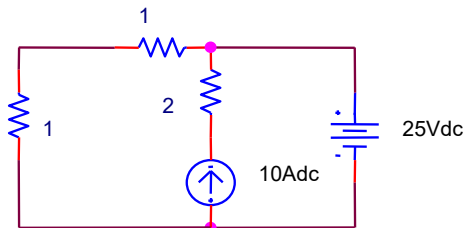
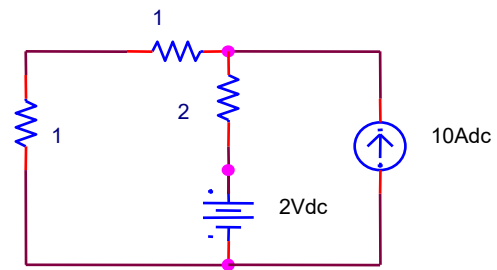
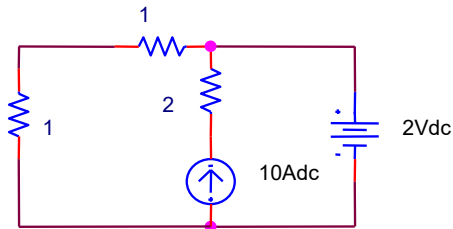


# *Circuit Analysis*

## Lesson #2

# Homework

- Voltage and Current division
  - How does the voltage divide across two capacitors in series? Show your results.
  - How does the current divide among two capacitors in parallel? Show your results.
- Calculate the Currents and Voltages for the following circuits:



## Homework Answers #2

- Voltage and Current division

Voltage division

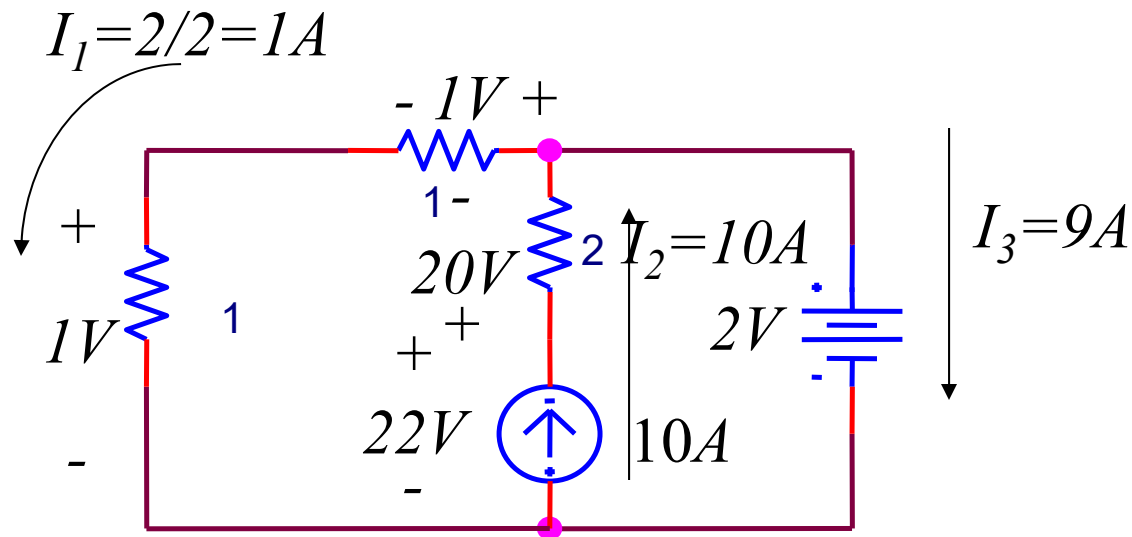
$$V_a = \frac{Z_b}{Z_a + Z_b} V_{in}; V_a = \frac{\frac{1}{j\omega C_a}}{\frac{1}{j\omega C_a} + \frac{1}{j\omega C_b}} V_{in} = \frac{\frac{1}{C_a}}{\frac{1}{C_a} + \frac{1}{C_b}} V_{in} = \frac{C_b}{C_a + C_b} V_{in}$$

Current division

$$I_a = \frac{\frac{1}{Z_b}}{\frac{1}{Z_a} + \frac{1}{Z_b}} I_{in}; I_a = \frac{j\omega C_b}{j\omega C_a + j\omega C_b} I_{in} = \frac{C_b}{C_a + C_b} I_{in}$$

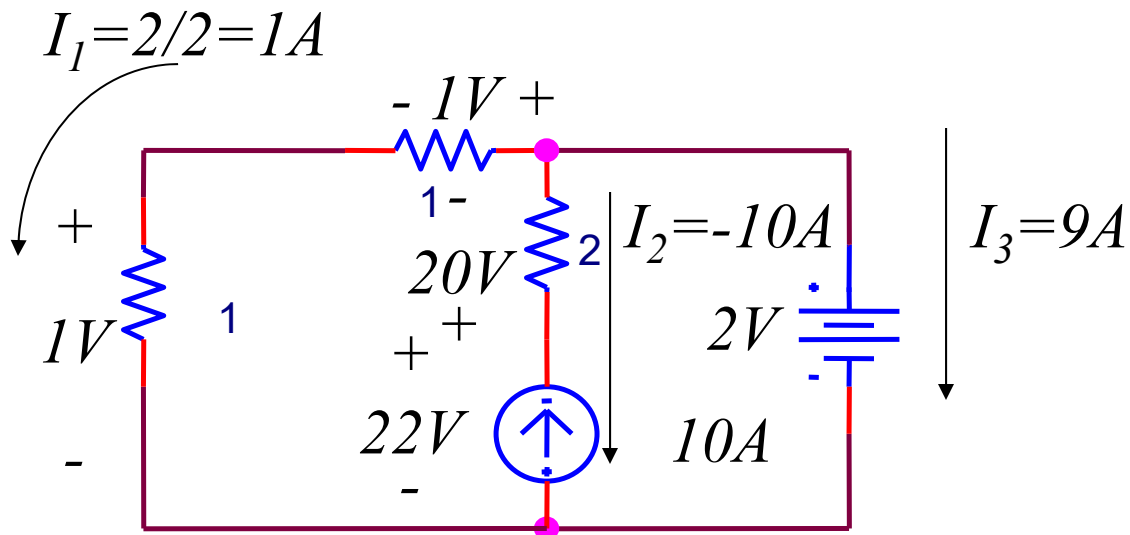
## Homework Answers #2

- Calculate the Currents and Voltages for the following circuits:



## Homework Answers #2

- Calculate the Currents and Voltages for the following circuits:



*Nodal*

$$I_1 + I_2 + I_3 = 0$$

Note:

$$I_2 = -10$$

$$\frac{V}{2} - 10 + I_3 = 0$$

Note:

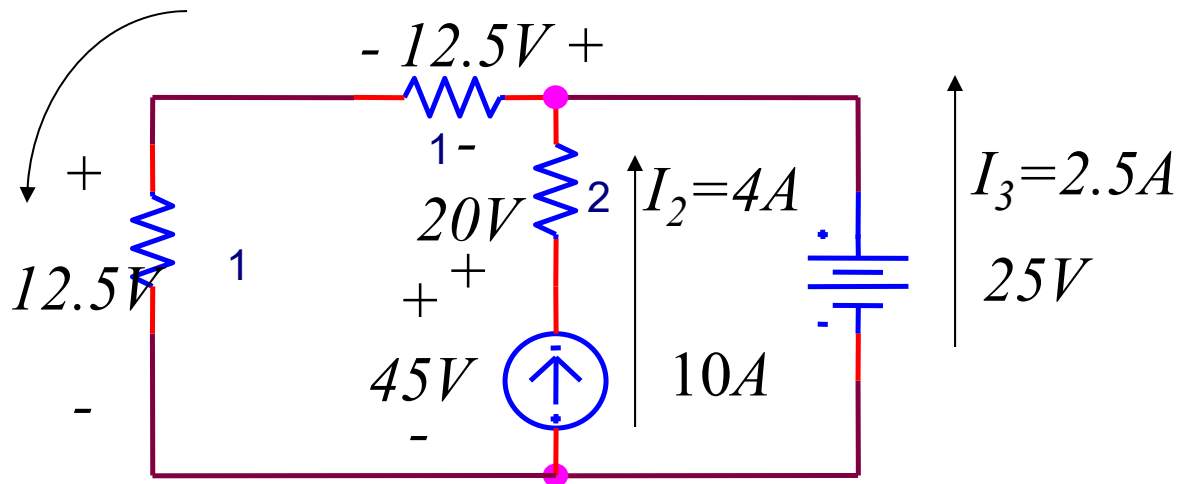
$$V = 2$$

$$1 - 10 + I_3 = 0 \Rightarrow I_3 = 9$$

## Homework Answers #2

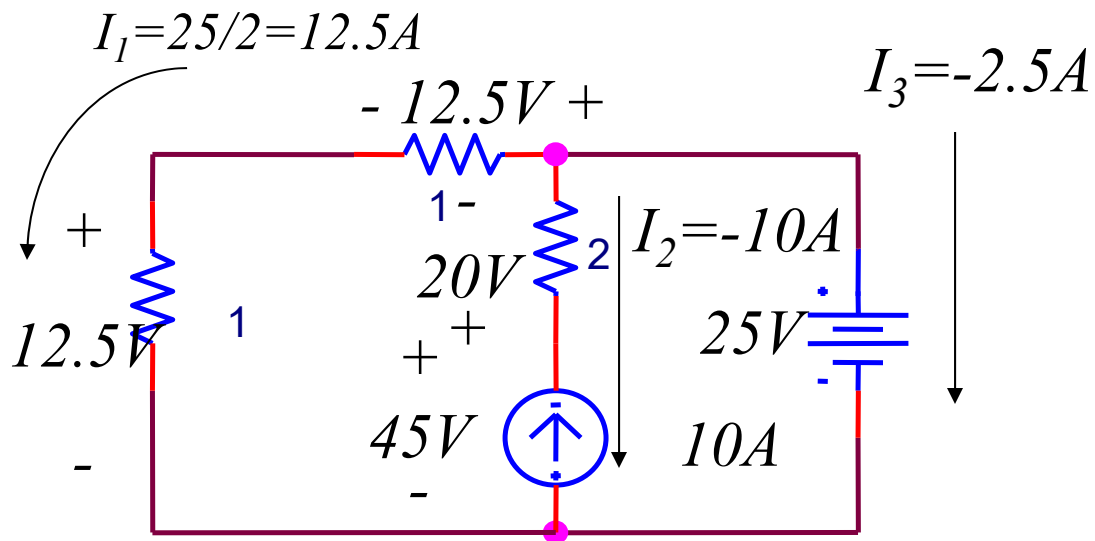
- Calculate the Currents and Voltages for the following circuits:

$$I_1 = 25/2 = 12.5A$$



## Homework Answers #2

- Calculate the Currents and Voltages for the following circuits:



*Nodal*

$$I_1 + I_2 + I_3 = 0$$

Note:

$$I_2 = -10$$

$$\frac{V}{2} - 10 + I_3 = 0$$

Note:

