BME 333 Biomedical Signals and Systems
Biomedical Signals and Systems Quiz #3

• Grading:
  – All questions are rated at 25%
  – You need to complete 4
    • Questions 4 & 5 is mandatory
  – Do all 5 for extra credit.
  – Show all work
1. Consider $N$ 64M Hz HD video signals which is to be quantized.
   
   a) Assuming the signal is sampled using $256^2$ levels, what should the capacity (in Bytes/second) of the line be for $N=1, 10, 100$?
   
   b) How many messages can be transmitted if the channel capacity is 1MB/s.
   
   c) Repeat parts a) and b) for a channel capacity of 256 levels bits per second.
   
   d) At the channel capacity rate of 32MB/s, what is the maximum number levels the video signal can be coded?

2. Compute the Laplace Transforms for the following functions
   
   a) $f(t) = 1$, $0 < t < 10$
   
   b) $f(t) = t$, $0 < t < 10$
   
   c) $f(t) = te^{-t}$, $0 < t < 10$

3. A tomograph of the brain needs to be taken using 100 slices. The size of the image to be generated is 12 inches square. The finest detail to be imaged is 0.05 inch using a 100 pixel resolution.
   
   a) Calculate the total number of pixels required to achieve the image resolution
   
   b) Assuming a 65536 gray scale levels, calculate the size of the file needed to store this image.
   
   c) Assuming it takes 50 microseconds to take one measurement, calculate the total time to complete the full tomograph. Also assume that the calibration time is the same as the measurement time.
   
   d) What data transmission speed is needed to transmit the full tomograph in 2 minutes?

4. Find the inverse transforms for the following:
   
   a) $\frac{s^3 + s^2 + 6}{s(s + 1)(s + 3)}$; 
   b) $\frac{6s + 6}{(s^2 + 9)(s + 1)}$; 
   c) $\frac{s}{s^3 (s + 2)}$

5. Find a) the impulse response and b) the output $y(t)$ for $x(t) = e^{-2t}u(t)$ using Laplace Transforms.
   
   $$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = x(t)$$
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Quiz #3 Answers

1. Consider N 64M Hz HD video signals which is to be quantized.

   a) Assuming the signal is sampled using 256^2 levels, what should the capacity (in Bytes/second) of the line be for N=1, 10, 100?
   b) How many messages can be transmitted if the channel capacity is 1MB/s.
   c) Repeat parts a) and b) for a channel capacity of 256 levels bits per second.
   d) At the channel capacity rate of 10MB/s, what is the maximum number levels the video signal can be coded?

   a) Each signal is sampled at 2x64M= 128M samples per second. Assuming 256^2=65536 = 2^16 levels are used, 16 bits are needed for each sample. Therefore to send one (N=1) signal the line capacity should be 16x128M = 2048Mb/s or 2.56GB/s are needed. To send N=10, line capacity of 20.480Gb/s or 25.6GB/s is needed and 204.8Gbs or 25.6GB/s is needed for N=100.
   b) 1MB/s = 8Mb/s. Since each channel is 2048Mb/s, there are 0.0039 messages or NO messages can fit into this channel.
   c) For 256 = 2^8 levels, 8 bits are used. 8x128Mb/s = 1024Mb/s=128MB/s. For N=1, 128MB/s, N=10, 1280MB/s and N=100, 12.8GB/s. 1MB/s, there 0.007 messages.
   d) Each signal requires 128Msamples/s. Since the channel capacity is 32MBytes/s or 256Mbits/s, then 256Mbits/s / 128Msamples/s or 2 bit/sample is available in the channel for the signal. Therefore, 2^2=4 levels is the maximum number of levels.
2. Compute the Fourier and Laplace Transforms for the following function

a) \( f(t) = 1, \, 0 < t < 10 \)

\[ \mathcal{F}[f(t)] = \frac{1}{s} \]

Using the time shifting property

\[ \mathcal{F}[f(t)] = \mathcal{F}[u(t) - u(t-10)] \]

\[ = \frac{1}{s}(1 - e^{-10s}) \]

b) \( f(t) = t; \, 0 \leq t \leq 10 \)

\[ = tu(t) - tu(t-10) \]

\[ = tu(t) - (t-10)u(t-10) - 10u(t-10) \]

