Filters

Lesson 12

Homework

- Find the response to the $\delta(t)$ for
  - Bandpass filter with lower frequency f_1 and upper frequency $f_2=2 f_1$
  - Band-elimination filter:

    $H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$

    $= 0,\ \omega_1 < |\omega| < \omega_2$
- Using Matlab build an ideal bandpass filter and calculate the output of the filter for a square wave input. Show cases where the filter removes a single harmonic (e.g., 1st, 2nd and 3rd) and removes 2 or more harmonics. Plot both the time domain and frequency domain for these signals.
  - Do this calculation in the frequency domain Hint: calculate the frequency response of the filter, multiply this with input using its spectrum to get the output spectrum, and invert the output spectrum to get the time domain plot of the output. (You may need to use fft and ifft.)
  - Repeat this calculation in the time domain using the fdatool.

Homework

- Prove that for the impulse response to a real filter

$$Given: R = \sqrt{\frac{L}{C}};\ \omega_o^{\ 2} = \frac{1}{LC}$$

$$1) H(j\omega) = \frac{\omega_o^{\ 2}}{\omega_o^{\ 2} - \omega^2 + j\omega_o\omega}$$

$$2) \Im[v_2(t)] = \Im[\frac{2}{\sqrt{3}}\omega_o e^{-\omega_o t/2}\sin(\frac{\sqrt{3}}{2}\omega_o t)u(t)] = H(j\omega)$$

- Use Matlab to calculate a single pole low pass filter and a single pole high pass filter. Plot the Bode plots for each and then apply them to a square wave. Plot the input and output signal spectrum and time signal.

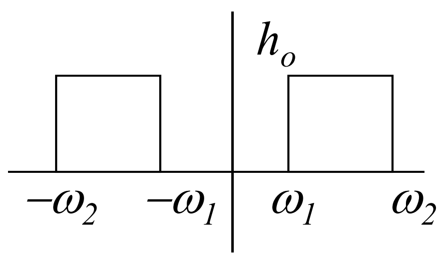
Homework Answers #1

- Find the response to the $\delta(t)$ for
  - Bandpass filter with lower frequency f_1 and upper frequency $f_2=2f_1$

Band Pass with cutoff frequencies ω_1 and ω_2

$H_{bp}(j\omega) = h_o e^{-j\omega T_d}$ for $\omega_2 > |\omega| > \omega_1$

0 elsewhere



$$v_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_2(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)V_1(j\omega)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)1e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_1}^{\omega_2} h_o e^{-j\omega T_d}e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{-\omega_2}^{-\omega_1} h_o e^{-j\omega T_d}e^{j\omega t}d\omega$$

$$= \frac{h_o}{2\pi}\left[\frac{e^{j\omega(t-T_d)}}{j(t-T_d)}\Big|_{\omega_1}^{\omega_2} + \frac{e^{j\omega(t-T_d)}}{j(t-T_d)}\Big|_{-\omega_2}^{-\omega_1}\right]$$

$$= \frac{h_o}{2\pi j(t-T_d)}\left[e^{j\omega_2(t-T_d)} - e^{j\omega_1(t-T_d)} + e^{-j\omega_1(t-T_d)} - e^{-j\omega_2(t-T_d)}\right]$$

$$= \frac{h_o}{\pi}\left[\frac{\omega_2\{e^{j\omega_2(t-T_d)} - e^{-j\omega_2(t-T_d)}\}}{2j\{\omega_2(t-T_d)\}} - \frac{\omega_1\{e^{j\omega_1(t-T_d)} - e^{-j\omega_1(t-T_d)}\}}{2j\{\omega_1(t-T_d)\}}\right]$$

$$= \frac{h_o\omega_1}{\pi}\left\{\frac{\omega_2}{\omega_1}Sa[\omega_2(t-T_d)] - Sa[\omega_1(t-T_d)]\right\}$$

$$= \frac{h_o\omega_1}{\pi}\{2Sa[2\omega_1(t-T_d)] - Sa[\omega_1(t-T_d)]\}$$

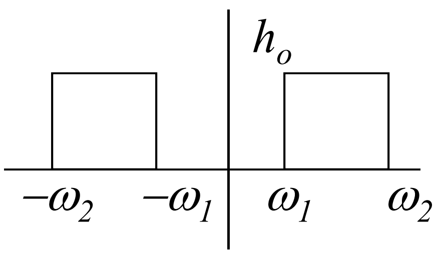
Homework Answers #2

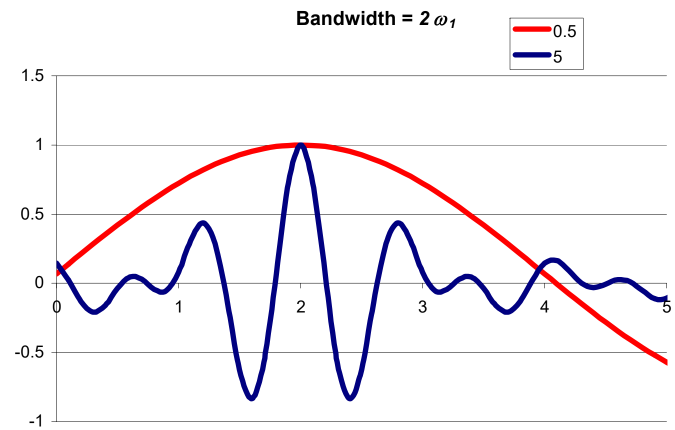
- Find the response to the $\delta(t)$ for
  - Bandpass filter with lower frequency f_1 and upper frequency $f_2=2f_1$

Band Pass with cutoff frequencies ω_1 and ω_2

$$H_{bp}(j\omega) = h_o e^{-j\omega T_d} \quad \text{for } \omega_2 > |\omega| > \omega_1$$

0 elsewhere





$$v_2(t) = \frac{h_o\omega_1}{\pi}\{2Sa[2\omega_1(t-T_d)] - Sa[\omega_1(t-T_d)]\}$$

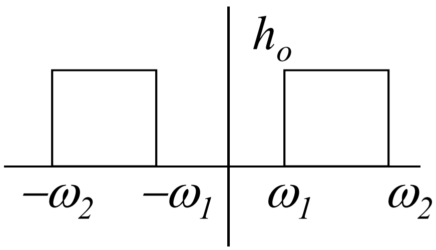
Homework Answers #3

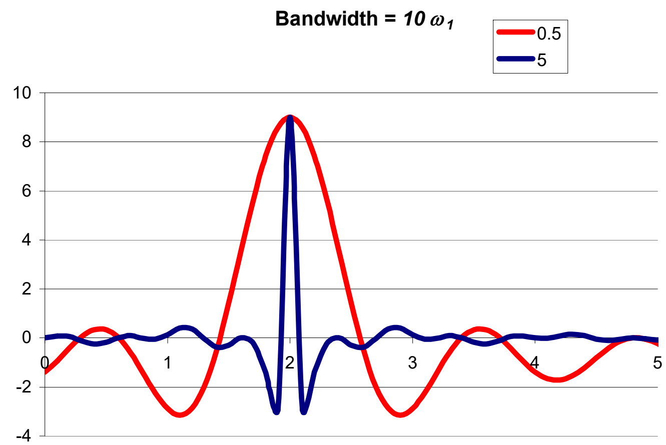
- Find the response to the $\delta(t)$ for
  - Bandpass filter with lower frequency f_1 and upper frequency $f_2=10\,f_1$

Band Pass with cutoff frequencies ω_1 and ω_2

$$H_{bp}(j\omega) = h_o e^{-j\omega T_d} \quad \text{for } \omega_2 > |\omega| > \omega_1$$

0 elsewhere



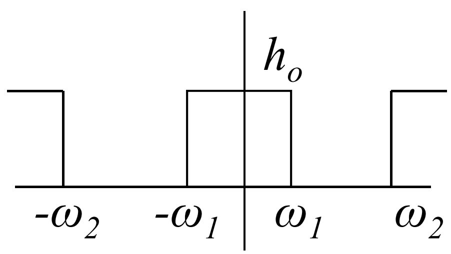


$$v_2(t) = \frac{h_o \omega_1}{\pi}\{10Sa[10\omega_1(t - T_d)] - Sa[\omega_1(t - T_d)]\}$$

Homework Answers #4

- Find the response to the $\delta(t)$ for
  - Band-elimination filter:

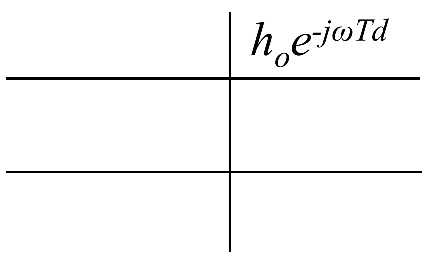
$$H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$$
$$= 0,\ \omega_1 < |\omega| < \omega_2$$



$$v_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V_2(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)V_1(j\omega)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} h_o e^{j\omega(t-T_d)}d\omega - \frac{h_o\omega_1}{\pi}\{\frac{\omega_2}{\omega_1}Sa[\omega_2(t-T_d)] - Sa[\omega_1(t-T_d)]\}$$

$$= h_o\delta(t-T_d) - \frac{h_o\omega_1}{\pi}\{\frac{\omega_2}{\omega_1}Sa[\omega_2(t-T_d)] - Sa[\omega_1(t-T_d)]\}$$



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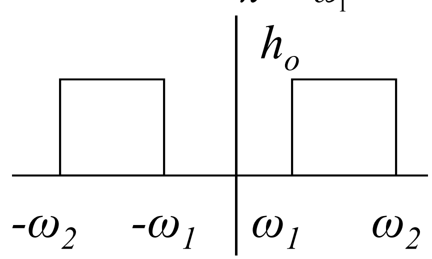
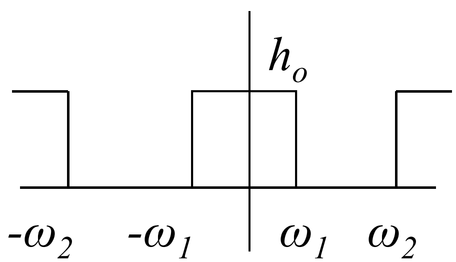
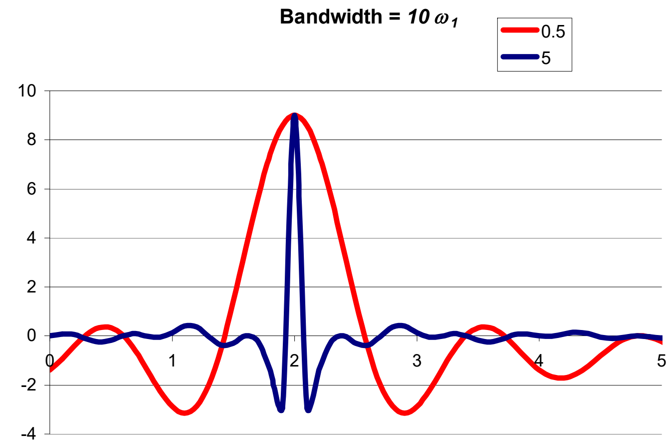