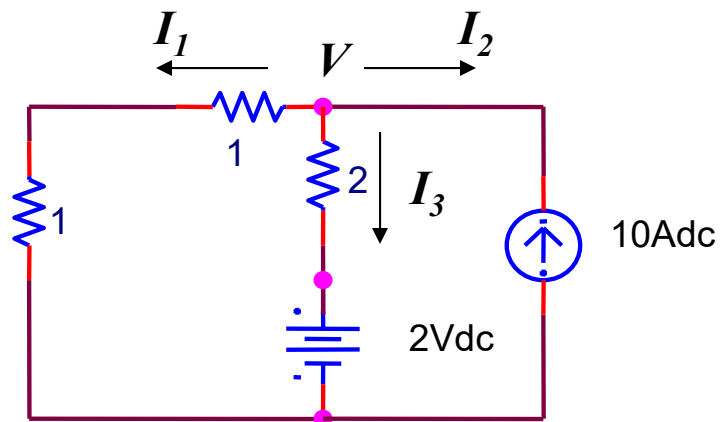
$$V = 25$$

$$12.5 - 10 + I_3 = 0 \Rightarrow I_3 = -2.5$$

$$V_{CS} = 25 - I_2 \cdot 2 = 25 - (-10) \times 2 = 45$$

## Homework Answers #3

- Calculate the Currents and Voltages for the following circuits:



Using Nodal Analysis:

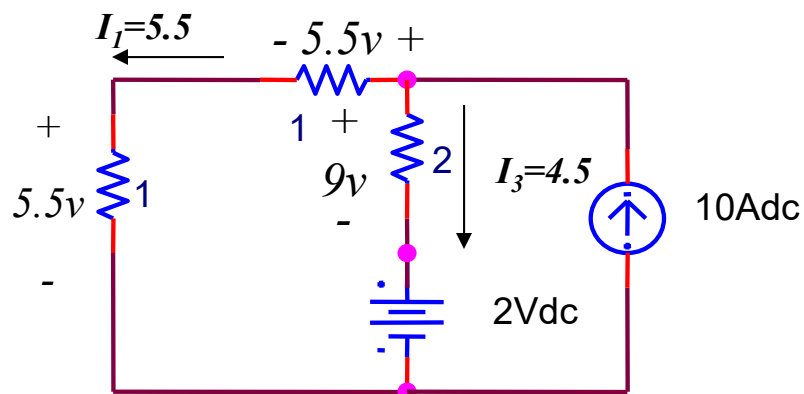
$$I_1 + I_2 + I_3 = 0$$

$$\frac{V}{2} + (-10) + \frac{V-2}{2} = 0$$

$$\frac{2V}{2} = 11 \Rightarrow V = 11$$

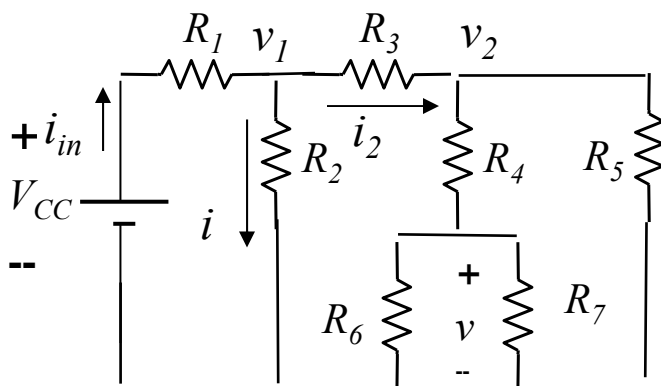
$$I_1 = \frac{11}{2} = 5.5A; I_3 = 10 - 5.5 = 4.5$$

$$V_{cs} = 11$$





# Homework



Calculate the current labeled  $i$  and the voltage labeled  $v$  in the following circuit

$$R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 1\Omega,$$

$$R_4 = 1\Omega, R_5 = 2\Omega, R_6 = 2\Omega,$$

$$R_7 = 2\Omega, V_{cc} = 4\text{v}$$

$$R'_1 = R_6 \parallel R_7 = \frac{R_6 R_7}{R_6 + R_7}$$

$$R'_2 = R_4 + R'_1 = R_4 + \frac{R_6 R_7}{R_6 + R_7}$$

$$R'_3 = R'_2 \parallel R_5 = \left( R_4 + \frac{R_6 R_7}{R_6 + R_7} \right) \parallel R_5 = \frac{\left( R_4 + \frac{R_6 R_7}{R_6 + R_7} \right) R_5}{\left( R_4 + \frac{R_6 R_7}{R_6 + R_7} \right) + R_5} = \frac{R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7}$$

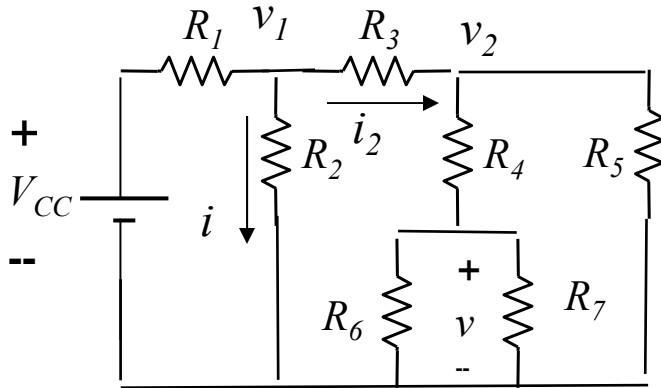
$$R'_4 = R'_3 + R_3 = \frac{R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7} + R_3$$

$$= \frac{R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7}$$

$$R'_5 = R_2 \parallel R'_4$$

$$R'_6 = R_1 + R'_5$$

# Homework



Calculate the current labeled  $i$  and the voltage labeled  $v$  in the following circuit

$$R_1 = 3\Omega, R_2 = 6\Omega, R_3 = 12\Omega,$$

$$R_4 = 4\Omega, R_5 = 2\Omega, R_6 = 2\Omega,$$

$$R_7 = 4\Omega, V_{cc} = 4v$$

$$R'_5 = R_2 \parallel R'_4 = R_2 \parallel \frac{R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7}$$

$$= \frac{R_2 \times \frac{R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7}}{R_2 + \frac{R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}{R_4 R_6 + R_4 R_7 + R_5 R_6 + R_5 R_7 + R_6 R_7}}$$

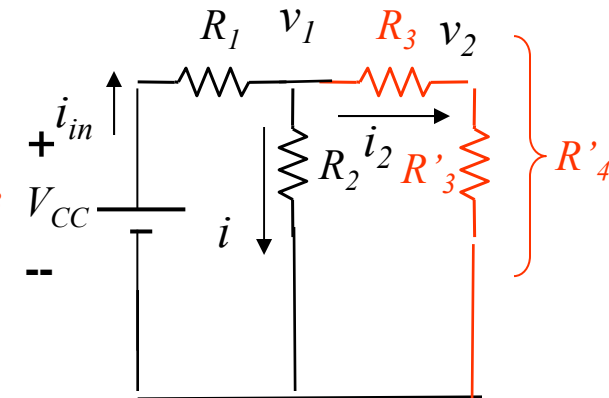
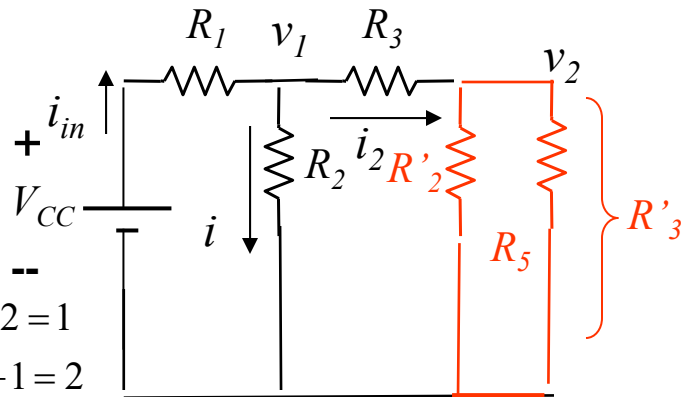
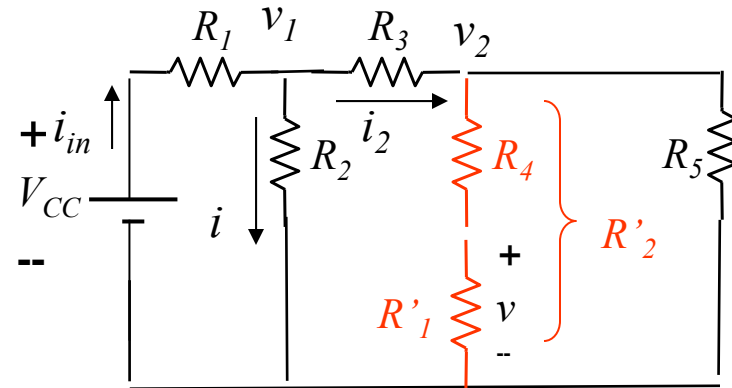
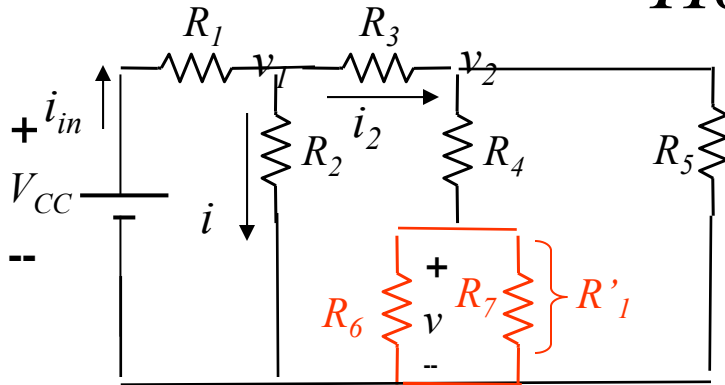
$$= \frac{R_2 R_3 R_4 R_6 + R_2 R_3 R_4 R_7 + R_2 R_3 R_5 R_6 + R_2 R_3 R_5 R_7 + R_2 R_3 R_6 R_7 + R_2 R_4 R_5 R_6 + R_2 R_4 R_5 R_7 + R_2 R_5 R_6 R_7}{R_2 R_4 R_6 + R_2 R_4 R_7 + R_2 R_5 R_6 + R_2 R_5 R_7 + R_2 R_6 R_7 + R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}$$

$$R'_6 = R_1 + R'_5 = R_1 + \frac{R_2 R_3 R_4 R_6 + R_2 R_3 R_4 R_7 + R_2 R_3 R_5 R_6 + R_2 R_3 R_5 R_7 + R_2 R_3 R_6 R_7 + R_2 R_4 R_5 R_6 + R_2 R_4 R_5 R_7 + R_2 R_5 R_6 R_7}{R_2 R_4 R_6 + R_2 R_4 R_7 + R_2 R_5 R_6 + R_2 R_5 R_7 + R_2 R_6 R_7 + R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}$$

$$= \frac{R_1 R_2 R_4 R_6 + R_1 R_2 R_4 R_7 + R_1 R_2 R_5 R_6 + R_1 R_2 R_5 R_7 + R_1 R_2 R_6 R_7 + R_1 R_3 R_4 R_6 + R_1 R_3 R_4 R_7 + R_1 R_3 R_5 R_6 + R_1 R_3 R_5 R_7 + R_1 R_3 R_6 R_7 + R_1 R_4 R_5 R_6 + R_1 R_4 R_5 R_7 + R_1 R_5 R_6 R_7}{R_2 R_4 R_6 + R_2 R_4 R_7 + R_2 R_5 R_6 + R_2 R_5 R_7 + R_2 R_6 R_7 + R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}$$

$$+ \frac{R_2 R_3 R_4 R_6 + R_2 R_3 R_4 R_7 + R_2 R_3 R_5 R_6 + R_2 R_3 R_5 R_7 + R_2 R_3 R_6 R_7 + R_2 R_4 R_5 R_6 + R_2 R_4 R_5 R_7 + R_2 R_5 R_6 R_7}{R_2 R_4 R_6 + R_2 R_4 R_7 + R_2 R_5 R_6 + R_2 R_5 R_7 + R_2 R_6 R_7 + R_3 R_4 R_6 + R_3 R_4 R_7 + R_3 R_5 R_6 + R_3 R_5 R_7 + R_3 R_6 R_7 + R_4 R_5 R_6 + R_4 R_5 R_7 + R_5 R_6 R_7}$$