Using the time shifting property

\[ \mathcal{F}[tu(t)] = \frac{1}{s^2} \]

\[ \mathcal{F}[(t-10)u(t-10)] = \frac{e^{-10s}}{s^2} \]

\[ \mathcal{F}[f(t)] = \mathcal{F}[tu(t) - (t-10)u(t-10) - 10u(t-10)] \]

\[ = \left[ \frac{1}{s^2} - \frac{e^{-10s}}{s^2} - \frac{10e^{-10s}}{s} \right] \]
2. Compute the Fourier and Laplace Transforms for the following function: \( f(t) = te^{-t}, 0 < t < 10 \)

\( c) f(t) = te^{-t}; 0 \leq t \leq 10 \)

Using the time shifting and frequency shifting property

\[
= te^{-t}u(t) - te^{-t}u(t-10) = te^{-t}u(t) - \frac{te^{-t}e^{10}}{e^{10}}u(t-10)
\]

\[
= te^{-t}u(t) - \frac{te^{-(t-10)}}{e^{10}}u(t-10)
\]

\[
= te^{-t}u(t) - \frac{(t-10)e^{-(t-10)}}{e^{10}}u(t-10) - \frac{10e^{-(t-10)}}{e^{10}}u(t-10)
\]

\[
f(t) = te^{-t}u(t) - \frac{(t-10)e^{-(t-10)}}{e^{10}}u(t-10) - \frac{10e^{-(t-10)}}{e^{10}}u(t-10)
\]

\[
\mathcal{L}[f(t)] = \left[ \frac{1}{(s+1)^2} - \frac{e^{-10(s+1)}}{(s+1)^2} - \frac{10e^{-10(s+1)}}{(s+1)} \right]
\]

OR

\[
f(t) = te^{-t}; 0 \leq t \leq 10
\]

\[
= g(t)e^{-t}; \text{ where } g(t) = tu(t) - (t-10)u(t-10) - 10u(t-10)
\]

From Part b)

\[
G(s) = \left[ \frac{1}{s^2} - \frac{e^{-10s}}{s^2} - \frac{10e^{-10s}}{s} \right]
\]

\[
\mathcal{L}[f(t)] = \mathcal{L}[g(t)e^{-t}] = G(s+1) = 6\left[ \frac{1}{(s+1)^2} - \frac{e^{-10(s+1)}}{(s+1)^2} - \frac{10e^{-10(s+1)}}{(s+1)} \right]
\]
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3. A tomograph of the brain needs to be taken using 100 slices. The size of the image to be generated is 12 inches square. The finest detail to be imaged is 0.05 inch using a 100 pixel resolution.

i. Calculate the total number of pixels required to achieve the image resolution.

\[
\left( \frac{12 \text{ inches}}{0.05 \text{ inch}} \times 100 \text{ pixels} \right)^2 = (240 \times 100 \text{ pixels})^2 = (24000 \text{ pixels})^2 = 576,000,000 \text{ pixels} = 576 \text{M pixels}
\]

ii. Assuming a 65536 gray scale levels, calculate the size of the file needed to store this image.

\[
65536 = 16 \text{ bits} \\
576,000,000 \text{ pixels} \times 16 \text{ bits} = 9,216,000,000 \text{ bits} = 1.152 \text{ GB}
\]

iii. Assuming it takes 50 microseconds to take one measurement, calculate the total time to complete the full tomograph. Also assume that the calibration time is the same as the measurement time.

\[
576,000,000 \text{ measurements need to be made at 50 microseconds} = 28,800 \text{ seconds per slice} = 480 \text{ min/slice} \\
480 \text{ min/slice} \times 100 \text{ slices} \times 2 \text{ (measurement + calibration)} = 96,000 \text{ minutes} = 1600 \text{ hours}
\]

iv. What data transmission speed is needed to transmit the full tomograph in 2 minutes?