Homework Answers #5

- Find the response to the $\delta(t)$ for
  - Band-elimination filter:

$$H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$$
$$= 0,\ \omega_1 < |\omega| < \omega_2$$



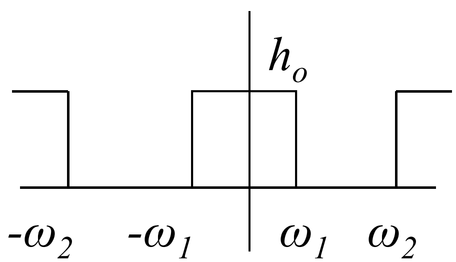


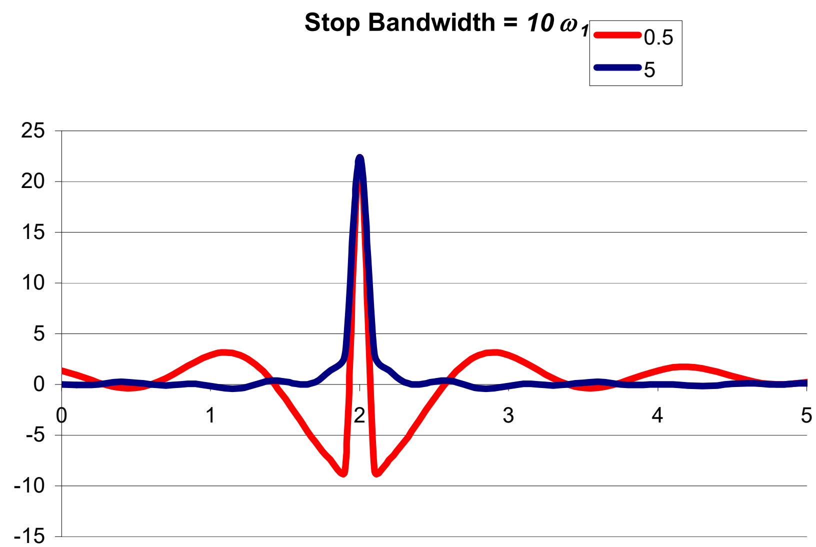
$$v_2(t) = h_o\delta(t-T_d) - \frac{h_o\omega_1}{\pi}\{\frac{\omega_2}{\omega_1}Sa[\omega_2(t-T_d)] - Sa[\omega_1(t-T_d)]\}$$

Homework Answers #6

- Find the response to the $\delta(t)$ for
  - Band-elimination filter:

$$H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$$
$$= 0,\ \omega_1 < |\omega| < \omega_2$$



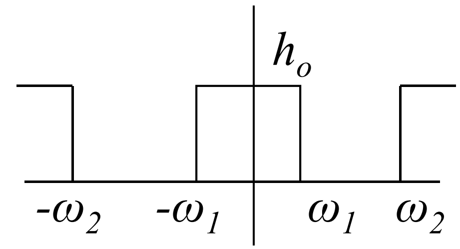


$$v_2(t) = h_o\delta(t - T_d) - \frac{h_o\omega_1}{\pi}\{\frac{\omega_2}{\omega_1} Sa[\omega_2(t - T_d)] - Sa[\omega_1(t - T_d)]\}$$

Homework Answers #4

- Find the response to the $\delta(t)$ for
  - Band-elimination filter:

$$H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$$
$$= 0,\ \omega_1 < |\omega| < \omega_2$$



Another method:

$$v_2(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}V_2(j\omega)e^{j\omega t}d\omega=\frac{1}{2\pi}\int_{-\infty}^{\infty}H(j\omega)V_1(j\omega)e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}h_o e^{j\omega(t-T_d)}d\omega=\frac{1}{2\pi}\int_{-\infty}^{-\omega_2}h_o e^{j\omega(t-T_d)}d\omega+\frac{1}{2\pi}\int_{-\omega_1}^{\omega_1}h_o e^{j\omega(t-T_d)}d\omega+\frac{1}{2\pi}\int_{\omega_2}^{\infty}h_o e^{j\omega(t-T_d)}d\omega$$

$$=\frac{h_o}{2\pi j(t-T_d)}\{e^{j\omega(t-T_d)}\Big|_{-\infty}^{-\omega_2}+e^{j\omega(t-T_d)}\Big|_{-\omega_1}^{\omega_1}+e^{j\omega(t-T_d)}\Big|_{\omega_2}^{\infty}\text{ s}$$

$$=\frac{h_o}{2\pi j(t-T_d)}\{e^{j\omega(t-T_d)}\Big|_{\omega=-\omega_2}-e^{j\omega(t-T_d)}\Big|_{\omega=-\infty}+e^{j\omega(t-T_d)}\Big|_{\omega=\omega_1}-e^{j\omega(t-T_d)}\Big|_{\omega=-\omega_1}+e^{j\omega(t-T_d)}\Big|_{\omega=\infty}-e^{j\omega(t-T_d)}\Big|_{\omega=\omega_2}\}$$

$$=\frac{h_o}{2\pi j(t-T_d)}\{[e^{j\omega(t-T_d)}\Big|_{\omega=\infty}-e^{j\omega(\text{[redacted]}T_d)}\Big|_{\omega=-\infty}]+[e^{j\omega(t-T_d)}\Big|_{\omega=\omega_1}-e^{j\omega(t-T_d)}\Big|_{\omega=-\omega_1}]+[e^{j\omega(t-T_d)}\Big|_{\omega=-\omega_2}-e^{j\omega(t-T_d)}\Big|_{\omega=\omega_2}]\}$$

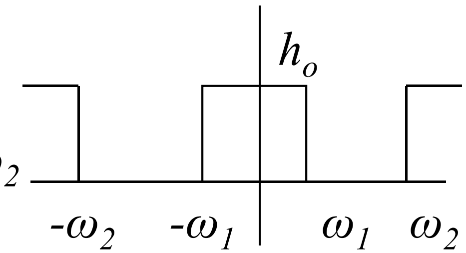
Recall

$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega t}d\omega$$

Homework Answers #4

- Find the response to the $\delta(t)$ for
  - Band-elimination filter:

$$H(j\omega)=h_o e^{-j\omega Td},\ 0 < |\omega| < \omega_1 \text{ and } |\omega| > \omega_2$$
$$= 0,\ \omega_1 < |\omega| < \omega_2$$



Another method: CONTINUED

$$v_2(t)=\frac{h_o}{2\pi j(t-T_d)}\{[e^{j\omega(t-T_d)}|_{\omega=\infty}-e^{j\omega(t-T_d)}|_{\omega=-\infty}]+[e^{j\omega(t-T_d)}|_{\omega=\omega_1}-e^{j\omega(t-T_d)}|_{\omega=-\omega_1}]+[e^{j\omega(t-T_d)}|_{\omega=-\omega_2}-e^{j\omega(t-T_d)}|_{\omega=\omega_2}]\}$$

Recall

$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega t}d\omega$$

$$\delta(t-T_d)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{j\omega(t-T_d)}d\omega=\frac{1}{2\pi j(t-T_d)}e^{j\omega(t-T_d)}\Big|_{\omega=-\infty}^{\omega=\infty}=\frac{1}{2\pi j(t-T_d)}[e^{j\omega(t-T_d)}|_{\omega=\infty}-e^{j\omega(t-T_d)}|_{\omega=-\infty}]$$

$$v_2(t)=h_o\delta(t-T_d)+\frac{h_o}{2\pi j(t-T_d)}\{[e^{j\omega_1(t-T_d)}-e^{-j\omega_1(t-T_d)}]+[e^{-j\omega_2(t-T_d)}-e^{j\omega_2(t-T_d)}]\}$$

$$=h_o\delta(t-T_d)+\frac{h_o\omega_1}{2\pi j(t-T_d)}[e^{j\omega_1(t-T_d)}-e^{-j\omega_1(t-T_d)}]-\frac{h_o\omega_2}{2\pi j(t-T_d)}[e^{j\omega_2(t-T_d)}-e^{-j\omega_2(t-T_d)}]$$

$$=h_o\delta(t-T_d)+\frac{h_o\omega_1}{\pi}[Sa(\omega_1(t-T_d))]-\frac{h_o\omega_2}{\pi}[Sa(\omega_2(t-T_d))]$$