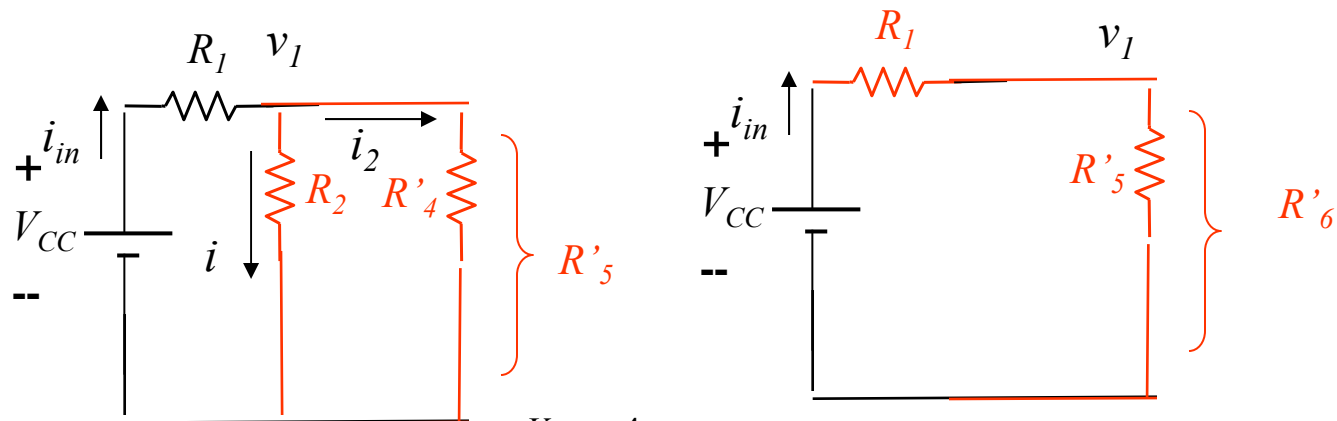
# Homework



$$\begin{aligned}
 R'_1 &= R_6 \parallel R_7 = 2 \parallel 2 = 1 \\
 R'_2 &= R_4 + R'_1 = 1 + 1 = 2 \\
 R'_3 &= R'_2 \parallel R_5 = 2 \parallel 2 = 1 \\
 R'_4 &= R'_3 + R_3 = 1 + 1 = 2 \\
 R'_5 &= R_2 \parallel R'_4 = 2 \parallel 2 = 1 \\
 R'_6 &= R_1 + R'_5 = 1 + 1 = 2
 \end{aligned}$$

$$\begin{aligned}
 i_{in} &= \frac{V_{CC}}{R'_6}; i = i_{in} \frac{R'_4}{R'_4 + R_2}; i_2 = i_{in} \frac{R_2}{R'_4 + R_2}; \\
 v_1 &= i_2 R'_4; v_2 = v_1 \frac{R'_3}{R'_3 + R_3}; v = v_2 \frac{R'_1}{R'_1 + R_4}
 \end{aligned}$$

# Homework



$$i_{in} = \frac{V_{CC}}{R'_6} = \frac{4}{2} = 2;$$

$$R'_1 = R_6 \parallel R_7 = 2 \parallel 2 = 1$$

$$R'_2 = R_4 + R'_1 = 1 + 1 = 2$$

$$R'_3 = R'_2 \parallel R_5 = 2 \parallel 2 = 1$$

$$R'_4 = R'_3 + R_3 = 1 + 1 = 2$$

$$R'_5 = R_2 \parallel R'_4 = 2 \parallel 2 = 1$$

$$R'_6 = R_1 + R'_5 = 1 + 1 = 2$$

$$i = i_{in} \frac{R'_4}{R'_4 + R_2} = 2 \times \frac{2}{2 + 2} = 1$$

$$i_2 = i_{in} \frac{R_2}{R'_4 + R_2} = 1 \times \frac{2}{2 + 2} = 0.5$$

$$v_1 = i_2 R'_4 = 0.5 \times 2 = 1$$

$$v_2 = v_1 \frac{R'_3}{R'_3 + R_3} = 1 \times \frac{1}{1 + 1} = 0.5$$

$$v = v_2 \frac{R'_1}{R'_1 + R_4} = 0.5 \times \frac{1}{1 + 1} = 0.25$$

# Homework

$$R'_1 = R_6 \parallel R_7 = 2 \parallel 2 = 1$$

$$R'_2 = R_4 + R'_1 = 1 + 1 = 2$$

$$R'_3 = R'_2 \parallel R_5 = 2 \parallel 2 = 1$$

$$R'_4 = R'_3 + R_3 = 1 + 1 = 2$$

$$R'_5 = R_2 \parallel R'_4 = 2 \parallel 2 = 1$$

$$R'_6 = R_1 + R'_5 = 1 + 1 = 2$$

$$i_{in} = \frac{V_{cc}}{R'_6} = \frac{4}{2} = 2;$$

$$i = i_{in} \frac{R'_4}{R'_4 + R_2} = 2 \times \frac{2}{2 + 2} = 1$$

$$i_2 = i_{in} \frac{R_2}{R'_4 + R_2} = 2 \times \frac{2}{2 + 2} = 1$$

$$v_1 = i_2 R'_4 = 1 \times 2 = 2$$

$$v_2 = v_1 \frac{R'_3}{R'_3 + R_3} = 2 \times \frac{1}{1 + 1} = 1$$

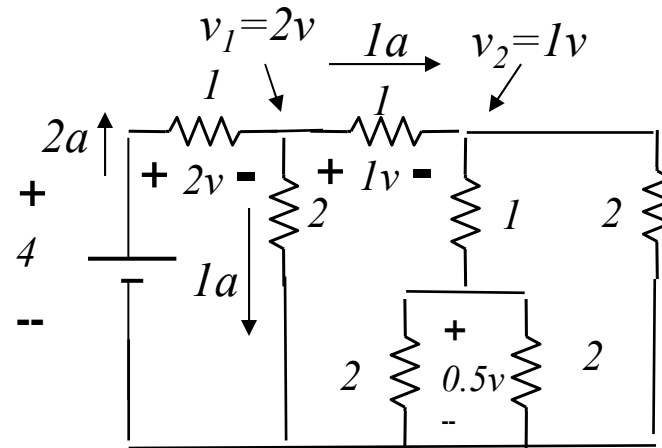
$$v = v_2 \frac{R'_1}{R'_1 + R_4} = 1 \times \frac{1}{1 + 1} = 0.5$$

Calculate the current labeled  $i$  and the voltage labeled  $v$  in the following circuit

$$R_1 = 1\Omega, R_2 = 2\Omega, R_3 = 1\Omega,$$

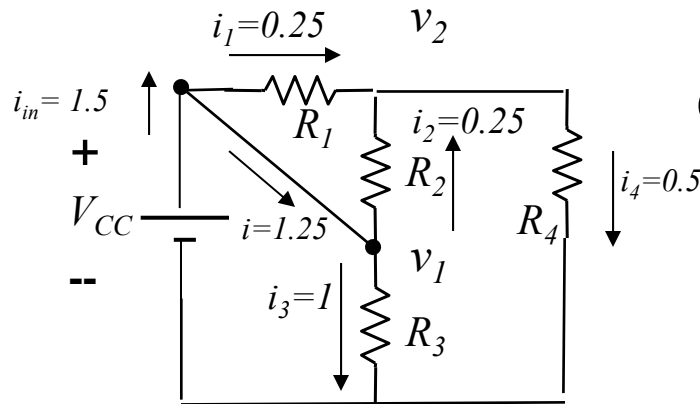
$$R_4 = 1\Omega, R_5 = 2\Omega, R_6 = 2\Omega,$$

$$R_7 = 2\Omega, V_{cc} = 4v$$



**CHECK YOUR  
ANSWERS!**

# Homework



Calculate the current labeled,  $i$ .

$$R_1 = 2\Omega, R_2 = 2\Omega, R_3 = 2\Omega, R_4 = 3\Omega, V_{cc} = 2v$$

$$R'_1 = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = 2 \parallel 2 = 1$$

$$R'_2 = R_4 + R'_1 = R_4 + \frac{R_1 R_2}{R_1 + R_2} = 3 + 1 = 4$$

$$R'_3 = R'_2 \parallel R_3 = \left( R_4 + \frac{R_1 R_2}{R_1 + R_2} \right) \parallel R_3 = \frac{\left( R_4 + \frac{R_1 R_2}{R_1 + R_2} \right) R_3}{\left( R_4 + \frac{R_1 R_2}{R_1 + R_2} \right) + R_3}$$

$$= \frac{R_1 R_3 R_4 + R_2 R_3 R_4 + R_1 R_2 R_3}{R_1 R_4 + R_2 R_4 + R_1 R_3 + R_2 R_3 + R_1 R_2}$$

$$= 4 \parallel 2 = \frac{8}{6} = \frac{4}{3}$$

$$i_{in} = \frac{V_{cc}}{R'_3} = \frac{2}{\frac{4}{3}} = \frac{3}{2};$$

$$i_3 = i_{in} \frac{R'_2}{R'_2 + R_3} = \frac{3}{2} \times \frac{4}{4+2} = \frac{3}{2} \times \frac{4}{6} = \frac{2}{2} = 1; v_1 = 2v$$

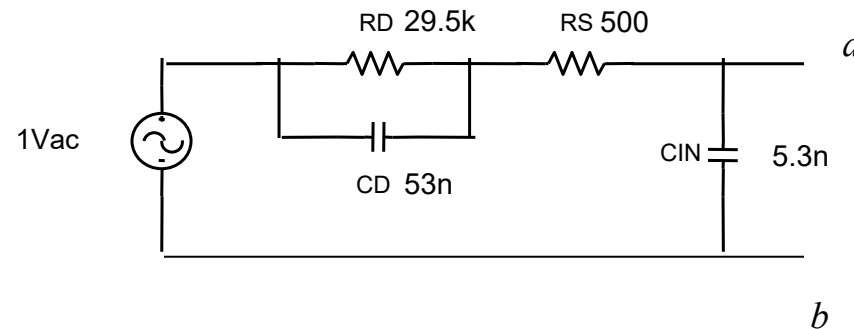
$$i_4 = i_{in} \frac{R_3}{R'_2 + R_3} = \frac{3}{2} \times \frac{2}{6} = \frac{1}{2}; v_1 = \frac{3}{2}v$$

$$i_1 = i_4 \frac{R_2}{R_2 + R_1} = \frac{1}{2} \times \frac{2}{4+2} = \frac{1}{4}$$

$$i_2 = i_4 \frac{R_1}{R_2 + R_1} = \frac{1}{2} \times \frac{2}{4+2} = \frac{1}{4}$$

$$i = i_{in} - i_1 = \frac{3}{2} - \frac{1}{4} = \frac{5}{4}$$

# Homework



An electrode is connected to an oscilloscope which has a purely capacitance input impedance,  $C_{IN}$ . Find and plot the output voltage  $V_{ab}(j\omega)$  as function of  $\omega$ .  
Use Matlab to perform the plot.