\[
9,216,000,000 \text{ bits}/120 \text{ sec} \times 100 \text{ slices} = 7.68 \text{ Gb/s}
\]
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4a) $\mathcal{L}^{-1} \left[ \frac{s^3+s^2+6}{s(s+1)(s+3)} \right] = \mathcal{L}^{-1} \left[ 1 + \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+3} \right]

K_1 = \frac{s^3+s^2+6}{(s+1)(s+3)} \bigg|_{s=0} = \frac{6}{(1)(3)} = 2

K_2 = \frac{s^3+s^2+6}{s(s+3)} \bigg|_{s=-1} = \frac{(-1)+1+6}{(-1)(2)} = \frac{6}{-2} = -3

K_3 = \frac{s^3+s^2+6}{s(s+1)} \bigg|_{s=-3} = \frac{-27+9+6}{-3(-2)} = \frac{-12}{6} = -2

$\mathcal{L}^{-1} \left[ \frac{s^3+s^2+9}{s(s+3)(s+5)} \right] = \delta(t) + [2 - 3e^{-t} - 2e^{-3t}]u(t)$

A Check

\[
1 + \frac{2}{s} + \frac{-3}{s+1} + \frac{-2}{s+3} = \frac{s(s+1)(s+3)+2(s+1)(s+3)-3s(s+3)-2s(s+1)}{s(s+1)(s+3)}
\]

\[
= \frac{s^3+4s^2+3s+2s^2+8s+6-3s^2-9s-2s^2-2s}{s(s+1)(s+3)}
\]

\[
= \frac{s^3+s^2+6}{s(s+1)(s+3)}
\]
4b) \[ \frac{6s + 6}{(s^2 + 9)(s + 1)} = \frac{6(s + 1)}{(s^2 + 9)(s + 1)} = \frac{6}{(s^2 + 9)} \]

\[ = \frac{K_1}{s - j3} + \frac{K_1^*}{s + j3} \]

\[ K_1 = \frac{6}{(s + j3)} \mid_{s = -j3} = \frac{6}{(j6)} = -j \angle -\frac{\pi}{2} \]

\[ K_1^* = 1 \angle +\frac{\pi}{2} \]

\[ \mathcal{L}^{-1}\left[ \frac{6s + 6}{(s^2 + 9)(s + 1)} \right] = \mathcal{L}^{-1}\left[ \frac{6}{(s^2 + 9)} \right] = \mathcal{L}^{-1}\left[ \frac{1 \angle -\frac{\pi}{2}}{s - j3} + \frac{1 \angle \frac{\pi}{2}}{s + j3} \right] \]

\[ = [2 \cos(3t - \frac{\pi}{2})]_{t(t)} = 2 \sin(3t)u(t) \]

OR

\[ \mathcal{L}^{-1}\left[ \frac{6s + 6}{(s^2 + 9)(s + 1)} \right] = \frac{K_1}{s - j1} + \frac{K_1^*}{s + j1} + \frac{K_2}{(s + 1)} \]

\[ K_2 = \frac{6s + 6}{(s^2 + 9)} \mid_{s = -j3} = \frac{-6 + 6}{(1 + 9)} = 0 \]

\[ K_1 = \frac{6s + 6}{(s + j3)(s + 1)} \mid_{s = j3} = \frac{18j + 6}{(j6)(3j + 1)} = \frac{6(3j + 1)}{(j6)(3j + 1)} = \frac{6}{(j6)} = -j \angle -\frac{\pi}{2} \]
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\[
4b
\]

A Check

\[
\frac{1 \angle -\frac{\pi}{2}}{s - j3} + \frac{1 \angle \frac{\pi}{2}}{s + j3} = \frac{-j}{s - j3} + \frac{j}{s + j3}
\]

\[
= \frac{-j(s + j3) + j(s - j3)}{(s - j3)(s + j3)}
\]

\[
= \frac{-js + 3 + js + 3}{(s - j3)(s + j3)} = \frac{6}{(s - j3)(s + j3)}
\]

\[
= \frac{6}{(s^2 + 9)}
\]
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4c) \[ \mathcal{L}^{-1}\left[ \frac{s}{s^3(s+2)} \right] = \mathcal{L}^{-1}\left[ \frac{1}{s^2(s+2)} \right] = \frac{K_1}{s+2} + \frac{M_0}{s^2} + \frac{M_1}{s} \]