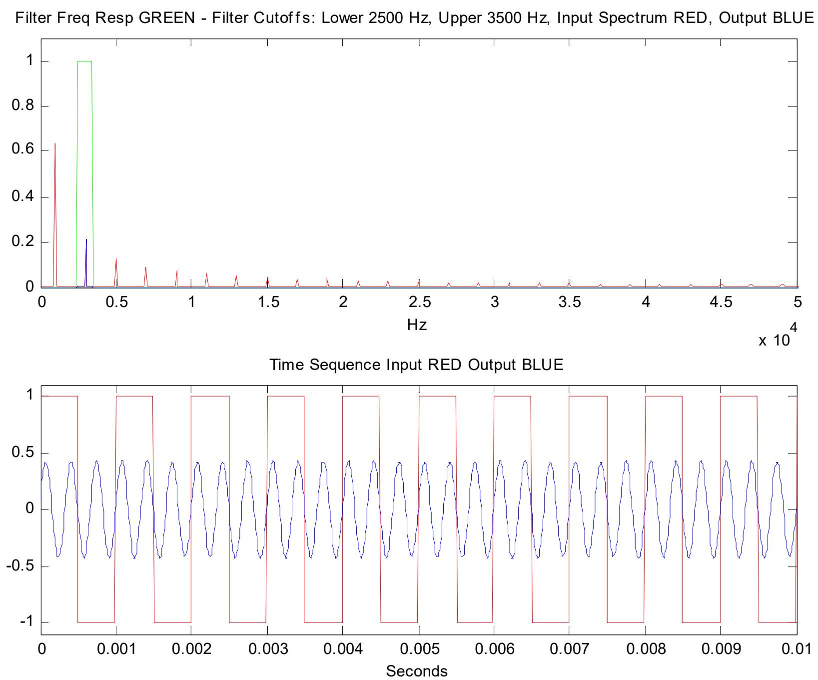
Matlab Code
Frequency Domain

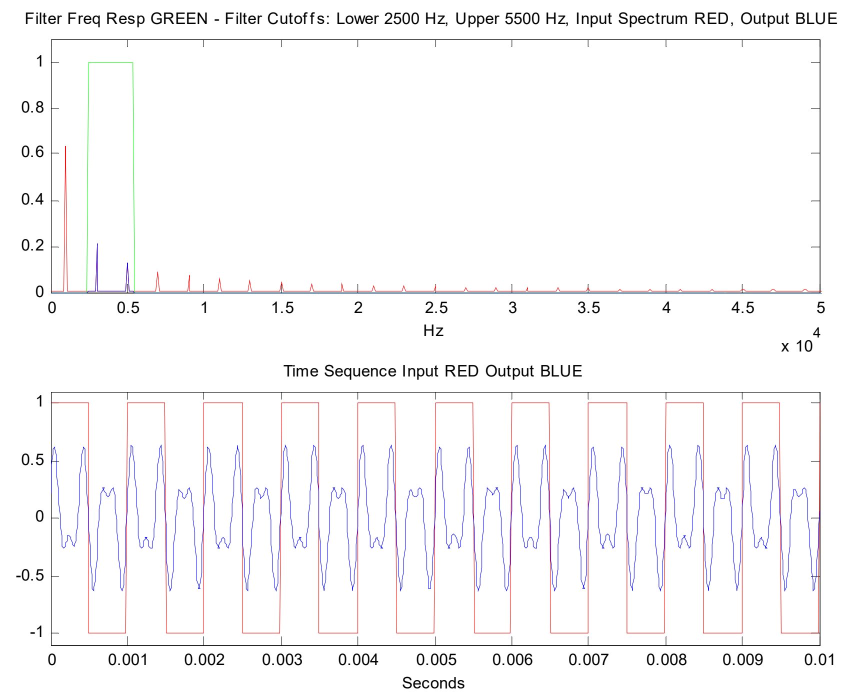
```
clear all;
fc=1000;fs=100*fc;N=1000;
fo=fs/N;ts=1/fs;tmax=N*ts;
time=(0:ts:tmax);freqs=(0:fo:fs);fmax=length(freqs);
fupper=3500;flower=2500;
filtrs=passfilter(freqs,flower,fupper,fs,fmax);
figure(1);
filtrs;
plot(freqs,filtrs);
x=square(2*pi*fc*time);
%plot(time,x);
s=fft(x)/N;
s1=s.*filtrs;
% s1=s;
x1=ifft(s1)*N
figure(2)
subplot(2,1,2);
plot(freqs,abs(s),'r',freqs,filtrs,'g',freqs,abs(s1),'b');
set(gca,'FontSize',7);
title('Spectrum');
xlabel('Hz');
axis([0 fs/2 0 1.1*max([max(s),max(filtrs),max(s1)])]);
subplot(2,1,1)
plot(time,x,'r',time,real(x1),'b');
set(gca,'FontSize',7);
title('Time Sequence');
xlabel('Seconds');
axis([0 tmax 1.1*min([min(x),min(abs(x1).*cos(angle(x1)))]) 1.1*max([max(x),max(abs(x1).*cos(angle(x1)))])]);
```

Matlab Code
Frequency Domain

```
function filtrs=passfilter(freqs,flower,fupper,fs,fmax)
for i=1:fmax
    if freqs(i)<flower
        filtrs(i)=0;
    else
        if freqs(i)<fupper
            filtrs(i)=1;
        else
            if freqs(i)<fs-fupper
                filtrs(i)=0;
            else
                if freqs(i)<fs-flower
                    filtrs(i)=1;
                else
                    filtrs(i)=0;
                end
            end
        end
    end
end
```

Matlab Frequency Domain

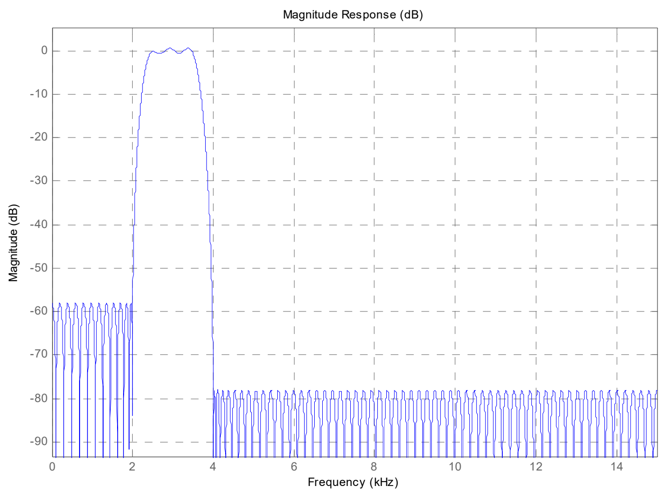


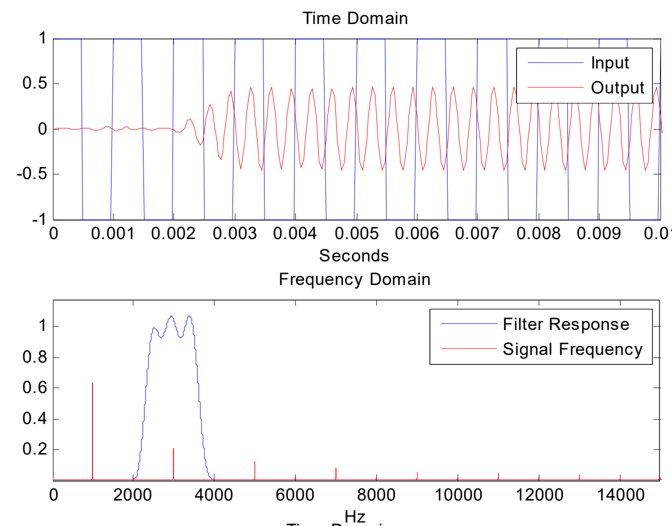


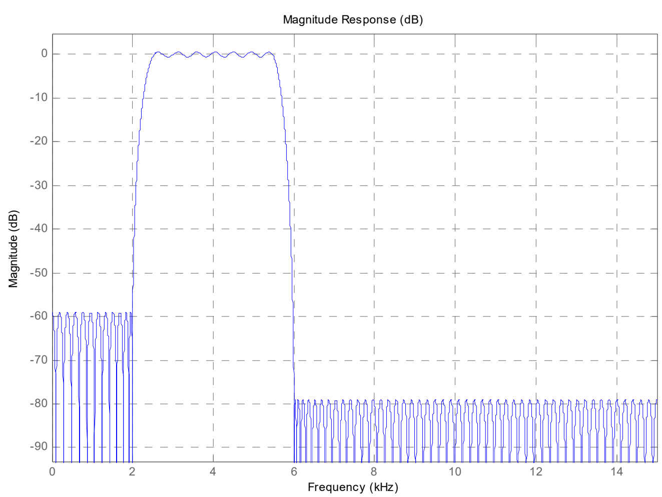
Matlab Code Time Domain

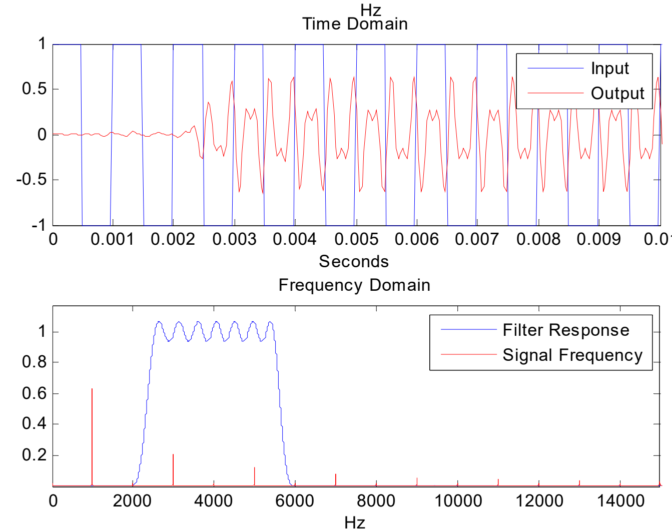
```
fs=30000;
ts=1/fs;
fmax=fs/2;
cycles=100;
fi=1000;
hightime=cycles/100;
time=(0:ts:hightime);
N=length(time);
omega=2*pi*fi;
in=square(omega*time);
out=filter(Num,1,in);
% out2=filter(Num2,1,in);
subplot(2,1,1)
plot(time,in,'b',time,out,'r')
axis([0 10/fi min(in) max(in)])
title('Time Domain');
xlabel('Seconds');
legend('Input','Output');
bmax=length(Num);
b=Num;
omegahat=(0:2*pi/N:2*pi-(2*pi)/N);
for i=1:length(omegahat)
    realmag(i)=0;
    imagmag(i)=0;
    for j=1:bmax
        realmag(i)=realmag(i)+b(j)*cos(-omegahat(i)*(j-1));
        imagmag(i)=imagmag(i)+b(j)*sin(-omegahat(i)*(j-1));
    end
    mag(i)=sqrt(realmag(i)^2+imagmag(i)^2);
    angle(i)=atan2(imagmag(i),realmag(i));
end
subplot(2,1,2)
freq=omegahat*fs/(2*pi);
ak=fft(in)/N;
plot(freq,mag,'b',freq,abs(ak),'r')
title('Frequency Domain');
xlabel('Hz');
legend('Filter Response','Signal Frequency')
axis([ freq(1) freq(length(freq))/2 min(mag) +1.1*max(mag)]);
```