# Electrode Impedance

RD=29.5k CD=53nf RS=500 Cin= 5.3nf

$$Z_E = \frac{R_D + R_S + j\omega R_D R_S C_D}{1 + j\omega R_D C_D}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C_{in}}}{\frac{1}{j\omega C_{in}} + Z_E} = \frac{1}{1 + j\omega C_{in} Z_E} = \frac{1}{1 + j\omega C_{in} \frac{R_D + R_S + j\omega R_D R_S C_D}{1 + j\omega R_D C_D}}$$

$$= \frac{1 + j\omega R_D C_D}{1 + j\omega R_D C_D + j\omega C_{in}(R_D + R_S + j\omega R_D R_S C_D)}$$

$$= \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D R_S C_D C_{in} + j\omega(R_D C_D + C_{in}(R_D + R_S))}$$

$$\left. \frac{V_{out}}{V_{in}} \right|_{\omega=0} = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D R_S C_D C_{in} + j\omega(R_D C_D + C_{in}(R_D + R_S))} \Bigg|_{\omega=0} = 1$$

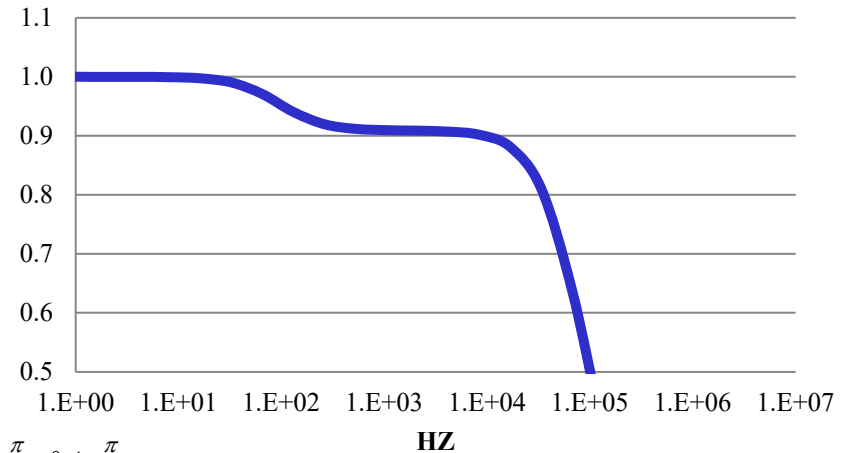
$$\left. \frac{V_{out}}{V_{in}} \right|_{\omega \rightarrow \infty} = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D R_S C_D C_{in} + j\omega(R_D C_D + C_{in}(R_D + R_S))} \Bigg|_{\omega \rightarrow \infty} \rightarrow \frac{j\omega R_D C_D}{-\omega^2 R_D R_S C_D C_{in}} = \frac{1}{\omega R_S C_{in}} \angle -\frac{\pi}{2} = 0 \angle -\frac{\pi}{2}$$

$$\left. \frac{V_{out}}{V_{in}} \right|_{\omega = \frac{1}{\sqrt{R_D R_S C_D C_{in}}}} = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D R_S C_D C_{in} + j\omega(R_D C_D + C_{in}(R_D + R_S))} \Bigg|_{\omega = \frac{1}{\sqrt{R_D R_S C_D C_{in}}}} = \frac{1 + j \frac{1}{\sqrt{R_D R_S C_D C_{in}}} R_D C_D}{j \frac{1}{\sqrt{R_D R_S C_D C_{in}}} (R_D C_D + C_{in}(R_D + R_S))}$$

$$= \frac{\sqrt{R_D R_S C_D C_{in}} + j R_D C_D}{j(R_D C_D + C_{in}(R_D + R_S))} = \frac{\sqrt{R_D R_S C_D C_{in}} + j R_D C_D}{(R_D C_D + C_{in}(R_D + R_S))} \angle \tan^{-1}\left(\frac{R_D C_D}{\sqrt{R_D R_S C_D C_{in}}}\right) - \frac{\pi}{2}$$

$$\frac{V_{out}}{V_{in}} = \frac{1 + j \frac{f}{f_p}}{1 - \left(\frac{f}{f_R}\right)^2 + j f \left(\frac{1}{f_p} + \frac{1}{f_m}\right)} = \frac{\sqrt{1 + \left(\frac{f}{f_p}\right)^2}}{\sqrt{\left(1 - \left(\frac{f}{f_R}\right)^2\right)^2 + \left(f \left(\frac{1}{f_p} + \frac{1}{f_m}\right)\right)^2}} \angle \tan^{-1}\left(\frac{f}{f_p}\right) - \tan^{-1}\left(\frac{f \left(\frac{1}{f_p} + \frac{1}{f_m}\right)}{1 - \left(\frac{f}{f_R}\right)^2}\right)$$

$$f = \frac{\omega}{2\pi}, f_p = \frac{1}{2\pi R_D C_D}, f_R = \frac{1}{2\pi \sqrt{R_D R_S C_D C_{in}}}, f_m = \frac{1}{2\pi C_{in}(R_D + R_S)}$$





# Electrode Impedance

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{1 + j\omega R_D C_D}{1 - \omega^2 R_D R_S C_D C_{in} + j\omega(R_D C_D + C_{in}(R_D + R_S))}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + sR_D C_D}{1 + s^2 R_D R_S C_D C_{in} + s(R_D C_D + C_{in}(R_D + R_S))} = \frac{sR_D C_D + 1}{s^2 R_D R_S C_D C_{in} + s(R_D C_D + C_{in}(R_D + R_S)) + 1} = \frac{R_D C_D}{R_D R_S C_D C_{in}} \frac{s + \frac{1}{R_D C_D}}{s^2 + s\left(\frac{R_D C_D + C_{in}(R_D + R_S)}{R_D R_S C_D C_{in}}\right) + \frac{1}{R_D R_S C_D C_{in}}}$$

$$= \frac{1}{R_S C_{in}} \frac{s + \frac{1}{R_D C_D}}{s^2 + s\left(\frac{R_D C_D + C_{in}(R_D + R_S)}{R_D R_S C_D C_{in}}\right) + \frac{1}{R_D R_S C_D C_{in}}} = \frac{1}{R_S C_{in}} \frac{s + s_z}{(s + s_{p_1})(s + s_{p_2})}$$

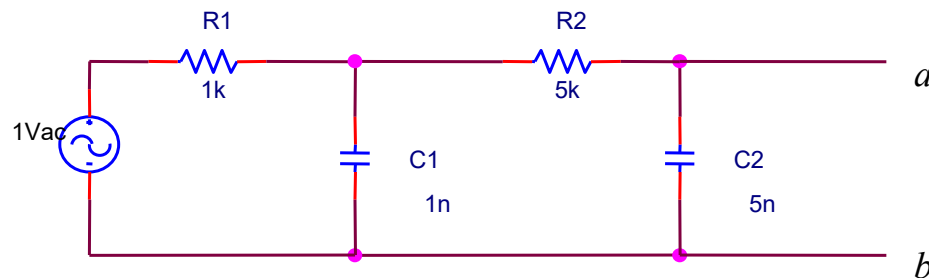
$$s_z = -\frac{1}{R_D C_D}; s_{p_1, p_2} = \frac{-\frac{R_D C_D + C_{in}(R_D + R_S)}{R_D R_S C_D C_{in}} \pm \sqrt{\left(\frac{R_D C_D + C_{in}(R_D + R_S)}{R_D R_S C_D C_{in}}\right)^2 - 4 \frac{1}{R_D R_S C_D C_{in}}}}{2} = \frac{-R_D C_D + C_{in}(R_D + R_S) \pm \sqrt{(R_D C_D + C_{in}(R_D + R_S))^2 - 4 R_D R_S C_D C_{in}}}{2 R_D R_S C_D C_{in}}$$

$$s_z = -640; s_{p_1, p_2} = -581, 4.2 \times 10^5$$

$$f_z = 102 \text{ Hz}; f_{p_1, p_2} = 92.5 \text{ Hz}, 6.6 \times 10^4 \text{ Hz}$$

## Homework

- Repeat the analysis of this circuit using Mesh and Nodal Analysis. That is find  $V_{ab}$  as a function of frequency



## Homework Answers #7

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

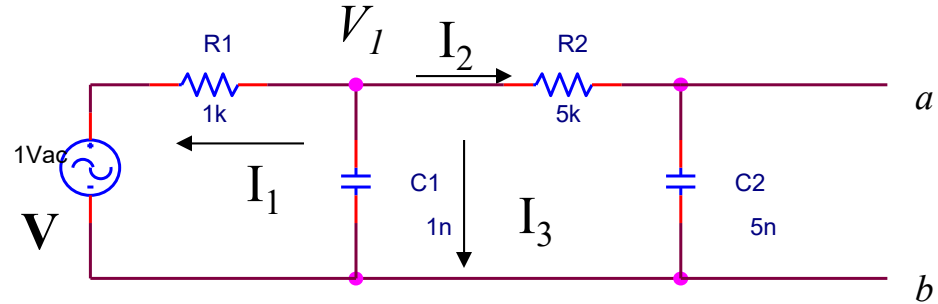
$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_1 - V}{Z_{R_1}} + \frac{V_1}{Z_{R_2} + Z_{C_2}} + \frac{V_1}{Z_{C_1}} = 0$$

$$\frac{V_1}{Z_{R_1}} + \frac{V_1}{Z_{R_2} + Z_{C_2}} + \frac{V_1}{Z_{C_1}} = \frac{V}{Z_{R_1}}$$

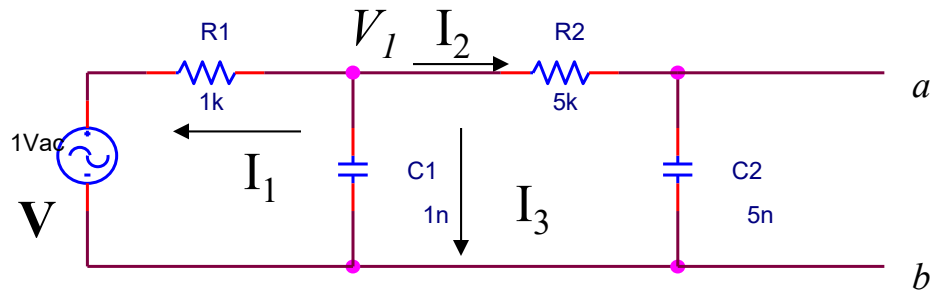
$$\left( \frac{1}{Z_{R_1}} + \frac{1}{Z_{R_2} + Z_{C_2}} + \frac{1}{Z_{C_1}} \right) V_1 = \frac{V}{Z_{R_1}}$$

$$\left( \frac{(Z_{R_2} + Z_{C_2})Z_{C_1} + Z_{R_1}Z_{C_1} + (Z_{R_2} + Z_{C_2})Z_{R_1}}{Z_{R_1}(Z_{R_2} + Z_{C_2})Z_{C_1}} \right) V_1 = \frac{V}{Z_{R_1}}$$



