COVARIANCE AND PORTFOLIO RISK - Applications in Finance

In portfolio management, the financial manager is interested in the relationship between two variables. The covariance is a descriptive measure of the linear association between two variables. When more than two variables are present in most applications, correlation analysis and the regression model offer a wider range of linear and nonlinear relationships.

(1) The **Covariance** σ_{XY} between two discrete random variables X and Y is

$$\sigma_{XY} = \sum (X - E[X])(Y - E[Y])P(XY)$$

where

      X   = discrete random variable X

      Y   = discrete random variable Y

      P(XY) = probability of occurrence of X and Y

A portfolio is a group of assets for investors to combine securities to reduce their risk. By diversifying their investments, the objective is to maximize the return while minimizing the risk. Each investment is weighted by the proportion of assets assigned to that investment. The portfolio expected return and the portfolio risk will indicate the risk exposure of the portfolio.

(2) **Portfolio Expected Return** of Two Random Variables

The portfolio expected return for a two-asset investment is equal to the weight assigned to asset X multiplied by the expected return of asset X plus the weight assigned to asset Y multiplied by the expected return of asset Y.

        Portfolio Expected Return = w E (X)   + (1 - w) E (Y)

where

      E (P)   = portfolio expected return

      w   = the proportion of portfolio value assigned to asset X

      (1- w) = the proportion of portfolio value assigned to asset Y

      E (X)   = expected return of asset X

      E (Y)   = expected return of asset Y

(3) **Portfolio Risk**

$$portfolio\ risk = \sqrt{w^2 \operatorname{var}(X) + (1 - w)^2 \operatorname{var}(Y) + 2w(1 - w)\sigma_{XY}}$$

(4)   **Coefficient of Variation (CV)**

The coefficient of variation shows the risk per unit of return, and it provides a more meaningful basis for comparison when the expected returns on two alternatives are not the same. The smaller the coefficient of variation, the lower risk factor occurs. Note that the coeffecient of variation is only appropriate when comparing the risk of investments that are not held as part of a portfolio.

Coefficient of variation = standard deviation / expected return

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EXAMPLE:

Suppose an investor is deciding between two alternative investments for the coming year.   The first investment is a mutual fund whose portfolio consists of a combination of stocks that make up the Dow Jones Industrial Average.   The second investment consists of shares of a growth stock. Using the following returns (per $1,000 investment) under three economic conditions, each with an estimated probability of occurrence from past experience:

P(XY)	Economic Conditions	Dow Jones Fund	Growth Stock
0.2	Recession	−$ 100	−$ 200
0.5	Stable economy	+$100	+$ 50
0.3	Expanding economy	+$ 250	+$350

Compute the expected value and the standard deviation for each investment and the covariance of the two investments.   Interpret your results

Suppose a portfolio of both of these two investments that consists of an equal investment in each of these two assets is established.   Compute the portfolio expected return and the portfolio risk. Is this a portfolio for a conservative investor?   Explain.

Microsoft Excel - covariance solution.xls

	A	B	C	D	E	F	G	H	I	J	K
1	Probability	Economic Conditions	Dow Jones (X)	Growth (Y)	E(X)	E(Y)	VAR(X)	VAR(Y)	COVARIANCE		
2	0.2	Recession	-100	-200	-20	-40	8405	16820	11890		
3	0.5	Stable economy	100	50	50	25	12.5	800	100		
4	0.3	Expanding economy	250	350	75	105	6307.5	20280	11310		
5											
6											
7	Expected Value of X		$105.00								
8	Expected Value of Y		$90.00				Coefficient of Variation on X			1.155682	
9	Variance of X		14725				Coefficient of Variation on Y			2.163102	
10	Variance of Y		37900								
11	Standard deviation of X		$121.35				Based on the low risk criterion, choose Stock X.				
12	Standard deviation of Y		$194.68								
13	Covariance		23300								
14											
15											
16	weight assigned to X		0.5								
17	Portfolio Expected Return		$97.50								
18	Portfolio risk		$157.50								
19											
20	Coefficient of Variation		1.615384615								
21											
22											
23											
24											

example on covariance / EXAM 2 / assignment on covariance #1 / ass

Ready

Project.

All problems are based on the LOW RISK criterion.

I.   You are trying to develop a strategy for investing in two different stocks.   The anticipated annual return for a $1,000 investment in each stock has the following probability distribution:

Probability	Returns of Stock X	Returns of Stock Y
0.1	- $ 100	$ 50
0.3	$ 0	$100
0.3	-$80	$20
0.3	$150	$100

Compute the

(a)   expected return for stock X.

(b)   expected return for stock Y.

(c)   standard deviation for stock X.

(d)   standard deviation for stock y.

(e)   covariance of stock X and stock Y.

(f)   Do you think you will invest in stock X or stock Y?   Explain.

(g)   Suppose you wanted to create a portfolio that consists of stock X and stock Y.   Compute the portfolio expected return and portfolio risk for each of the following proportions in stock X.

   (1)   0.10

   (2)   0.30

   (3)   0.50

   (4)   0.70

   (5)   0.90

   Which portfolio would you recommend?   Explain.

II.   To set up a portfolio that consists of a corporate bond fund and a common stock fund, the following information about the annual return (per $1,000) of each of these investments under different economic conditions is available along with the probability that each of these economic conditions will occur.

Probability	State of the Economy	Corporate Bonds	Common Stocks
0.10	Recession	- $ 30	- $150
0.15	Stagnation	$50	- $20
0.35	Slow growth	$90	$120
0.30	Moderate growth	$100	$160
0.10	High growth	$110	$250

Compute the

   (i)   expected return for corporate bonds.

   (ii)   expected return for common stocks.

   (iii)   standard deviation for corporate bonds.

(iv)   standard deviation for common stocks.

(v)   covariance of corporate bonds and common stocks.

(vi)   Do you think you will invest in corporate bonds or common stocks?   Explain.

(vii)   Suppose you wanted to create a portfolio that consists of corporate bonds or common stocks.   Compute the portfolio expected return and portfolio risk for each of the following proportions in corporate bonds.

      (1)   0.10

      (2)   0.30

      (3)   0.50

      (4)   0.70

      (5)   0.90

      Which portfolio would you recommend?   Explain.