Matlab Code Time Domain

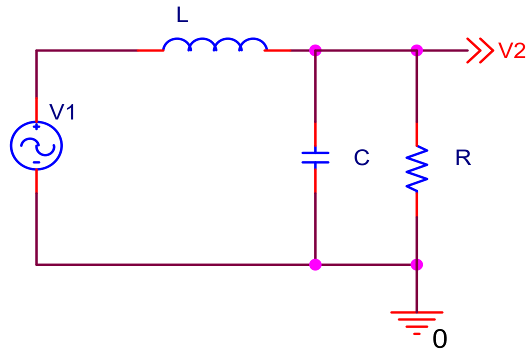








Response to a Real LP Filter



$$v_o = \frac{R \parallel Z_C}{R \parallel Z_C + Z_L} v_{in}$$

$$R \parallel Z_C = \frac{RZ_C}{R + Z_C} = \frac{R\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{j\omega CR + 1}$$

$$\frac{v_o}{v_{in}} = H(j\omega) = \frac{\frac{R}{j\omega CR+1}}{\frac{R}{j\omega CR+1} + j\omega L} = \frac{R}{R + j\omega L(j\omega CR+1)} = \frac{1}{1 - \omega^2 LC + j\omega\frac{L}{R}}$$

$$R = \sqrt{\frac{L}{C}}; \Rightarrow \frac{L}{R} = \frac{L}{\sqrt{\frac{L}{C}}} = \sqrt{LC};\ \omega_o{}^2 = \frac{1}{LC}$$

$$H(j\omega) = \frac{1}{1 - \omega^2 LC + j\omega\frac{L}{R}} = \frac{1}{1 - \frac{\omega^2}{\omega_o{}^2} + j\frac{\omega}{\omega_o}}$$

$$H(j\omega) = \omega_o{}^2 \frac{1}{\omega_o{}^2 - \omega^2 + j\omega_o\omega}$$

$$v_2(t) = \frac{2}{\sqrt{3}}\omega_o e^{-\omega_o t/2} \sin\left(\frac{\sqrt{3}}{2}\omega_o t\right)u(t)$$

Response to a Real LP Filter

$$v_2(t) = \frac{2}{\sqrt{3}}\omega_o e^{-\omega_o t/2}\sin(\frac{\sqrt{3}}{2}\omega_o t)u(t) = \frac{1}{\sqrt{3}j}\omega_o e^{-\frac{\omega_o}{2}t}\{e^{j(\frac{\sqrt{3}}{2}\omega_o t)} - e^{-j(\frac{\sqrt{3}}{2}\omega_o t)}\}u(t)$$

$$= \frac{1}{\sqrt{3}j}\omega_o\{e^{-(\frac{1}{2}-j\frac{\sqrt{3}}{2})\omega_o t} - e^{-(\frac{1}{2}+j\frac{\sqrt{3}}{2})\omega_o t)}\}u(t)$$

$$\Im[f(t)e^{-\alpha t}] = F(\alpha + j\omega)$$

$$\Im[u(t)] = \frac{1}{j\omega}$$

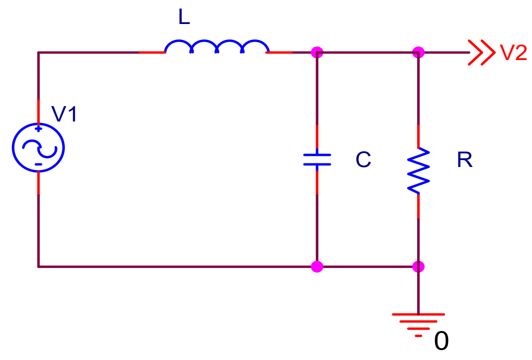
$$\Im[v_2(t)] = \frac{1}{\sqrt{3}j}\omega_o\{\frac{1}{(\frac{1}{2}-j\frac{\sqrt{3}}{2})\omega_o + j\omega} - \frac{1}{(\frac{1}{2}+j\frac{\sqrt{3}}{2})\omega_o + j\omega}\}$$

$$= \frac{1}{\sqrt{3}j}\omega_o\{\frac{(\frac{1}{2}+j\frac{\sqrt{3}}{2})\omega_o + j\omega - [(\frac{1}{2}-j\frac{\sqrt{3}}{2})\omega_o + j\omega]}{[\frac{1}{2}\omega_o + j(\omega - \frac{\sqrt{3}}{2}\omega_o)][\frac{1}{2}\omega_o + j(\omega + \frac{\sqrt{3}}{2}\omega_o)]}\}$$

$$= \frac{1}{\sqrt{3}j}\omega_o\{\frac{j\sqrt{3}\omega_o}{[\frac{1}{4}\omega_o^2 + j\frac{\omega_o}{2}(\omega - \frac{\sqrt{3}}{2}\omega_o) + j\frac{\omega_o}{2}(\omega + \frac{\sqrt{3}}{2}\omega_o) + j(\omega - \frac{\sqrt{3}}{2}\omega_o)j(\omega + \frac{\sqrt{3}}{2}\omega_o)]}\}$$

$$= \frac{\omega_o^2}{[\frac{1}{4}\omega_o^2 + j\omega_o\omega - (\omega - \frac{\sqrt{3}}{2}\omega_o)(\omega + \frac{\sqrt{3}}{2}\omega_o)]}$$

$$= \frac{\omega_o^2}{[\frac{1}{4}\omega_o^2 + j\omega_o\omega - (\omega^2 - \frac{3}{4}\omega_o^2)]} = \frac{\omega_o^2}{[\omega_o^2 - \omega^2 + j\omega_o\omega]}$$



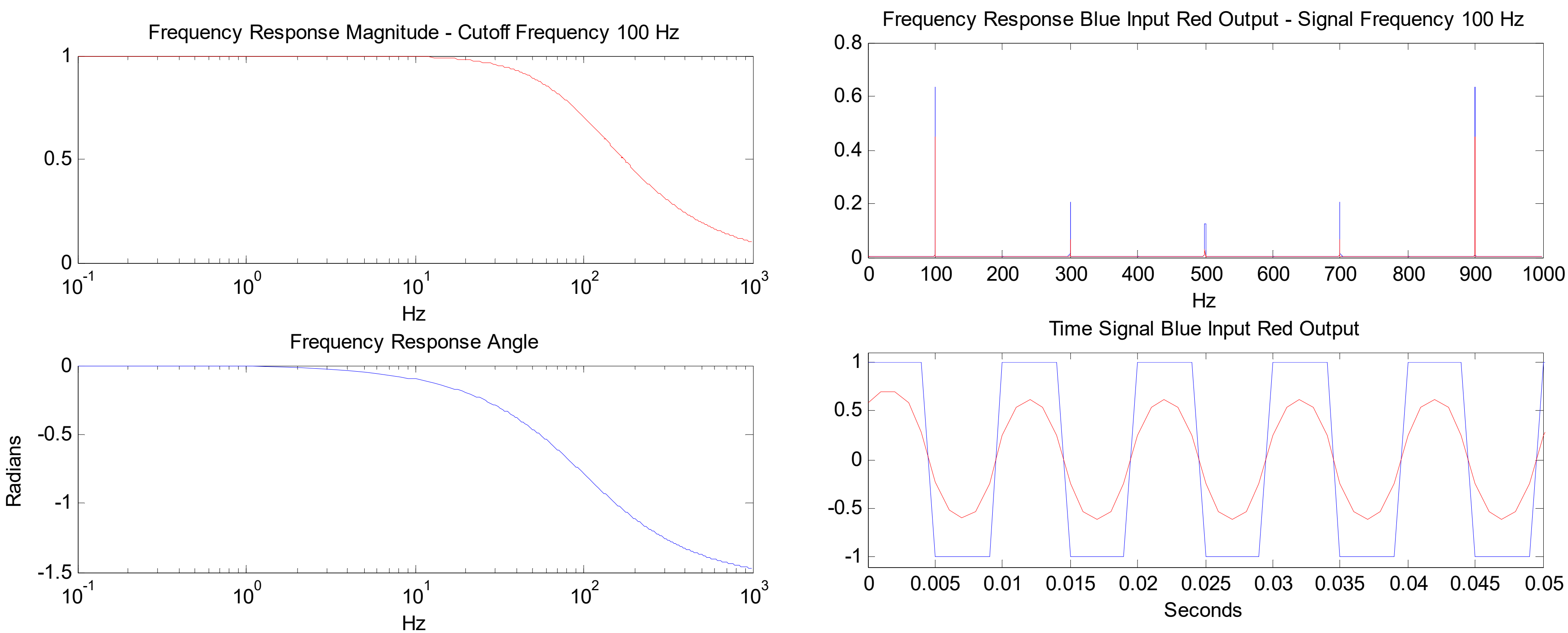
Matlab Code

```
clear all
type=1;
freqs=(.1:1:10^3);maxfreqs=length(freqs);omega=2*pi*freqs;fco=100;omegao=2*pi*fco;
[fd angle]=filterac(omega,omegao,type);
figure(1)
subplot(2,1,1)
semilogx(freqs,fd,'r');
title(['Frequency Response Magnitude - Cutoff Frequency ',num2str(fco),' Hz'])
xlabel('Hz')
subplot(2,1,2)
semilogx(freqs,angle,'b');
title('Frequency Response Angle')
xlabel('Hz')
ylabel('Radians')
fc=100;fs=fc*10;ts=1/fs;tp=1/fc;maxtime=10;
time=(0:ts:maxtime);N=length(time);fo=fs/N;
x=square(2*pi*fc*time);
freqsqu=(0:fo:fs-fo);
x1=fft(x)/N;
omegasqu=2*pi*freqsqu;
for i=1:N
    if i<N/2
    [fd(i) angle(i)]=filterac(omegasqu(i),omegao,type);
    else
    [fd(i) angle(i)]=filterac(omegasqu(N+1-i),omegao,type);
    end
   fr(i)=complex(fd(i)*cos(angle(i)),fd(i)*sin(angle(i)));
end
y=x1.*fr;
figure(2)
subplot(2,1,1)
plot(freqsqu,abs(x1),'b',freqsqu,abs(y),'r');
title(['Frequency Response Blue Input Red Output - Signal Frequency ',num2str(fc),' Hz'])
xlabel('Hz')
y2=ifft(y)*N;
subplot(2,1,2)
plot(time,x,'b',time,real(y2),'r');
title('Time Signal Blue Input Red Output')
xlabel('Seconds')
axis([0 5*tp 1.1*min(real(x)) 1.1*max(real(x))])
```