## Homework Answers #8

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



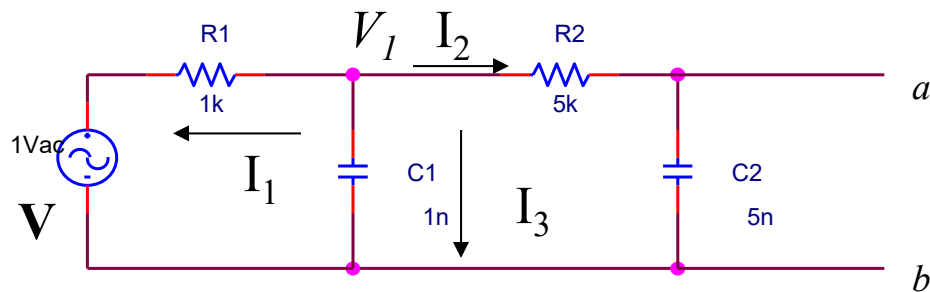
$$V_1 = \frac{(Z_{R_2} + Z_{C_2})Z_{C_1}}{Z_{R_2}Z_{C_1} + Z_{C_2}Z_{C_1} + Z_{R_1}Z_{C_1} + Z_{R_2}Z_{R_1} + Z_{C_2}Z_{R_1}} V$$

$$V_{ab} = \frac{Z_{C_2}}{(Z_{R_2} + Z_{C_2})} V_1 = \frac{Z_{C_2}}{(Z_{R_2} + Z_{C_2})} \frac{(Z_{R_2} + Z_{C_2})Z_{C_1}}{Z_{R_2}Z_{C_1} + Z_{C_2}Z_{C_1} + Z_{R_1}Z_{C_1} + Z_{R_2}Z_{R_1} + Z_{C_2}Z_{R_1}} V$$

$$= \frac{Z_{C_1}Z_{C_2}}{Z_{R_2}Z_{C_1} + Z_{C_2}Z_{C_1} + Z_{R_1}Z_{C_1} + Z_{R_2}Z_{R_1} + Z_{C_2}Z_{R_1}} V$$

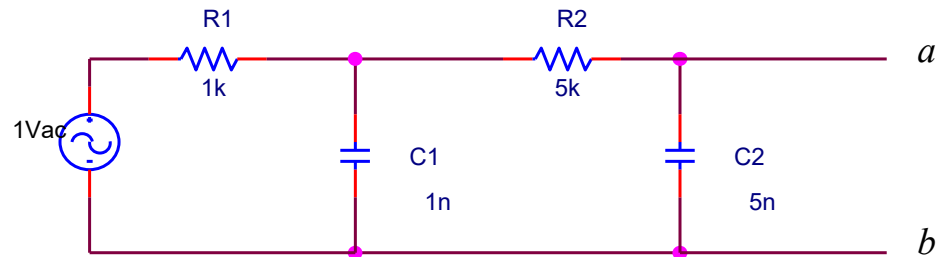
## Homework Answers #8

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



$$\begin{aligned}
 V_{ab} &= \frac{Z_{C_1} Z_{C_2}}{Z_{R_2} Z_{C_1} + Z_{C_2} Z_{C_1} + Z_{R_1} Z_{C_1} + Z_{R_2} Z_{R_1} + Z_{C_2} Z_{R_1}} V \\
 &= \frac{\frac{1}{j\omega C_1} \frac{1}{j\omega C_2}}{R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2} + R_1 \frac{1}{j\omega C_1} + R_2 R_1 + \frac{1}{j\omega C_2} R_1} \\
 &= \frac{1}{j\omega C_2 R_2 + 1 + j\omega C_2 R_1 - \omega^2 C_1 C_2 R_2 R_1 + j\omega C_2 R_1} \\
 &= \frac{1}{1 - \omega^2 C_1 C_2 R_2 R_1 + j\omega(C_2 R_2 + C_2 R_1 + C_2 R_1)}
 \end{aligned}$$

## Homework Answers #6



$$\frac{V_{ab}}{V} = \frac{1}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]}$$

$$= \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}}$$

$$= \frac{1}{\sqrt{[1 - (\omega 5 \times 10^{-6})^2]^2 + \omega^2 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\omega 31 \times 10^{-6}}{1 - (\omega 5 \times 10^{-6})^2} \right]$$

$$\frac{V_{ab}}{V} \Big|_{\omega=0} = \frac{1}{\sqrt{[1 - (0 \times 5 \times 10^{-6})^2]^2 + 0 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{0 \times 31 \times 10^{-6}}{1 - (0 \times 5 \times 10^{-6})^2} \right]$$

$$= \frac{1}{\sqrt{1}} \angle -\tan^{-1} \left[ \frac{0}{1} \right] = 1 \angle 0$$

$$\frac{V_{ab}}{V} \Big|_{\omega=\frac{1}{5 \times 10^{-6}}} = \frac{1}{\sqrt{[1 - (\frac{1}{5 \times 10^{-6}} \times 5 \times 10^{-6})^2]^2 + \frac{1}{5 \times 10^{-6}}^2 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\frac{1}{5 \times 10^{-6}} \times 31 \times 10^{-6}}{1 - (\frac{1}{5 \times 10^{-6}} \times 5 \times 10^{-6})^2} \right] = \frac{1}{\sqrt{[0]^2 + \frac{1}{5 \times 10^{-6}}^2 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\frac{1}{5 \times 10^{-6}} \times 31 \times 10^{-6}}{0} \right] = \frac{1}{\sqrt{38.44}} \angle -\tan^{-1} \left[ \frac{6.2}{0} \right]$$

$$= \frac{1}{6.2} \angle -\frac{\pi}{2}$$

$$\frac{V_{ab}}{V} \Big|_{\omega \rightarrow \infty} = \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}} \Big|_{\omega \rightarrow \infty} \rightarrow \frac{1}{-(\omega 5 \times 10^{-6})^2} \rightarrow 0 \angle -\pi$$

# Homework Answers #6

$$\frac{V_{ab}}{V_{a'b}} = \frac{1}{\frac{1}{j\omega C_2} + R_2} = \frac{1}{1 + j\omega R_2 C_2}$$

$$\frac{V_{a'b}}{V} = \frac{Z_{a'b}}{Z_{a'b} + R_1}$$

$$Z_{a'b} = \frac{\left(\frac{1}{j\omega C_2} + R_2\right) \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_2} + R_2 + \frac{1}{j\omega C_1}} = \frac{\left(\frac{1 + j\omega C_2 R_2}{j\omega C_2}\right) \frac{1}{j\omega C_1}}{\left(\frac{1 + j\omega C_2 R_2}{j\omega C_2}\right) + \frac{1}{j\omega C_1}}$$

$$= \frac{1 + j\omega C_2 R_2}{j\omega C_1(1 + j\omega C_2 R_2) + j\omega C_2} = \frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)}$$

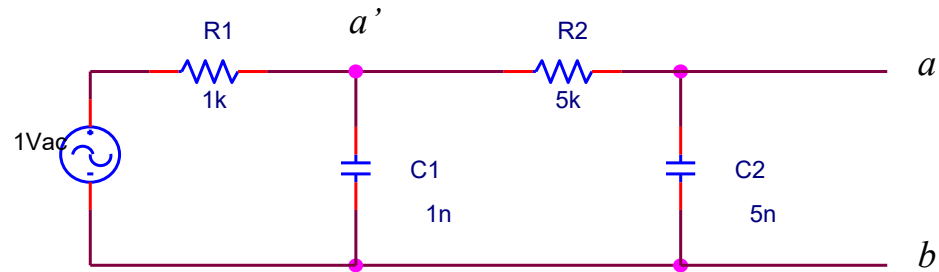
$$\frac{V_{a'b}}{V} = \frac{Z_{a'b}}{Z_{a'b} + R_1} = \frac{\frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)}}{\frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)} + R_1}$$

$$= \frac{1 + j\omega C_2 R_2}{1 + j\omega C_2 R_2 + R_1(-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2))}$$

$$= \frac{1 + j\omega C_2 R_2}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$\frac{V_{ab}}{V} = \frac{V_{ab}}{V_{a'b}} \frac{V_{a'b}}{V} = \frac{1}{1 + j\omega R_2 C_2} \frac{1 + j\omega C_2 R_2}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$= \frac{1}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

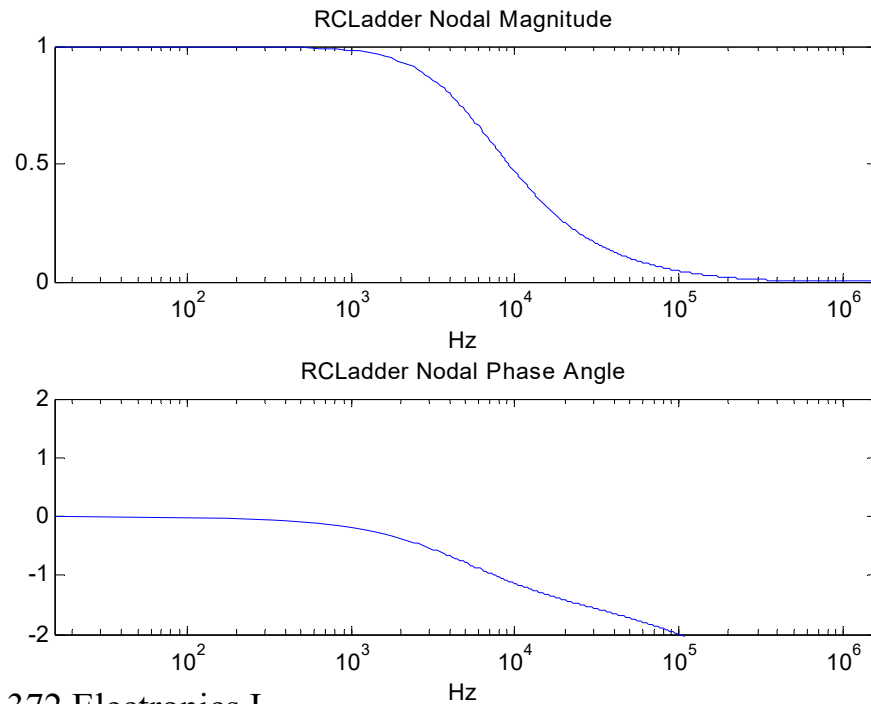


$$\frac{V_{ab}}{V} = \frac{1}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$= \frac{1}{\sqrt{(1 - \omega^2 C_2 R_2 C_1 R_1)^2 + (\omega(R_1 C_1 + R_1 C_2 + C_2 R_2))^2}} \angle \tan^{-1}\left(\frac{\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}{1 - \omega^2 C_2 R_2 C_1 R_1}\right)$$

# Matlab Code

```
clear all;
R1=1e3;C1=1e-9;R2=5e3;C2=5e-9;
omega=(100:1000:10^7);maxomega=length(omega);
z1=R1;GIN=[1/R1];
for i=1:maxomega
    zC1=1/complex(0,omega(i)*C1);
    zC2=1/complex(0,omega(i)*C2);
    z3=R2+zC2;
    G=1/R1+1/zC1+1/z3;
    V1=G\GIN;
    VOUT(i)=V1*(zC2/z3);
end
f=omega/(2*pi);
subplot(2,1,1);
semilogx(f,abs(VOUT));
title('RCLadder Nodal Magnitude');
xlabel('Hz');
axis([f(1) f(maxomega) 0 1]);
subplot(2,1,2);
semilogx(f,atan2(imag(VOUT),real(VOUT)));
title('RCLadder Nodal Phase Angle');
xlabel('Hz');
axis([f(1) f(maxomega) -2 2]);
```





