
L=1;C=1;R=1;C=1;
omega=(.01:.01:100);maxomega=length(omega);
for i=1:maxomega
   ZL=complex(0,omega(i)*L);
   ZC=1/complex(0,(omega(i)*C));
   z(i)=1/(1/R+1/ZL+1/ZC);
end
f=omega/(2*pi);
subplot(2,1,1);
semilogx(f,abs(z));
title('Parallel RLC Magnitude');
xlabel('Hz');
axis([f(1) f(maxomega) 0 2]);
subplot(2,1,2);
semilogx(f,atan2(imag(z),real(z)));
title('Parallel RLC Phase Angle');
xlabel('Hz');
axis([f(1) f(maxomega) -2 2]);
```



Homework



a

Find and plot the impedance $Z_{ab}(j\omega)$ as a function of ω

b

Let's define:

$$R_1 = Z_1 = 1k = R; R_2 = Z_3 = 5k = 5R;$$

$$Z_2 = \frac{1}{j\omega C_1} = \frac{1}{j\omega 10^{-9}} = \frac{1}{j\omega C}; Z_4 = \frac{1}{j\omega C_2} = \frac{1}{j\omega 5 10^{-9}} = \frac{1}{j\omega 5C};$$

$$Z_{ab}(j\omega) = (Z_1 \parallel Z_2 \Leftrightarrow Z_3) \parallel Z_4$$

$$Z_1 \parallel Z_2 = \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = \frac{R_1}{1 + j\omega R_1 C_1}$$

$$Z_1 \parallel Z_2 \Leftrightarrow Z_3 = \frac{R_1}{1 + j\omega R_1 C_1} + R_2 = \frac{R_1 + R_2(1 + j\omega R_1 C_1)}{1 + j\omega R_1 C_1}$$

$$= \frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{1 + j\omega R_1 C_1}$$

$$Z_{ab}(j\omega) = (Z_1 \parallel Z_2 \Leftrightarrow Z_3) \parallel Z_4 = \frac{\frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{1 + j\omega R_1 C_1} \times \frac{1}{j\omega C_2}}{\frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{1 + j\omega R_1 C_1} + \frac{1}{j\omega C_2}}$$

$$= \frac{\frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{j\omega C_2(1 + j\omega R_1 C_1)}}{\frac{j\omega C_2(R_1 + R_2 + j\omega R_2 R_1 C_1) + 1 + j\omega R_1 C_1}{j\omega C_2(1 + j\omega R_1 C_1)}}$$

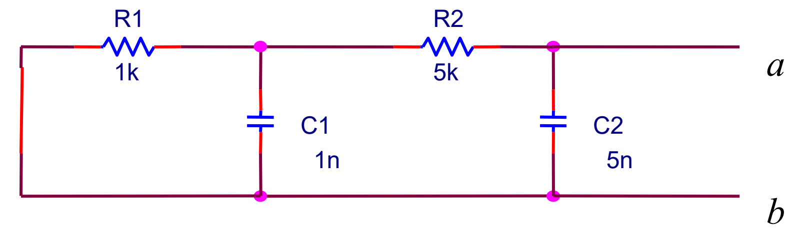
$$Z_{ab}(j\omega) = \frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{j\omega C_2(R_1 + R_2 + j\omega R_2 R_1 C_1) + 1 + j\omega R_1 C_1}$$

$$= \frac{R_1 + R_2 + j\omega R_2 R_1 C_1}{1 - \omega^2 R_2 R_1 C_1 C_2 + j\omega[C_2(R_1 + R_2) + R_1 C_1]}$$