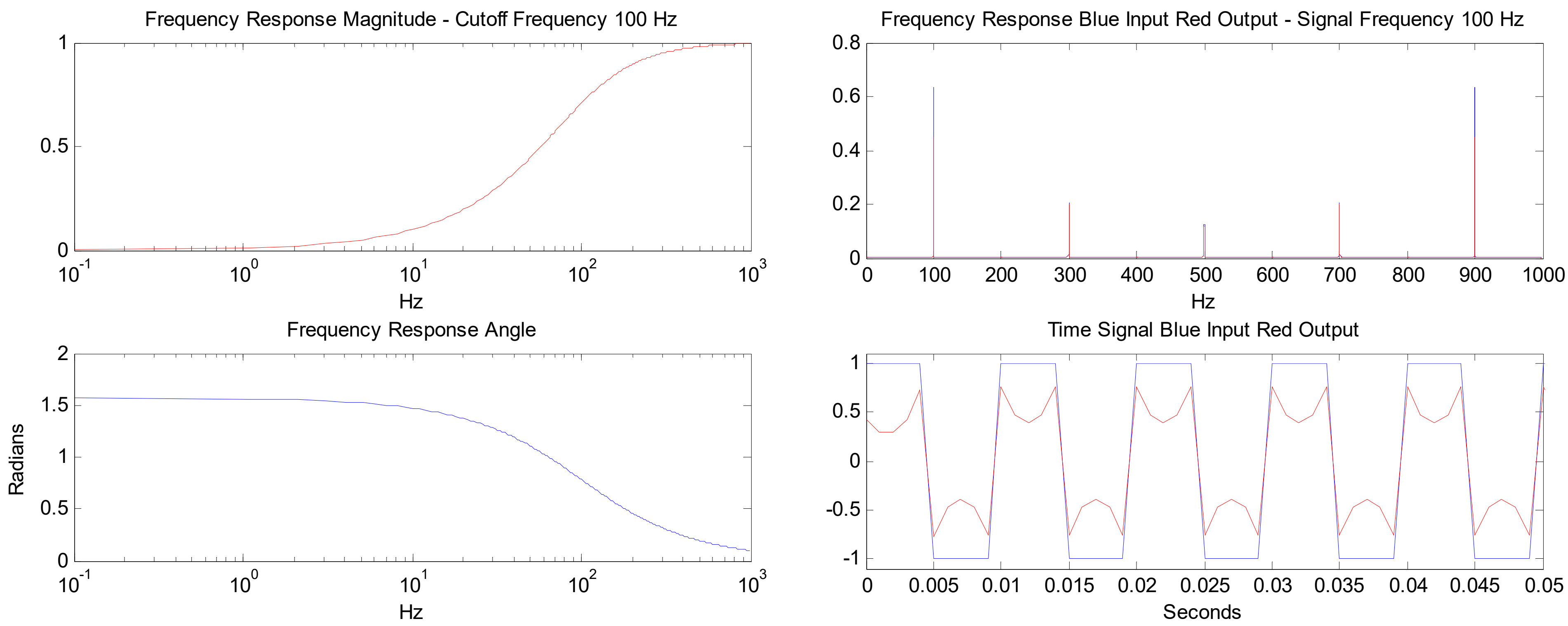
Matlab Code

```
function [fd angle]=filterac(omega,cutoff,type)
maxomega=length(omega);
if type==1
   for i=1:maxomega
      fd(i)=1/sqrt(1+(omega(i)/cutoff)^2);
      angle(i)=-atan(omega(i)/cutoff);
   end
else
   for i=1:maxomega
      fd(i)=omega(i)/cutoff/sqrt(1+(omega(i)/cutoff)^2);
      angle(i)=pi/2-atan(omega(i)/cutoff);
   end
end
```

Matlab Code


Frequency Response Magnitude - Cutoff Frequency 100 Hz
1
0.5
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Hz
Frequency Response Angle
Radians
0
-0.5
-1
-1.5
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Hz
Frequency Response Blue Input Red Output - Signal Frequency 100 Hz
0.8
0.6
0.4
0.2
0
0
100
200
300
400
500
600
700
800
900
1000
Hz
Time Signal Blue Input Red Output
1
0.5
0
-0.5
-1
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Seconds

Matlab Code


Frequency Response Magnitude - Cutoff Frequency 100 Hz
Hz
Frequency Response Angle
Radians
Hz
Frequency Response Blue Input Red Output - Signal Frequency 100 Hz
Hz
Time Signal Blue Input Red Output
Seconds

Lesson 12a

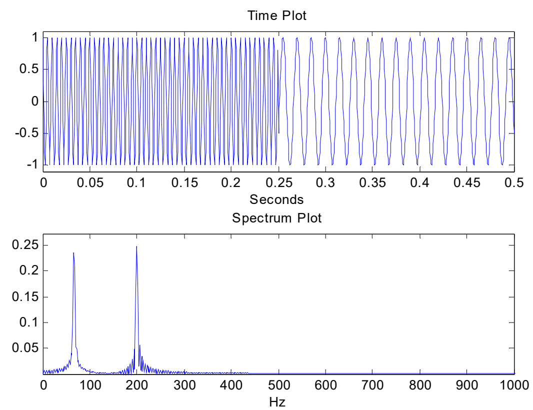
Short Term Fourier Transform

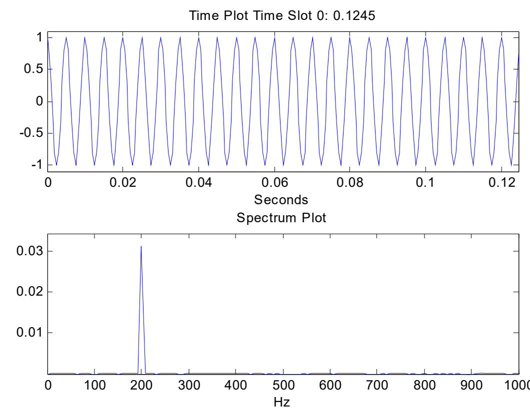
- Using the sequential sine wave signal you created in the homework, design a Matlab program which will perform a Short Term Fourier Transform. Divide the full time sequence into 4 equally spaced time windows and calculate and plot the time signal and spectrum for each window to show how the spectrum changes as a function of time.

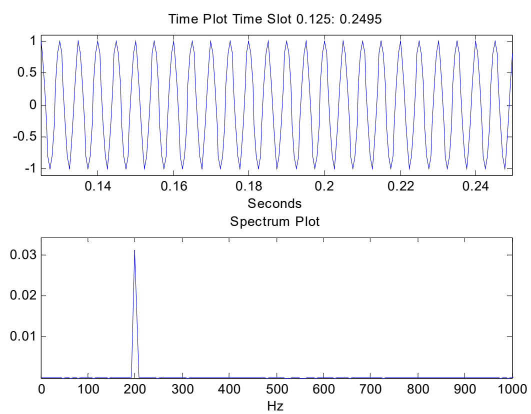
Short Term Fourier Transform

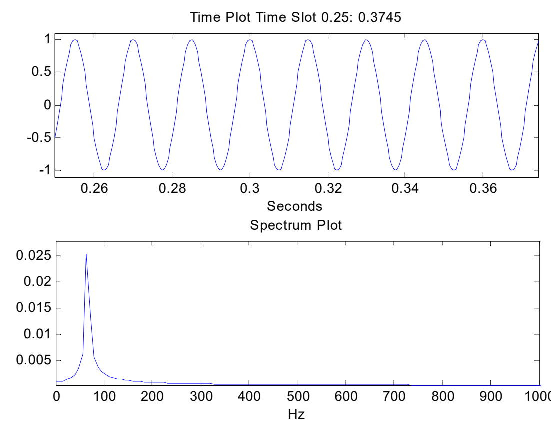
```
clear all;
fc=200;to=1/fc;fs=10*fc;ts=1/fs;cycles=100;time=(0:ts:cycles*to);N=length(time);
Wind=4;fo=fs/N;
for i=1:N
   if i<N/2
      x(i)=cos(2*pi*fc*time(i));
   else
      x(i)=cos(2*pi*fc/3*time(i));
   end
end
figure(1);subplot(2,1,1);
plot(time,x);title('Time Plot');xlabel('Seconds');axis([0 cycles*to 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));title('Spectrum Plot');xlabel('Hz');axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
for i=1:Wind
   figure(i+1)
   subplot(2,1,1)
   timeslot=time((i-1)*(N-1)/Wind+1:(i)*(N-1)/Wind);
   maxtimeslot=length(timeslot);
   xx=x((i-1)*(N-1)/Wind+1:(i)*(N-1)/Wind);
   plot(timeslot,xx);title(['Time Plot Time Slot ',num2str(timeslot(1)),':
      ',num2str(timeslot(maxtimeslot))]);xlabel('Seconds');axis([timeslot(1) timeslot(maxtimeslot) 1.1*min(x) 1.1*max(x)]);
   f=fft(xx)/N/Wind;
   freqs=(0:fs/((N-1)/Wind):fs-fo);
   subplot(2,1,2);
   plot(freqs,abs(f));title('Spectrum Plot');xlabel('Hz');axis([0 fs/2 1.1*min(abs(f)) 1.1*max(abs(f))]);
end
```

Short Term Fourier Transform







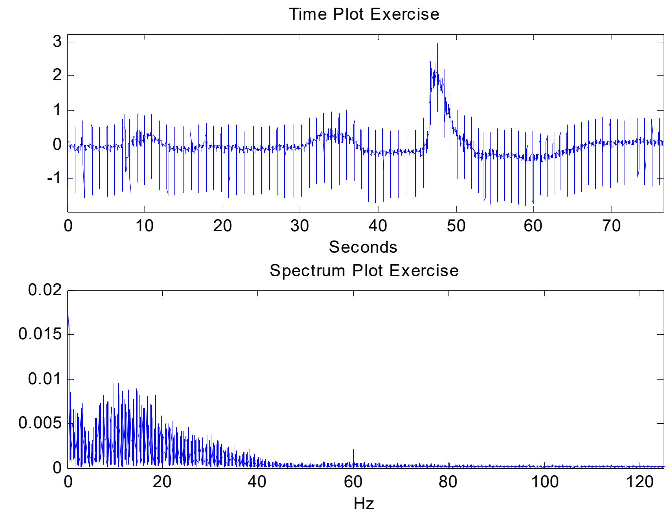


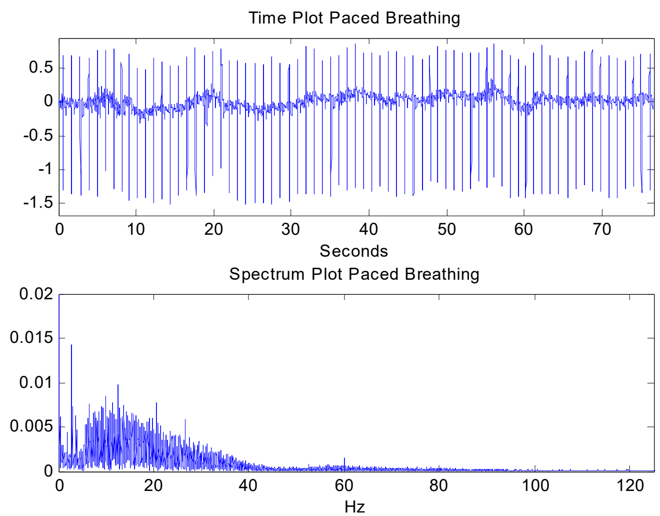
Spectrum of an Actual ECG

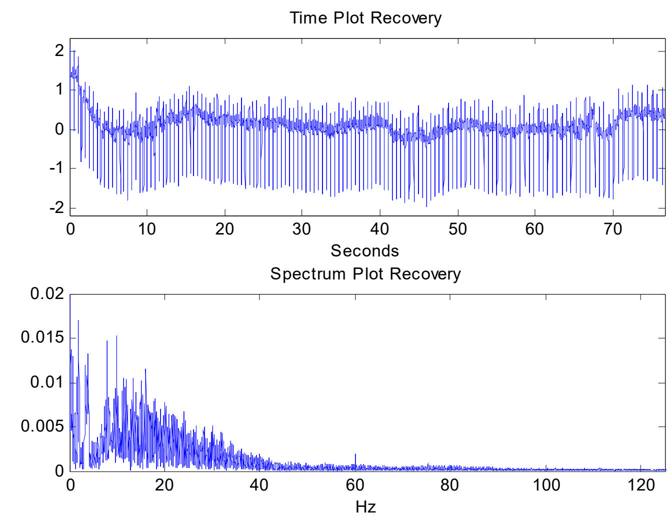
- Using the three files labeled Paced, Exercise, and Recovery, use Matlab and its "fft" function to calculate and plot the time signal and the Spectrum.