# Homework Answers #6

$$\frac{V_{ab}}{V_{a'b}} = \frac{1}{\frac{1}{j\omega C_2} + R_2} = \frac{1}{1 + j\omega R_2 C_2}$$

$$\frac{V_{a'b}}{V} = \frac{Z_{a'b}}{Z_{a'b} + R_1}$$

$$Z_{a'b} = \frac{\left(\frac{1}{j\omega C_2} + R_2\right) \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_2} + R_2 + \frac{1}{j\omega C_1}} = \frac{\left(\frac{1 + j\omega C_2 R_2}{j\omega C_2}\right) \frac{1}{j\omega C_1}}{\left(\frac{1 + j\omega C_2 R_2}{j\omega C_2}\right) + \frac{1}{j\omega C_1}}$$

$$= \frac{1 + j\omega C_2 R_2}{j\omega C_1(1 + j\omega C_2 R_2) + j\omega C_2} = \frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)}$$

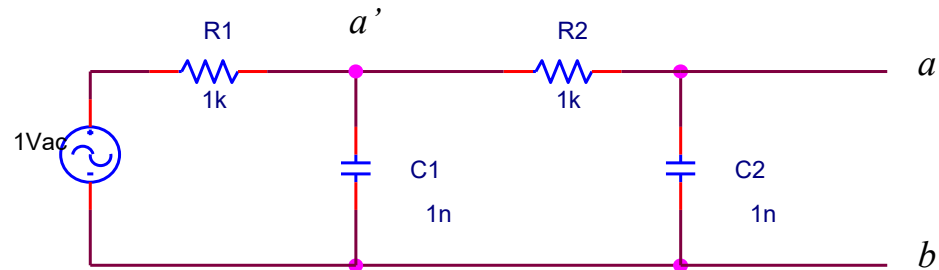
$$\frac{V_{a'b}}{V} = \frac{Z_{a'b}}{Z_{a'b} + R_1} = \frac{\frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)}}{\frac{1 + j\omega C_2 R_2}{-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2)} + R_1}$$

$$= \frac{1 + j\omega C_2 R_2}{1 + j\omega C_2 R_2 + R_1(-\omega^2 C_2 R_2 C_1 + j\omega(C_1 + C_2))}$$

$$= \frac{1 + j\omega C_2 R_2}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$\frac{V_{ab}}{V} = \frac{V_{ab}}{V_{a'b}} \frac{V_{a'b}}{V} = \frac{1}{1 + j\omega R_2 C_2} \frac{1 + j\omega C_2 R_2}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$= \frac{1}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$



$$\frac{V_{ab}}{V} = \frac{1}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

$$= \frac{1}{\sqrt{(1 - \omega^2 C_2 R_2 C_1 R_1)^2 + (\omega(R_1 C_1 + R_1 C_2 + C_2 R_2))^2}} \angle -\tan^{-1}\left(\frac{\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}{1 - \omega^2 C_2 R_2 C_1 R_1}\right)$$

# Homework Answers #6

$$\frac{V_{ab}}{V} = \frac{1}{1 - \omega^2 C_2 R_2 C_1 R_1 + j\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}$$

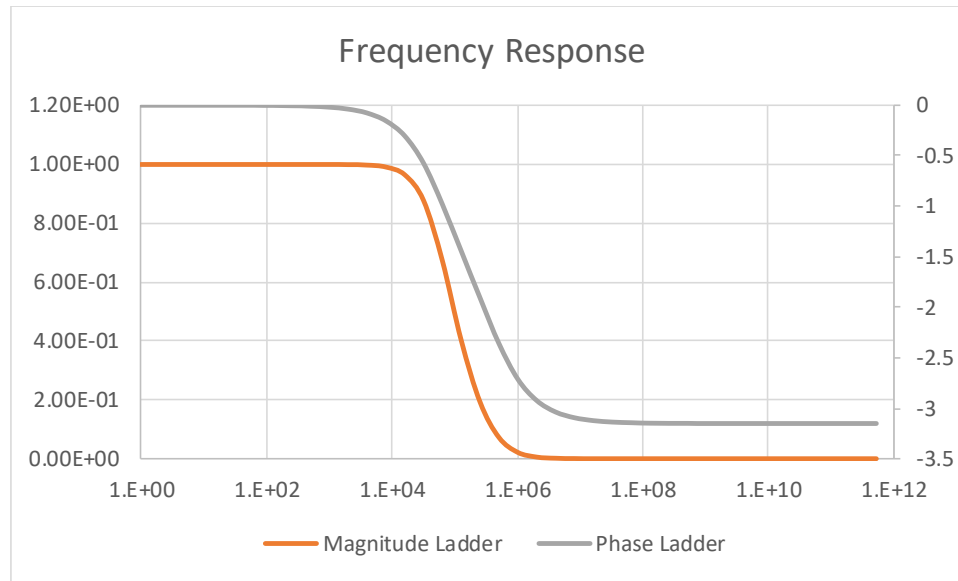
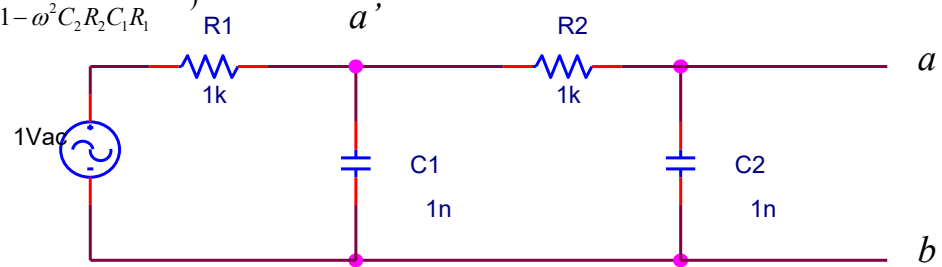
$$= \frac{1}{\sqrt{(1 - \omega^2 C_2 R_2 C_1 R_1)^2 + (\omega(R_1 C_1 + R_1 C_2 + C_2 R_2))^2}} \angle -\tan^{-1}\left(\frac{\omega(R_1 C_1 + R_1 C_2 + C_2 R_2)}{1 - \omega^2 C_2 R_2 C_1 R_1}\right)$$

$$\frac{V_{ab}}{V} \Big|_{\omega=0} = \frac{1}{(1-0) + j0} = 1 \angle 0$$

$$\frac{V_{ab}}{V} \Big|_{\omega \rightarrow \infty} \rightarrow \frac{1}{-\omega^2 C_2 C_1 R_1 R_2} \Big|_{\omega \rightarrow \infty} = 0 \angle \pm \pi = 0 \angle -\pi$$

$$\frac{V_{ab}}{V} \Big|_{\omega = \frac{1}{\sqrt{C_2 C_1 R_1 R_2}}} = \frac{1}{(0) + j \frac{1}{\sqrt{C_2 C_1 R_1 R_2}} [C_2(R_1 + R_2) + C_1 R_1]}$$

$$= \frac{\sqrt{C_2 C_1 R_1 R_2}}{C_2(R_1 + R_2) + C_1 R_1} \angle -\frac{\pi}{2}$$



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J.Schesser

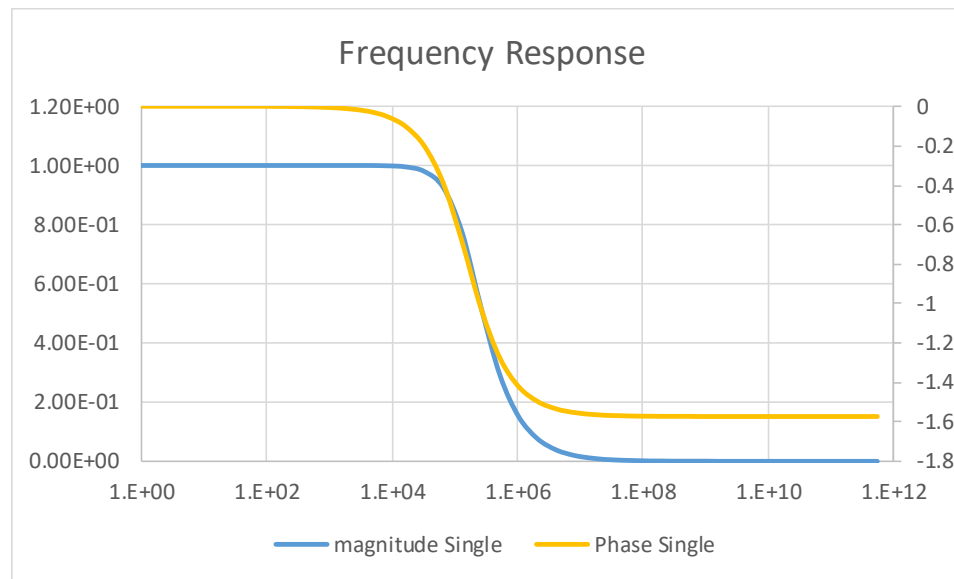
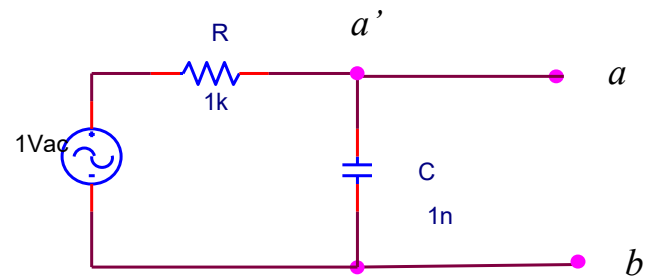
# Homework Answers #6

$$\frac{V_{ab}}{V} = \frac{j\omega C}{\frac{1}{j\omega C} + R} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}(\omega RC)$$

