Angle Modulation

Lesson 15
Angle Modulation

• Two types: Frequency and Phase both less susceptible to noise but more difficult to construct and reconstruct.

\[ x(t) = A_o \cos(\omega_o t + \phi(t)) \]

• Let \( m(t) \) be a BL signal and \( f_M << \omega_o/2\pi \)

• If \( \phi(t) = k_\phi m(t) \) then we have phase modulation (PM):

\[ x(t) = A_o \cos(\omega_o t + k_\phi m(t)) \]

with modulation index \( \phi_m = |k_\phi m(t)|_{peak} \)

• If \( \phi(t) = \int_{-\infty}^{t} k_\omega m(\tau)d\tau \) then we have frequency modulation (FM):

\[ x(t) = A_o \cos(\omega_o t + \int_{-\infty}^{t} k_\omega m(\tau)d\tau) \]

with instantaneous radian frequency \( \omega(t) = d\phi(t)/dt = \omega_o + k_\omega m(t) \),
with maximum frequency deviation \( \Delta \omega = |k_\omega m(t)|_{peak} \),

and with modulation index \( \phi_m = \left| k_\omega \int_{-\infty}^{t} m(\tau)d\tau \right|_{peak} \)
Spectrum of Angle Modulation

- Unlike AM there is no single theory that can handle the general PM and FM cases
- Let’s look at a simple example where

\[ m(t) = m_o \cos \omega_m t \]

PM \( \Rightarrow \) \( A_o \cos(\omega_o t + k_\phi m_o \cos \omega_m t) \)

FM \( \Rightarrow \) \( A_o \cos[\omega_o t + (k_\omega m_o / \omega_m) \sin \omega_m t] \)

- Let’s look at the following case:

\[ v(t) = A_o \cos[\omega_o t + \phi_m \sin \omega_m t] \]
Spectrum Continued

\[ v(t) = A_o \cos[\omega_o t + \phi_m \sin \omega_m t] \]

\[ = A_o [\cos(\omega_o t)\cos(\phi_m \sin \omega_m t) - \sin(\omega_o t)\sin(\phi_m \sin \omega_m t)] \]

Note: \( e^{j\phi_m \sin \omega_m t} = \cos(\phi_m \sin \omega_m t) + j \sin(\phi_m \sin \omega_m t) \)

\[ = \sum_{-\infty}^{\infty} a_k e^{jk\omega_m t} \]

and \( a_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\phi_m \sin \omega_m t - k\omega_m t)} d(\omega_m t) \)

The solution of this integral is called a Bessel function of the 1st kind and order \( k \) and is denoted as:

\[ J_k(\phi_m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\phi_m \sin \omega_m t - k\omega_m t)} d(\omega_m t) \]

and \( J_k(\phi_m) = (-1)^k J_{-k}(\phi_m) \)
Bessel Functions
**Spectrum of an Angle Modulated Signal**

\[ e^{j\phi_m \sin \omega_m t} = \sum_{-\infty}^{\infty} J_k(\phi_m) e^{jk\omega_m t} \]

It can be shown that

\[ \cos[\omega_o t + \phi_m \sin \omega_m t] = J_0(\phi_m) \cos \omega_o t \]
\[ + J_1(\phi_m)[\cos(\omega_0 + \omega_m) t - \cos(\omega_0 - \omega_m) t] \]
\[ + J_2(\phi_m)[\cos(\omega_0 + 2\omega_m) t + \cos(\omega_0 - 2\omega_m) t] \]
\[ + J_3(\phi_m)[\cos(\omega_0 + 3\omega_m) t - \cos(\omega_0 - 3\omega_m) t] \]
\[ + K \]

- When look at AM, there are only 2 bands; however, for PM/FM there are an infinite number of frequencies.
- We say that PM/FM disperses the spectrum of modulating signal, \( m(t) \).
- Note that if \( \phi_m < 0.20 \), then \( J_0(\phi_m) \approx 0.99 \) and \( J_1(\phi_m) < J_1(\phi_m) < 0.1 \), then we call the narrow band FM.
- It can be shown that for narrow band FM, there are two side bands.
Construction of FM signal and reconstruction of $m(t)$

- Construction of FM signal can be achieved using a voltage controlled oscillator.

- Reconstruction of $m(t)$ can be achieved using an FM discriminator followed by an envelope detector.
Homework

- TBD