Spectrum of an Actual ECG

```
clear all;
load paced;
x=paced;
plotecg(x,1,'Time Plot Paced Breathing','Spectrum Paced Breathing');
load exercise;
x=exercise;
plotecg(x,1,'Time Plot Exercise','Spectrum Exercise');
load recovery;
x=recovery;
plotecg(x,1,'Time Plot Recovery','Spectrum Recovery');
```







Spectrum of an Actual ECG

```
function plotecg(filename,fig,titletime,titlespec)
x=filename;
maxx=length(x);
fs=250;ts=1/fs;
maxtime=maxx*ts;
time=(0:ts:maxtime-ts);
N=length(time);
fo=fs/N;
figure(fig);
subplot(2,1,1);
plot(time,x);
title(titletime);
xlabel('Seconds');
axis([0 maxtime-ts 1.1*min(x) 1.1*max(x)]);
f=fft(x)/N;
freqs=(0:fo:fs-fo);
subplot(2,1,2);
plot(freqs,abs(f));
title(titlespec);
xlabel('Hz');
axis([0 fs/2 0 0.02]);
```

Homework

- Using Matlab
  1. Obtain the IBI from web page and calculate time domain measures
  2. Generate the IIBI (assume f_s = 4s/s)
  3. Calculate the Fourier Transform of the IIBI
  4. Plot the HRV spectrum
  5. Divide the time domain into 6 windows
     1. Calculate the Spectrum within each window
     2. Calculate the HF and LF averages for each window
     3. Plot HF and LF over the 6 windows to determine how the autonomic nervous system is functioning

Matlab Code

```
clear all;
load IBI20040414a;
ibi=IBI20040414a;
samplingperiod=.25;
figure(1);
[iibi,n,time,sequ]=ibiconvert(ibi,samplingperiod); %find the iibi and plot both the ibi and iibi
figure(2);
plotter(sequ,iibi,'IIBI','Seconds','Milliseconds',0,max(sequ),0,max(iibi)*(1.1),2,1);
[mag]=fttmag(iibi,n); %calculate the mag of the fft of the iibi
[f]=freqs(samplingperiod,n); %determine the frequencies to plot
plotter (f,mag,'Spectrum','Hz','',0,.5, 0, max(mag),3,1);
[hfm,lfm]=spectrum(mag,f,samplingperiod,n);hfmn=hfm/(hfm+lfm);lfmn=lfm/(hfm+lfm);lfratiom=lfm/hfm;
iibimean=mean(iibi);
iibi2=iibi-iibimean;
[mag]=fttmag(iibi2,n); %calculate the mag of the fft of the iibi minus the mean
plotter (f,mag,'Spectrum - Mean removed','Hz','',0,.5, 0, max(mag),4,1);
[hfnm,lfnm]=spectrum(mag,f,samplingperiod,n);hfnmn=hfnm/(hfnm+lfnm);lfnmn=lfnm/(hfnm+lfnm);lfrationm=lfnm/hfn
    m;
plottext(['HFn mean = ',num2str(hfmn)],['LFn mean = ',num2str(lfmn)],['LF/HF mean = ',num2str(lfratiom)],['HFn no mean
    = ',num2str(hfnmn)],['LFn no mean = ',num2str(lfnmn)],['LF/HF mean = ',num2str(lfrationm)],0,0,0,0);
%slicing starts
slice=60; % seconds
sliceorig=slice; %save original value
trep=ceil(max(sequ)/slice); %numper of repetitions/slices in sequence
%trep=1
tmaxiibi=length(iibi); %number of iibi samples
iibimax=max(iibi); %maximum value of the iibi
k=2;
```

CONTINUED

Matlab Code CONTINUED

```
k=2;
for ii=1:trep;
    figure(ii+2);
    tmin=slice*(ii-1)/samplingperiod; %start time of slice
    tmax=slice*ii/samplingperiod; %end time of slice
    if ii==trep % last slice roundoff
        tmax=tmaxiibi;
        slice=rem(max(sequ),slice);
    else;end;
    iibislice=iibi(tmin+1:tmax); %sliced iibi
    plotter(sequ(tmin+1:tmax),iibislice,['IBII slice from ',num2str((tmin)*samplingperiod),' to ',num2str((tmax)*samplingperiod), '
        Seconds'],'Seconds','Milliseconds',tmin*samplingperiod,sliceorig*ii,0,iibimax*1.1,k,1)
    [mag]=fttmag(iibislice,slice/samplingperiod);
    [f]=freqs(samplingperiod,slice/samplingperiod);
    [hfm,lfm]=spectrum(mag,f,samplingperiod,slice/samplingperiod);hfmn=hfm/(hfm+lfm);lfmn=lfm/(hfm+lfm);lfratiom=lfm/hfm;
    plotter (f,mag,['Spectrum slice from ',num2str((tmin)*samplingperiod),' to ',num2str((tmax)*samplingperiod), ' Seconds'],'Hz','',0,.5, 0, max(mag),k+1,1);
    iibimean=mean(iibislice);
    iibislice2=iibislice-iibimean;
    [mag]=fttmag(iibislice2,slice/samplingperiod);
    plotter (f,mag,['Spectrum - Mean removed slice from ',num2str((tmin)*samplingperiod),' to ',num2str((tmax)*samplingperiod), ' Seconds'],'Hz','',0,.5, 0,
        max(mag),k+2,1);
    [hfnm,lfnm]=spectrum(mag,f,samplingperiod,slice/samplingperiod);hfnmn=hfnm/(hfnm+lfnm);lfnmn=lfnm/(hfnm+lfnm);lfrationm=lfnm/hfnm;
    hft(ii)=hfnm;lft(ii)=lfnm;
    tthflf(ii)=ii/trep*max(sequ);
    plottext(['HFn mean = ',num2str(hfmn)],['LFn mean = ',num2str(lfmn)],['LF/HF mean = ',num2str(lfratiom)],['HFn no mean = ',num2str(hfnmn)],['LFn no mean =
        ',num2str(lfnmn)],['LF/HF mean = ',num2str(lfrationm)],0,0,0,0);
    k=k+4;
end;
figure(ii+3)
subplot(2,1,1)
plot(sequ,iibi)
title('IIBI')
xlabel('Seconds')
ylabel('Milliseconds')
axis([0,int16((max(sequ)+50)/50)*50,0,max(iibi)*(1.1)]);
subplot(2,1,2)
plot(tthflf,hft,'b',tthflf,lft,'r')
title('HF BLUE LF RED')
xlabel('Seconds')
ylabel('Milliseconds')
```

Matlab Code

```
function [iibi,t,time,sequ]=ibiconvert(ibi,samplingperiod)
l=length(ibi); %length of ibi sequence
ts=samplingperiod;
time=[ibi(1)]; %initialize time array
for i=1:l-1;
   time(i+1)=time(i)+ibi(i+1); %calculate time array for ibi
end;
time=time/1000; %convert milliseconds to seconds
% figure(1);
% stem(time,ibi);
% axis([0 50 0 max(ibi)]);
plotter(time,ibi,'IBI','Seconds','Milliseconds',0,max(time),0,max(ibi)*(1.1),1,2)
plotter(time,ibi,'IBI','Seconds','Milliseconds',0,time(10),0,max(ibi)*(1.1),3,2)
timemax=max(time); %maximum time of sequence
t=ceil(timemax/ts);
% t=length(ibi)
%maximum number of iibi samples
for i=0:ts:timemax;
   for j=1:l;
      if i<time(j)
         k=i/ts+1;
         iibi(k)=ibi(j);
         break; %build iibi
         else;
      end;
   end;
end;
for i=1:t;
   sequ(i)=ts*i;
   if sequ(i)<time(10);
      sequend=i;
   end
 end %calculate time array for iibi
% figure(2)
% plot(sequ,iibi);
% axis([0 50 0 max(ibi)]);
plotter(sequ,iibi,'IIBI','Seconds','Milliseconds',0,max(sequ),0,max(iibi)*(1.1),2,1);
plotter(sequ,iibi,'IIBI','Seconds','Milliseconds',0,sequ(sequend),0,max(iibi)*(1.1),4,1);
```

Matlab Code

```
function plotter(x,y,titles,titlex,titley,axisxmin,axisxmax,axisymin,axisymax,fig,typeplot);
fontsize=7;
%if fig<3;figure(fig);plot(x,y);else;
   fig;
   point=rem(fig,4);if point==0 point=4;end;
   subplot(2,2,point);
   if typeplot==1;
      plot(x,y);
   else
      if typeplot==2;
         stem(x,y);
      else;
         5
      end;
   end;
%end;
title(titles,'FontSize',fontsize);xlabel(titlex,'FontSize',fontsize);ylabel(titley,'FontSize',fontsiz
     e);
if (axisxmax>0) & (axisymax> 0) axis([axisxmin axisxmax axisymin axisymax]);end;
set(gca,'FontSize',fontsize-2,'XGrid','on');
```

Matlab Code

```
function [mag]=fttmag(iibi,t);
c=fft(iibi)/length(iibi);
mag=abs(c);
```