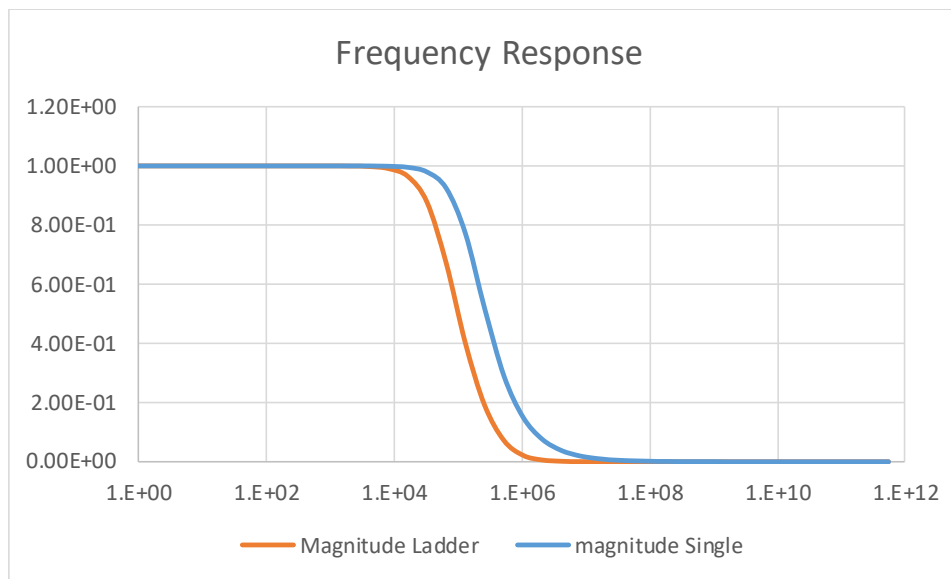
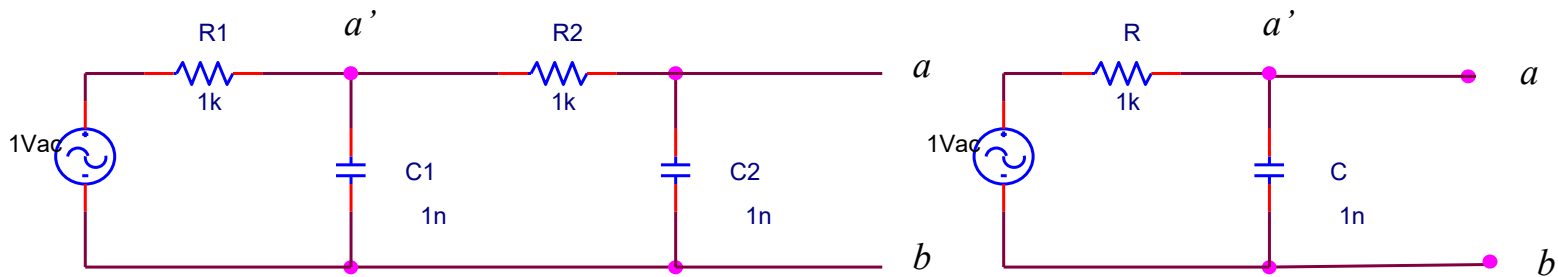
$$\frac{V_{ab}}{V} \Big|_{\omega=0} = \frac{1}{1 + j0} = 1 \angle 0$$

$$\frac{V_{ab}}{V} \Big|_{\omega \rightarrow \infty} \rightarrow \frac{1}{j\omega RC} = 0 \angle -\frac{\pi}{2}$$

$$\frac{V_{ab}}{V} \Big|_{\omega = \frac{1}{RC}} = \frac{1}{1 + j1} = \frac{1}{\sqrt{2}} \angle -\frac{\pi}{4}$$

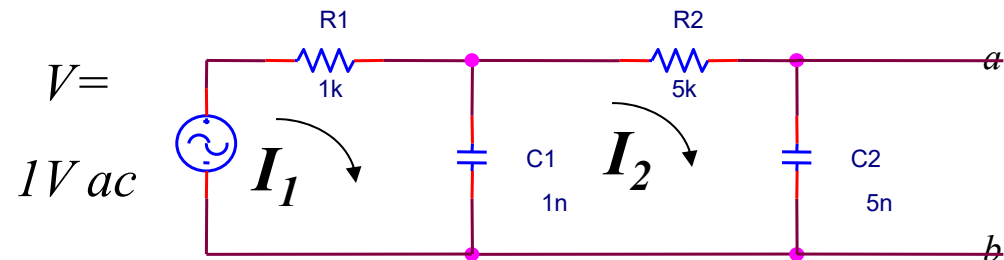


# Homework Answers #6



## Homework Answers #4

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



Mesh 1

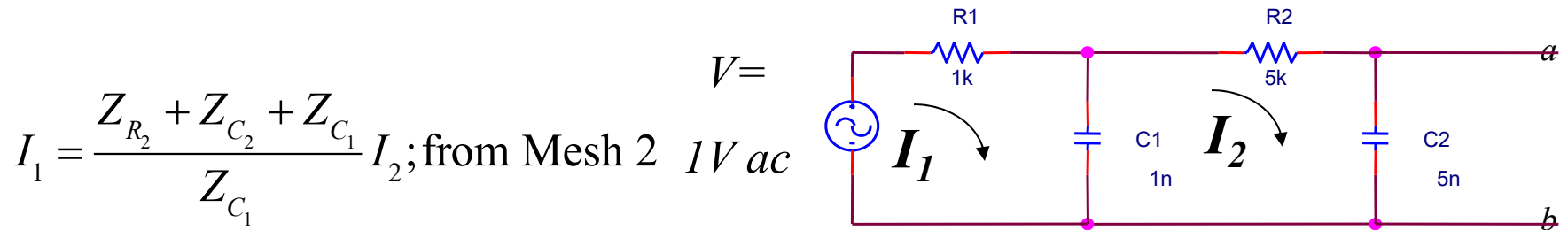
$$V = I_1 Z_{R_1} + Z_{C_1} (I_1 - I_2) = I_1 (Z_{R_1} + Z_{C_1}) - I_2 Z_{C_1}$$

Mesh 2

$$0 = I_2 (Z_{R_2} + Z_{C_2}) + Z_{C_1} (I_2 - I_1) = -Z_{C_1} I_1 + I_2 (Z_{R_2} + Z_{C_2} + Z_{C_1})$$

## Homework Answers #5

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



$$I_1 = \frac{Z_{R_2} + Z_{C_2} + Z_{C_1}}{Z_{C_1}} I_2; \text{from Mesh 2}$$

$$V = \left\{ \frac{Z_{R_2} + Z_{C_2} + Z_{C_1}}{Z_{C_1}} I_2 \right\} (Z_{R_1} + Z_{C_1}) - I_2 Z_{C_1}; \text{Substituting this into Mesh 1}$$

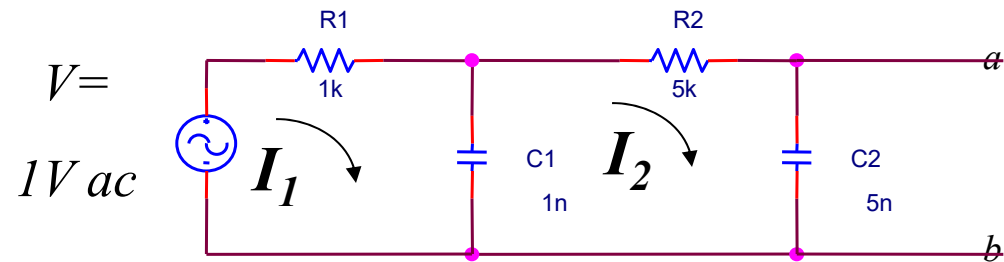
$$= \frac{(Z_{R_2} + Z_{C_2} + Z_{C_1})(Z_{R_1} + Z_{C_1}) - Z_{C_1}^2}{Z_{C_1}} I_2$$

$$= \frac{Z_{R_2} Z_{R_1} + Z_{C_2} Z_{R_1} + Z_{C_1} Z_{R_1} + Z_{R_2} Z_{C_1} + Z_{C_2} Z_{C_1} + Z_{C_1} Z_{C_1} - Z_{C_1}^2}{Z_{C_1}} I_2$$

$$= \frac{Z_{R_2} Z_{R_1} + Z_{C_2} Z_{R_1} + Z_{C_1} Z_{R_1} + Z_{R_2} Z_{C_1} + Z_{C_2} Z_{C_1}}{Z_{C_1}} I_2$$

## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



$$I_2 = \frac{Z_{C_1}}{Z_{R_2} Z_{R_1} + Z_{C_2} Z_{R_1} + Z_{C_1} Z_{R_1} + Z_{R_2} Z_{C_1} + Z_{C_2} Z_{C_1}} V$$

$$V_{ab} = I_2 Z_{C_2} = \frac{Z_{C_1} Z_{C_2}}{Z_{R_2} Z_{R_1} + Z_{C_2} Z_{R_1} + Z_{C_1} Z_{R_1} + Z_{R_2} Z_{C_1} + Z_{C_2} Z_{C_1}} V$$

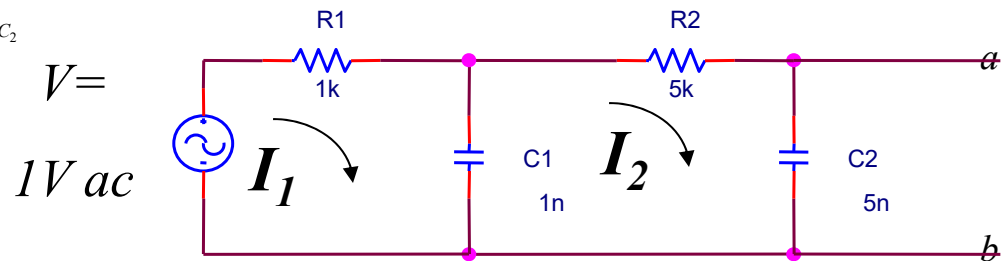
## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

$$\begin{aligned}
 I_2 &= \frac{\frac{1}{j\omega C_1}}{R_1 R_2 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2}} V \\
 &= \frac{1}{j\omega C_1 R_1 R_2 + \frac{C_1}{C_2} R_1 + R_1 + R_2 + \frac{1}{j\omega C_2}} V \\
 &= \frac{j\omega C_2}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega C_2 \frac{C_1}{C_2} R_1 + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} V
 \end{aligned}$$

$$\begin{aligned}
 V_{ab} &= I_2 Z_{C_2} = \frac{j\omega C_2}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} V Z_{C_2} \\
 &= \frac{j\omega C_2 \frac{1}{j\omega C_1}}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} V \\
 &= \frac{1}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} V
 \end{aligned}$$

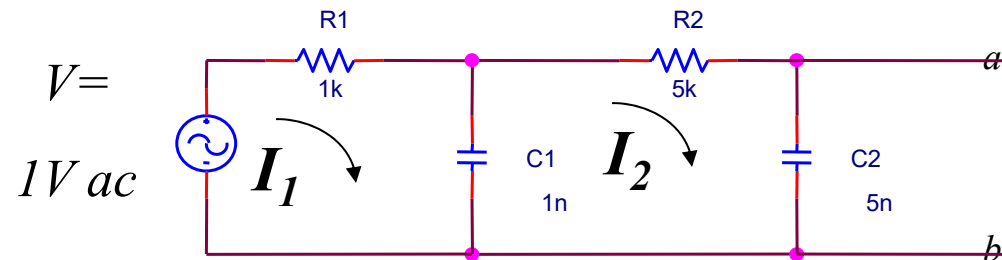
$$\begin{aligned}
 V_{ab} &= \frac{1}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} \\
 &= \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}} \\
 &= \frac{1}{\sqrt{[1 - (\omega 5 \times 10^{-6})^2]^2 + \omega^2 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\omega 31 \times 10^{-6}}{1 - (\omega 5 \times 10^{-6})^2} \right]
 \end{aligned}$$