\[ K_1 = \left. \frac{1}{s^2} \right|_{s=-2} = \frac{1}{4} \]

\[ M_0 = \left. \frac{1}{(s+2)} \right|_{s=0} = \frac{1}{2} \]

\[ M_1 = \frac{d}{ds} \left. \frac{1}{(s+2)} \right|_{s=0} = \frac{d(s+2)^{-1}}{ds} \bigg|_{s=0} = -(s+2)^{-2} \bigg|_{s=0} = -\frac{1}{(s+2)^2} \bigg|_{s=0} = -\frac{1}{4} \]

\[ \mathcal{L}^{-1}\left[ \frac{1}{s^2(s+1)} \right] = \mathcal{L}^{-1}\left[ \frac{1}{4} \left( \frac{1}{s+2} + \frac{2}{s^2} - \frac{1}{s} \right) \right] \]

\[ = \frac{1}{4} [e^{-2t} + 2t - 1] \delta(t) \]

4c) A Check

\[ \left[ \frac{1}{4} \left( \frac{1}{s+2} + \frac{2}{s^2} - \frac{1}{s} \right) \right] = \frac{1}{4} \frac{s^2 + 2(s+2) - s(s+2)}{s^2(s+2)} \]

\[ = \frac{1}{4} \frac{s^2 + 2s + 4 - s^2 + 2s}{s^2(s+2)} \]

\[ = \frac{1}{s^2(s+2)} \]
5) a) $\tilde{h}(t) + 5h(t) + 6h(t) = \delta(t)$

$(s^2 + 5s + 6)H(s) = 1$

$H(s) = \frac{1}{(s^2 + 5s + 6)} = \frac{1}{(s+2)(s+3)}$

$= \frac{K_1}{s+2} + \frac{K_2}{s+3}$

$K_1 = \frac{1}{(s+3)} \bigg|_{s=-2} = \frac{1}{(-2+3)} = 1$

$K_2 = \frac{1}{(s+2)} \bigg|_{s=-3} = \frac{1}{(-3+2)} = -1$

$h(t) = (e^{-2t} - e^{-3t})u(t)$

CHECK

$\left[ \frac{1}{(s+2)} - \frac{1}{(s+3)} \right] = \left[ \frac{s+3-(s+2)}{(s+2)(s+3)} \right]$

$= \frac{1}{(s+2)(s+3)}$

5) b) $y(t) + 5y(t) + 6y(t) = x(t) = e^{-2t}u(t)$

$(s^2 + 5s + 6)Y(s) = (s+2)(s+3)Y(s) = \frac{1}{s+2}$

$Y(s) = \frac{1}{(s+2)^2(s+3)} = \frac{K_1}{s+3} + \frac{K_2}{(s+2)^2} + \frac{K_3}{s+2}$

$K_1 = \frac{1}{(s+2)^2} \bigg|_{s=-3} = \frac{1}{(-3+2)^2} = 1$

$K_2 = \frac{1}{(s+3)} \bigg|_{s=-2} = \frac{1}{(-2+3)} = 1$

$K_3 = \frac{1}{s+2} \bigg|_{s=-2} = \frac{1}{(-2+3)} = 1$

$\frac{d}{ds} \left[ \frac{1}{(s+3)} + \frac{1}{(s+2)^2} + \frac{-1}{s+2} \right]$

$= \frac{1}{(s+3)} + \frac{1}{(s+2)^2} + \frac{-1}{s+2}$

$y(t) = (e^{-3t} + te^{-2t} - e^{-2t})u(t)$

CHECK

$\frac{1}{s+3} + \frac{1}{(s+2)^2} + \frac{-1}{(s+2)} = \frac{(s+2)^2 + (s+3) - (s+3)(s+2)}{(s+3)(s+2)^2}$

$= \frac{s^2 + 4s + 4 + s + 3 - s^2 - 5s - 6}{(s+3)(s+2)^2} = \frac{1}{(s+3)(s+2)^2}$