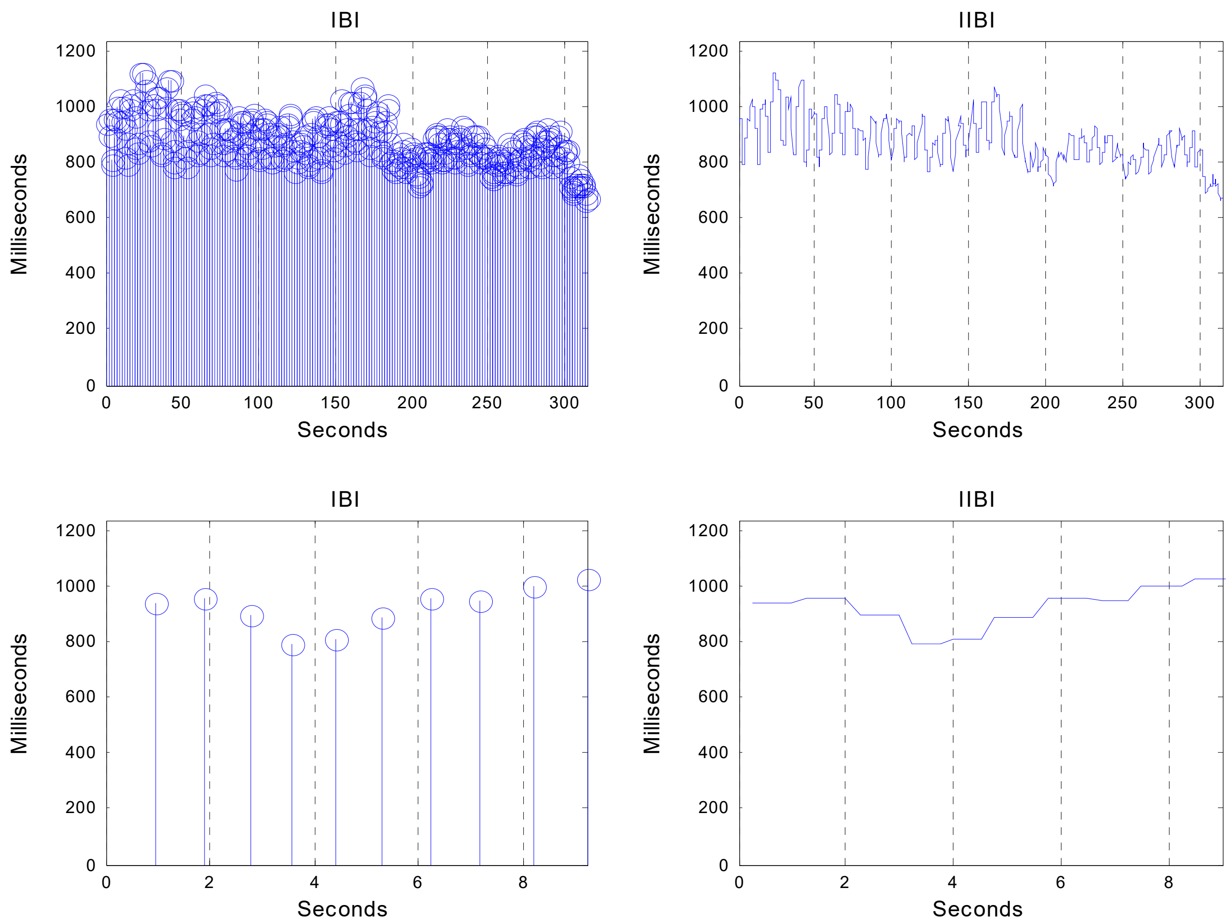
```
function [f]=freqs(samplingperiod,t);
fo=1/samplingperiod/t;
for i=1:t;
   f(i)=fo*i;
end
```

```
function [hf,lf]=spectrum(mag,f,ss,n);
lfmin=.04;lfmax=.15;hfmin=.15;hfmax=.7;
fo=1/ss/n;
lfminidx=ceil(lfmin/fo);lfmaxidx=ceil(lfmax/fo);hfminidx=ceil(hfmin/fo);hfmaxidx=ceil(hf
      max/fo);
lf=0;hf=0;
for i=lfminidx:lfmaxidx;lf=lf+(mag(i)^2)*fo;end;
for i=hfminidx:hfmaxidx;hf=hf+(mag(i)^2)*fo;end;
```

Matlab Code

```
function plottext(line1,line2,line3,line4,line5,line6,axisxmax,axisymin,axisymax,fig);
fontsize=7;
%if fig<3;figure(fig);plot(x,y);else;
%fig;
%point=rem(fig,4);if point==0 point=4;end;
x=[1];y=[1];subplot(2,2,1);plot(x,y);
axis([0 10 -10 0]);axis off;
text(0,0,'0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz','FontSize',fontsize);
text(0,-2,line1,'FontSize',fontsize);
text(0,-3,line2,'FontSize',fontsize);
text(0,-4,line3,'FontSize',fontsize);
text(0,-6,line4,'FontSize',fontsize);
text(0,-7,line5,'FontSize',fontsize);
text(0,-8,line6,'FontSize',fontsize);
axis off;
%end;
%title(titles,'FontSize',fontsize);xlabel(titlex,'FontSize',fontsize);ylabel(titley,'FontSize',fontsize);
%if (axisxmax>0) & (axisymax> 0) axis([axisxmin axisxmax axisymin axisymax]);end;
%set(gca,'FontSize',fontsize-2);
```

IBI vs IIBI



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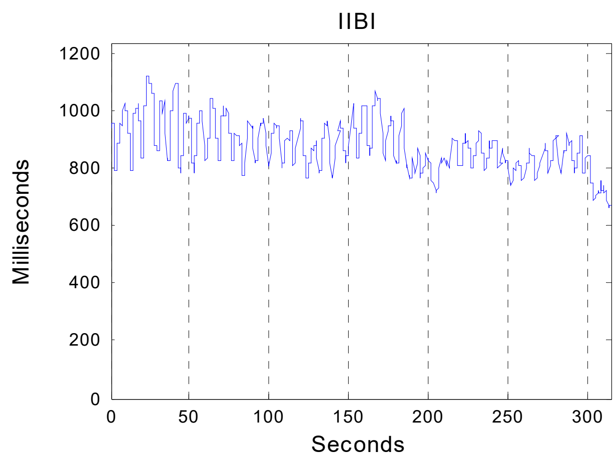
- J.Schesser

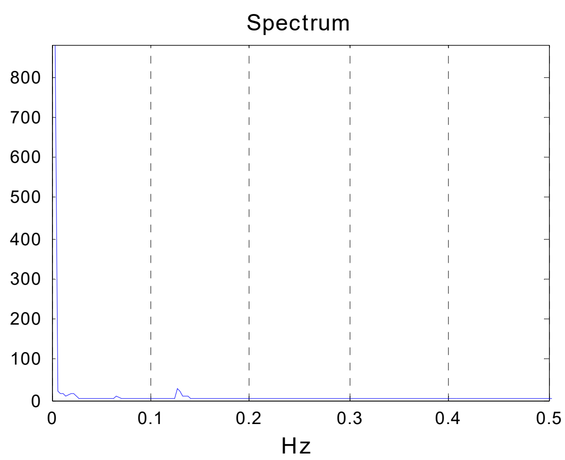
Over the Entire Time Domain

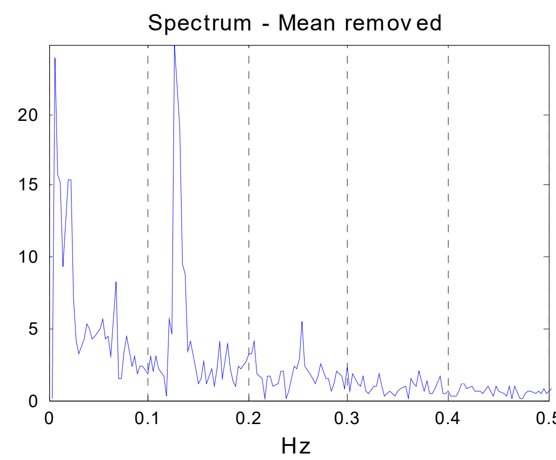
0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz

HFn mean = 0.15717
LFn mean = 0.84283
LF/HF mean = 5.3627

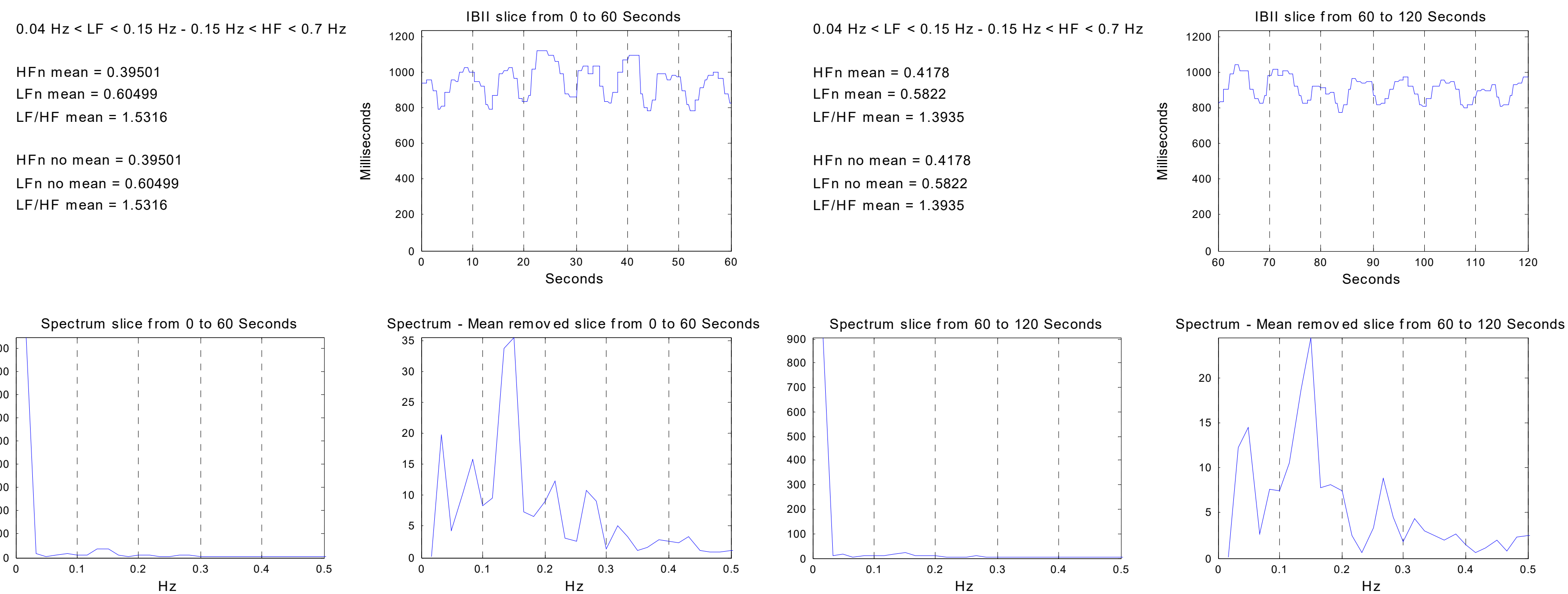
HFn no mean = 0.15717
LFn no mean = 0.84283
LF/HF mean = 5.3627