## Homework Answers #4

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



Mesh 1

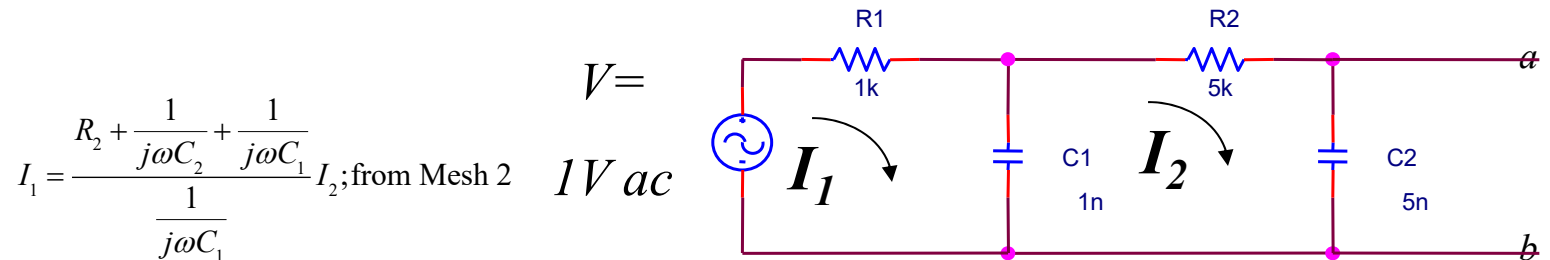
$$V = I_1 R_1 + \frac{1}{j\omega C_1} (I_1 - I_2) = I_1 \left( R_1 + \frac{1}{j\omega C_1} \right) - I_2 \frac{1}{j\omega C_1}$$

Mesh 2

$$0 = I_2 \left( R_2 + \frac{1}{j\omega C_2} \right) + \frac{1}{j\omega C_1} (I_2 - I_1) = -\frac{1}{j\omega C_1} I_1 + I_2 \left( R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} \right)$$

## Homework Answers #5

- Repeat the analysis of this circuit using Mesh and Nodal Analysis



$$I_1 = \frac{R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}} I_2; \text{from Mesh 2}$$

$$V = \left\{ \frac{R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1}} I_2 \right\} \left( R_1 + \frac{1}{j\omega C_1} \right) - I_2 \frac{1}{j\omega C_1}; \text{Substituting this into Mesh 1}$$

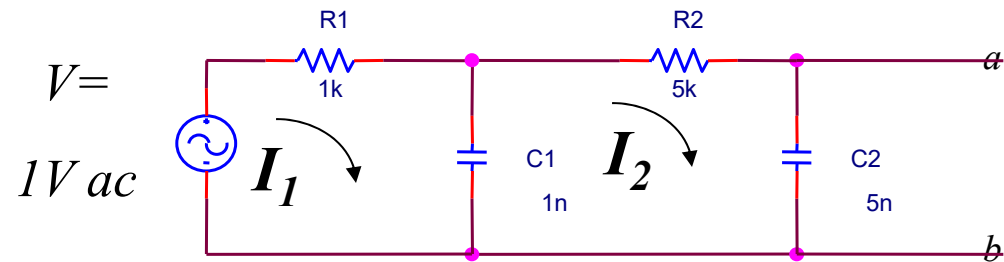
$$= \frac{\left( R_2 + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} \right) \left( R_1 + \frac{1}{j\omega C_1} \right) - \left( \frac{1}{j\omega C_1} \right)^2}{\frac{1}{j\omega C_1}} I_2$$

$$= \frac{R_2 R_1 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_1} - \left( \frac{1}{j\omega C_1} \right)^2}{\frac{1}{j\omega C_1}} I_2$$

$$= \frac{R_2 R_1 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1}} I_2$$

## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

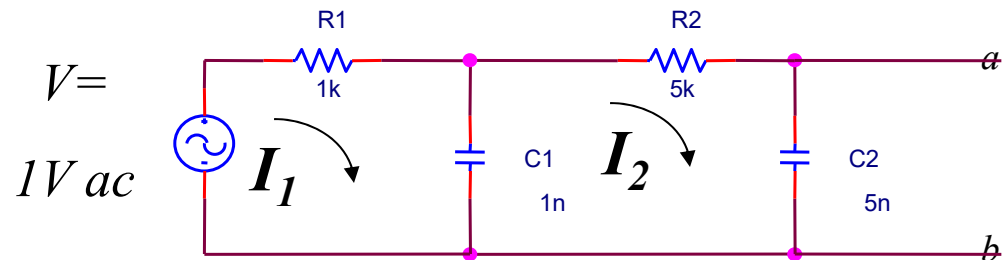


$$I_2 = \frac{1}{j\omega C_1} \frac{V}{R_2 R_1 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2}}$$

$$V_{ab} = I_2 \frac{1}{j\omega C_2} = \frac{\frac{1}{j\omega C_1} \frac{1}{j\omega C_2}}{R_2 R_1 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2}} V$$

## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

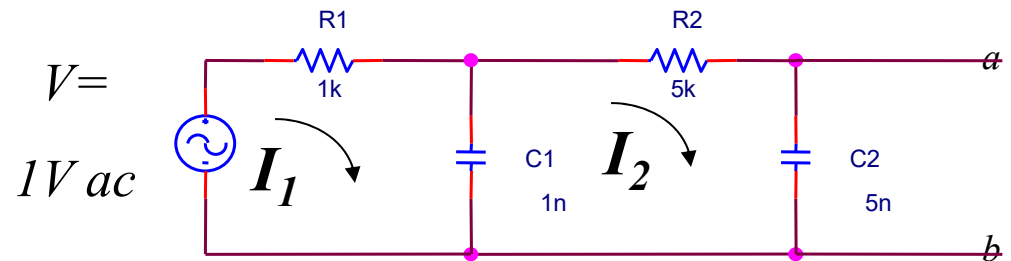


$$\begin{aligned}
 \frac{V_{ab}}{V} &= \frac{\frac{1}{j\omega C_1} \frac{1}{j\omega C_2}}{R_2 R_1 + \frac{1}{j\omega C_2} R_1 + \frac{1}{j\omega C_1} R_1 + R_2 \frac{1}{j\omega C_1} + \frac{1}{j\omega C_1} \frac{1}{j\omega C_2}} \\
 &= \frac{1}{(j\omega C_1)(j\omega C_2)R_2 R_1 + (j\omega C_1)(j\omega C_2)\frac{1}{j\omega C_2} R_1 + (j\omega C_1)(j\omega C_2)\frac{1}{j\omega C_1} R_1 + (j\omega C_1)(j\omega C_2)R_2 \frac{1}{j\omega C_1} + (j\omega C_1)(j\omega C_2)\frac{1}{j\omega C_1} \frac{1}{j\omega C_2}} \\
 &= \frac{1}{(-\omega^2 C_1 C_2 R_2 R_1) + j\omega C_1 R_1 + j\omega C_2 R_1 + j\omega C_2 R_2 + 1} \\
 &= \frac{1}{1 - \omega^2 C_1 C_2 R_2 R_1 + j\omega(C_1 R_1 + C_2 R_1 + C_2 R_2)}
 \end{aligned}$$

## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

$$\begin{aligned}
 V_{ab} &= \frac{1}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]} \\
 &= \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}} \\
 &= \frac{1}{\sqrt{[1 - (\omega 5 \times 10^{-6})^2]^2 + \omega^2 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\omega 31 \times 10^{-6}}{1 - (\omega 5 \times 10^{-6})^2} \right]
 \end{aligned}$$



## Homework Answers #6

- Repeat the analysis of this circuit using Mesh and Nodal Analysis

$$V_{ab} = \frac{1}{(1 - \omega^2 C_2 C_1 R_1 R_2) + j\omega [C_2 (R_1 + R_2) + C_1 R_1]}$$

$$= \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}}$$

$$= \frac{1}{\sqrt{[1 - (\omega 5 \times 10^{-6})^2]^2 + \omega^2 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\omega 31 \times 10^{-6}}{1 - (\omega 5 \times 10^{-6})^2} \right]$$

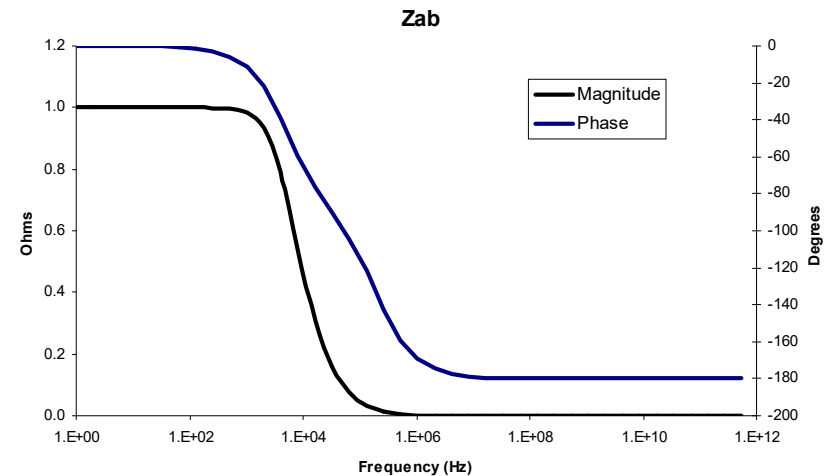
$$V_{ab} |_{\omega=0} = \frac{1}{\sqrt{[1 - (0 \times 5 \times 10^{-6})^2]^2 + 0 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{0 \times 31 \times 10^{-6}}{1 - (0 \times 5 \times 10^{-6})^2} \right]$$

$$= \frac{1}{\sqrt{1}} \angle -\tan^{-1} \left[ \frac{0}{1} \right] = 1 \angle 0$$

$$V_{ab} |_{\omega = \frac{1}{5 \times 10^{-6}}} = \frac{1}{\sqrt{[1 - (\frac{1}{5 \times 10^{-6}} \times 5 \times 10^{-6})^2]^2 + \frac{1}{5 \times 10^{-6}}^2 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\frac{1}{5 \times 10^{-6}} \times 31 \times 10^{-6}}{1 - (\frac{1}{5 \times 10^{-6}} \times 5 \times 10^{-6})^2} \right] = \frac{1}{\sqrt{[0]^2 + \frac{1}{5 \times 10^{-6}}^2 \times 961 \times 10^{-12}}} \angle -\tan^{-1} \left[ \frac{\frac{1}{5 \times 10^{-6}} \times 31 \times 10^{-6}}{0} \right] = \frac{1}{\sqrt{38.44}} \angle -\tan^{-1} \left[ \frac{6.2}{0} \right]$$

$$= \frac{1}{6.2} \angle -\frac{\pi}{2}$$

$$V_{ab} |_{\omega \rightarrow \infty} = \frac{1}{1 - (\omega 5 \times 10^{-6})^2 + j\omega 31 \times 10^{-6}} |_{\omega \rightarrow \infty} \rightarrow \frac{1}{-(\omega 5 \times 10^{-6})^2} \rightarrow 0 \angle -\pi$$



# Matlab Code

```
clear all;
R1=1e3;C1=1e-9;R2=5e3;C2=5e-9;
omega=(100:1000:10^7);maxomega=length(omega);
VIN=[1; 0];
for i=1:maxomega
    zC1=1/complex(0,omega(i)*C1);
    zC2=1/complex(0,omega(i)*C2);
    Z=[R1+zC1 -zC1;-zC1 R2+zC1+zC2];
    IC=Z\VIN;
    VOUT(i)=IC(2)*zC2;
end
f=omega/(2*pi);
subplot(2,1,1);
semilogx(f,abs(VOUT));
title('RCLadder Mesh Magnitude');
xlabel('Hz');
axis([f(1) f(maxomega) 0 1]);
subplot(2,1,2);
semilogx(f,atan2(imag(VOUT),real(VOUT)));
title('RCLadder Mesh Phase Angle');
xlabel('Hz');
axis([f(1) f(maxomega) -2 2]);
```