$$= \frac{6R + j\omega 5R^2 C}{1 - (\omega 5RC)^2 + j\omega 31RC} = \frac{6 \times 10^3 + j\omega 5 \times 10^{-3}}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}}$$

$$= \frac{\sqrt{36 \times 10^6 + \omega^2 25 \times 10^{-6}}}{\sqrt{[1 - (\omega 5 \times 10^{-6})^2]^2 + \omega^2 961 \times 10^{-12}}} \angle\{\tan^{-1}(\frac{\omega 5 \times 10^{-3}}{6 \times 10^3}) - \tan^{-1}[\frac{\omega 31 \times 10^{-6}}{1 - (\omega 5 \times 10^{-6})^2}]\}$$

Homework



$$Z_{ab}(j\omega)=\frac{R_1+R_2+j\omega R_2R_1C_1}{1-\omega^2R_2R_1C_1C_2+j\omega[C_2(R_1+R_2)+R_1C_1]}$$

$$=\frac{\sqrt{36\times10^6+\omega^2 25\times10^{-6}}}{\sqrt{[1-(\omega5\times10^{-6})^2]^2+\omega^2 961\times10^{-12}}}\angle\{\tan^{-1}(\frac{\omega5\times10^{-3}}{6\times10^3})-\tan^{-1}[\frac{\omega31\times10^{-6}}{1-(\omega5\times10^{-6})^2}]\}$$

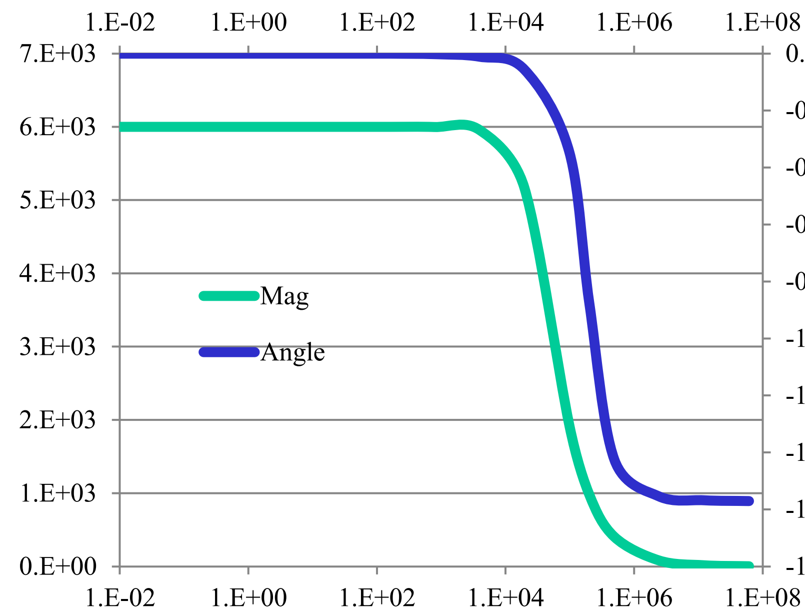
$$\omega=0;Z_{ab}(j\omega)=\frac{\sqrt{36\times10^6}}{\sqrt{[1]^2}}\angle\{\tan^{-1}(0)-\tan^{-1}[0]\}=6\times10^3\angle0$$

$$\omega\to\infty;Z_{ab}(j\omega)\to\frac{j\omega R_2R_1C_1}{-\omega^2R_2R_1C_1C_2}=\frac{j}{-\omega_1C_2}\to0\angle-\frac{\pi}{2}$$

$$\omega=\frac{1}{\sqrt{R_1R_2C_1C_2}}Z_{ab}(j\omega)=\frac{R_1+R_2+j\frac{1}{\sqrt{R_1R_2C_1C_2}}R_2R_1C_1}{+j\frac{1}{\sqrt{R_1R_2C_1C_2}}[C_2(R_1+R_2)+R_1C_1]}$$

$$=\frac{\sqrt{(R_1+R_2)^2+\frac{1}{R_1R_2C_1C}(R_2R_1C_1)^2}}{\frac{1}{\sqrt{R_1R_2C_1C_2}}[C_2(R_1+R_2)+R_1C_1]}\angle\tan^{-1}(\frac{\frac{1}{\sqrt{R_1R_2C_1C_2}}R_2R_1C_1}{R_1+R_2})-\frac{\pi}{2}$$

$$=981\angle0.88$$



Matlab Code

```
clear all;
R1=1e3;C1=1e-9;R2=5e3;C2=5e-9;
omega=(100:1000:10^7);maxomega=length(omega);
for i=1:maxomega
   zC1=1/complex(0,omega(i)*C1);
   zC2=1/complex(0,omega(i)*C2);
   z1(i)=1/(1/R1+1/zC1);
   z2(i)=z1(i)+R2;
   z(i)=1/(1/z2(i)+1/zC2);
end
f=omega/(2*pi);
subplot(2,1,1);
semilogx(f,abs(z));
title('RCLadder Magnitude');
xlabel('Hz');
axis([f(1) f(maxomega) 0 7500]);
subplot(2,1,2);
semilogx(f,atan2(imag(z),real(z)));
title('RCLadder Phase Angle');
xlabel('Hz');
axis([f(1) f(maxomega) -2 2]);
```