Windows

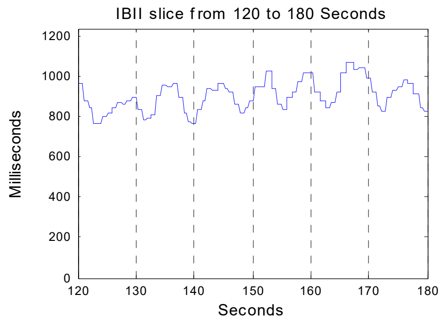

0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz
HFn mean = 0.39501
LFn mean = 0.60499
LF/HF mean = 1.5316
HFn no mean = 0.39501
LFn no mean = 0.60499
LF/HF mean = 1.5316
IBII slice from 0 to 60 Seconds
Milliseconds
Seconds
0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz
HFn mean = 0.4178
LFn mean = 0.5822
LF/HF mean = 1.3935
HFn no mean = 0.4178
LFn no mean = 0.5822
LF/HF mean = 1.3935
IBII slice from 60 to 120 Seconds
Milliseconds
Seconds
Spectrum slice from 0 to 60 Seconds
Hz
Spectrum - Mean removed slice from 0 to 60 Seconds
Hz
Spectrum slice from 60 to 120 Seconds
Hz
Spectrum - Mean removed slice from 60 to 120 Seconds
Hz

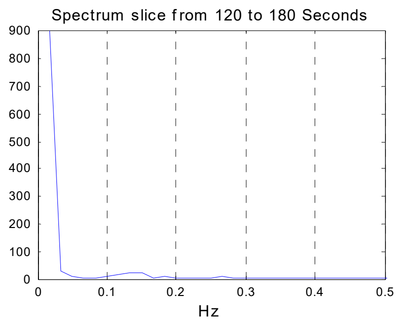
Windows

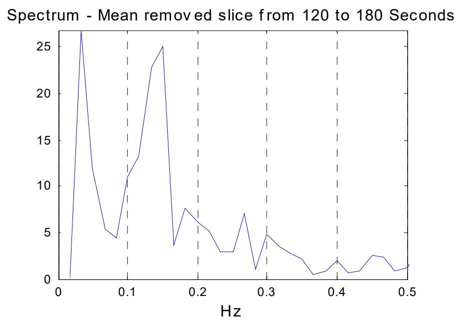
0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz

HFn mean = 0.36173
LFn mean = 0.63827
LF/HF mean = 1.7645

HFn no mean = 0.36173
LFn no mean = 0.63827
LF/HF mean = 1.7645



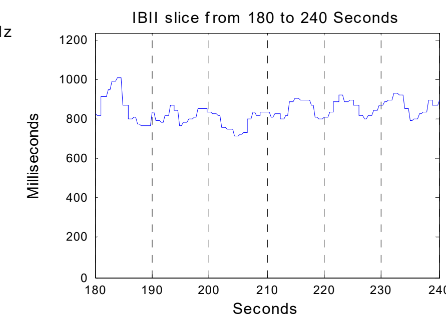


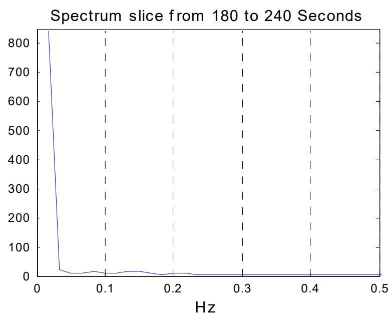


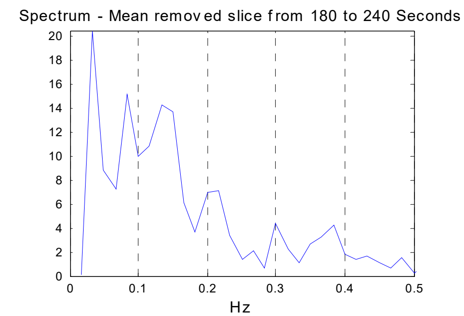
0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz

HFn mean = 0.31651
LFn mean = 0.68349
LF/HF mean = 2.1595

HFn no mean = 0.31651
LFn no mean = 0.68349
LF/HF mean = 2.1595





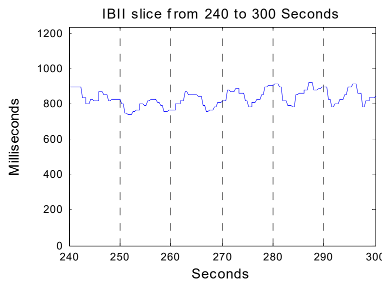


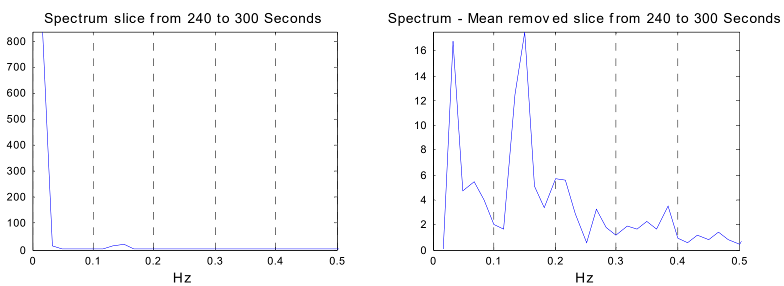
Windows

0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz

HFn mean = 0.47225
LFn mean = 0.52775
LF/HF mean = 1.1175

HFn no mean = 0.47225
LFn no mean = 0.52775
LF/HF mean = 1.1175

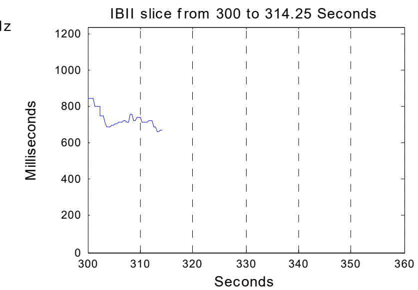


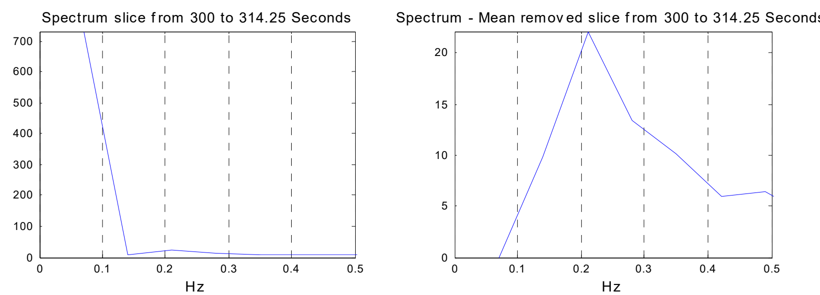


0.04 Hz < LF < 0.15 Hz - 0.15 Hz < HF < 0.7 Hz

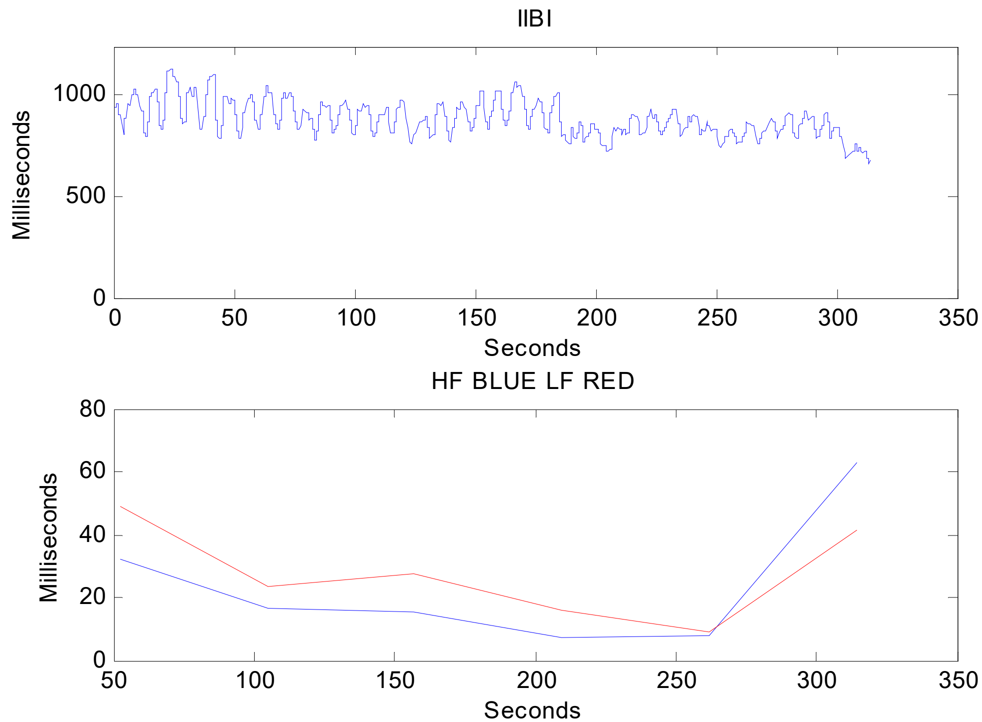
HFn mean = 0.0016675
LFn mean = 0.99833
LF/HF mean = 598.7152

HFn no mean = 0.60428
LFn no mean = 0.39572
LF/HF mean = 0.65487





HF and LF



IIBI
1000
500
0
Milliseconds
0
50
100
150
200
250
300
350
Seconds
HF BLUE LF RED
80
60
40
20
0
Milliseconds
50
100
150
200
250
300
350
Seconds